

# The South African Business Cycle and the application of Dynamic Stochastic General Equilibrium models

by

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*Dissertation presented for the degree of Doctor of Philosophy  
in Economics in the Faculty of Economic and Management  
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December 2014

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# Abstract

## **The South African Business Cycle and the application of Dynamic Stochastic General Equilibrium models**

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December 2014

This dissertation considers the use of Dynamic Stochastic General Equilibrium (DSGE) models for the analysis of South African macroeconomic business cycle phenomena. It includes four separate, but interrelated parts, which follow a logical sequence.

The first part motivates the use of these models before establishing the theoretical foundations for these models. The theoretical foundations are accompanied by detailed derivations that are used to construct a model for a small open economy.

The second part considers the properties of South African macroeconomic data that may be used to estimate the parameters in these models. It includes a discussion of the variables that may be included in such a model, as well as various methods that may be used to extract the business cycle. Thereafter, the sample size for the dataset is established, after investigating for possible structural breaks in the first two moments of the data, using various univariate and multivariate techniques. The final chapter of this part contains an investigation into the measures of core inflation, whereby a comparison of trimmed

means, dynamic factor models and various wavelet decompositions are applied to data for South Africa.

The third part considers the application of the dataset that was identified in part two, in a DSGE model that incorporates features that are typical of small open economies. It includes a discussion that relates to the role of the exchange rate in these models, which is found to contain key information. In addition, this part also includes a optimal policy investigation, which considers the reaction function of central bank.

The final part of this thesis considers more recent advances that have been applied to DSGE models for the South African economy. It includes an example of a nonlinear model that is estimated with the aid of a particle filter, which is then used for forecasting purposes. The forecasting results of both linear and nonlinear versions of the model are then compared with the results from various Vector Autoregression (VAR) and Bayesian VAR models.

# Uittreksel

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Hierdie proefskrif oorweeg die gebruik van Dinamiese Stogastiese Algemene Ewig (Engels: Dynamic Stochastic General Equilibrium (DSGE)) modelle vir die analise van besigheidsiklus gebeure in die Suid Afrikaanse makroekonomie. Dit bestaan uit vier aparte dog onderling verwante dele wat in 'n logiese ontwikkeling vorm.

Die eerste deel motiveer die gebruik van dié modelle en daarna word die teoretiese onderbou van die modelle daargestel. Die teoretiese onderbou word aangevul met gedetailleerde stappe van die afleiding van die verhoudings wat gebruik word om 'n model vir 'n klein oop ekonomie saam te stel.

Die tweede deel oorweeg die eienskappe van Suid Afrikaanse makroekonomiese data wat relevant is vir 'n ekonometriese model in hierdie konteks. Dit sluit 'n bespreking in van die veranderlikes wat vir so 'n model gebruik kan word, asook 'n bespreking van die verskeie metodes wat gebruik kan word om die besigheidsiklus uit die data te identifiseer. Die steekproefgrootte van die data word dan vasgestel, ná die moontlikheid van strukturele onderbrekings van tendens in die eerste en tweede momente van die data ondersoek is met behulp van verskeie enkel en meervoudige-veranderlike tegnieke. Die laaste

hoofstuk van dié deel is ’n studie van verskeie maatstawwe van kern inflasie (core inflation), waar ’n vergelyking getref word tussen die resultate van die volgende metodes toegepas op Suid Afrikaanse data: afgesnede gemiddeldes (trimmed means), dinamiese faktor modelle en verskeie golfvormige onderverdelings (wavelet decompositions).

Die derde deel gebruik die datastel, wat in deel twee ontwikkel is, in die passing van ’n DSGE model wat die tipiese eienskappe van ’n klein oop ekonomie inkorporeer. Dit sluit ’n bespreking in van die rol van die wisselkoers in hierdie tipe modelle, en daar word empiries bevind dat die wisselkoers belangrike inligting bevat. Hierdie deel sluit ook ’n ondersoek in van optimale beleid in terme van die reaksie funksie van die sentrale bank.

Die laaste deel van die proefskrif bestudeer die resultate van onlangse ontwikkelinge in DSGE modelle wat toegepas word op die Suid Afrikaanse ekonomie. Dit sluit ’n voorbeeld van ’n nie-liniêre model wat met behulp van ’n partikel filter (particle filter) geskat word en gebruik word vir vooruitskattings. Die vooruitskattings uit beide die liniêre en nie-liniêre modelle word dan vergelyk met dié verkry uit verskeie Vektor Outo-Regressie (Vector Autoregression (VAR)) en Bayesiaanse VAR modelle.

# Acknowledgements

A thesis that is this long in the making generates many debts of gratitude. Special mention should be made of the contribution of my extremely talented co-authors: Sami Alpanda, Mehmet Balcilar, Stan du Plessis, Rangan Gupta and Geoffrey Woglom. Many thanks for allowing me to learn from you. There have also been a number of other academics who have been ardent supporters of this project, and in this regard I would like to especially thank Ben Smit and Rudi Steinbach, as well as the staff and students at both the University of Cape Town and Stellenbosch University.

# Dedications

To Mom, Dad, Nicola, Cliff, Leigh, Jen, Hugh, James,  
Marylyn, Roger, Mike & Gail.

Thanks for your support, patience and understanding.



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# Chapter 1

## Introduction

## 1.1 Theoretical Macroeconomic Modelling

The theoretical contributions of Lucas (1976), Lucas and Sargent (1978), Kydland and Prescott (1982), and Prescott (1986) resulted in dramatic changes to the way in which empirical macroeconomic investigations are currently conducted. The importance of these contributions could perhaps be gauged by the fact that each of these authors has received the Nobel Prize in Economic Sciences.

Prior to the publication of these important works, most empirical analyses on macroeconomic business cycles made use of statistical models that were based on backward-looking linear regression models. The most ambitious of these was designed as part of the Cowles Commission, which took the form of a large multi-equation regression model that followed the general practices proposed by Tinbergen (1939).<sup>1</sup> Shortly before its demise in 1972, this model included nearly 400 equations and included a few structural non-linear representations.<sup>2</sup>

Whilst the Cowles Commission model survived ongoing criticism throughout its development, the Lucas (1976) critique provided compelling reasons for a new direction in business cycle research; whereby many of the practices that were involved in these large-scale linear regression models started to demise. The essence of the argument put forward by Lucas (1976) suggests that these large-scale linear regression models, that are based on historical macroeconomic data, would not be able to provide prudent insight into the effects of a change in economic policy. The reason for this is the change in policy would give rise to a new pattern of behaviour with associated shifts in the estimated parameters of the policy model. Or in the words of (Lucas, 1976, p. 41),

“Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.”

The new direction in modelling therefore included a procedure for deriving policy-invariant parameters, which would need to describe certain fundamental aspects of decision making (such as those that relate to preferences and

---

<sup>1</sup>For an early critique of this tradition of model building, which is largely still relevant today, see Keynes (1940).

<sup>2</sup>For a discussion on the usefulness of the Cowles Commission model, see Fair (1992).

constraints). These parameters are often termed ‘*structural*’ or ‘*deep*’ parameters as they relate to the essential elements that determine the choices that individual’s make, under any given policy framework.

Following the critique of Lucas (1976), a new breed of macroeconomic models emerged, that were based on microeconomic foundations.<sup>3</sup> Early proponents of this method included a group of *Real Business Cycle* (RBC) theorists who developed models around the concepts of utility and profit maximisation. These models incorporated many general equilibrium features and several stochastic properties, but excluded a number of important rigidities and other imperfections.<sup>4</sup>

To improve upon the RBC model’s characterisation of macroeconomic data a number of important studies, including Clarida *et al.* (1999) and Christiano *et al.* (2005), argued for the incorporation of various imperfections. These included nominal and real rigidities in the form of sticky prices and wages, amongst others. These characteristics seemed to be present in the data,<sup>5</sup> as shifts in the aggregate demand for goods and services tend to have a greater affect on output, than that which is generated by the Real Business Cycle’s perfectly competitive flexible-price economy (Blanchard, 2009).<sup>6</sup> These developments have been called *New-Keynesian* and the work of Galí (2008) provides a relatively extensive discussion on the foundations of New-Keynesian models.<sup>7</sup>

These theoretical models also incorporated forward-looking behaviour on the part of the individual, which necessitated the development of more advanced techniques for the approximation of the model solution.<sup>8</sup> Variants of these techniques for forward-looking linear difference equations have since been provided by Blanchard and Kahn (1980), Sims (2001), Uhlig (1999), and Klein (2000).<sup>9</sup> More recently, solution methods for forward-looking *nonlinear* differ-

---

<sup>3</sup>In a later paper, Lucas (1987) suggests that there should be very little difference between the study of microeconomics and macroeconomics.

<sup>4</sup>See, Kydland and Prescott (1982) and Prescott (1986) for early variants of these models. These models are also termed perfectly competitive *flexible-price* models in the more recent literature.

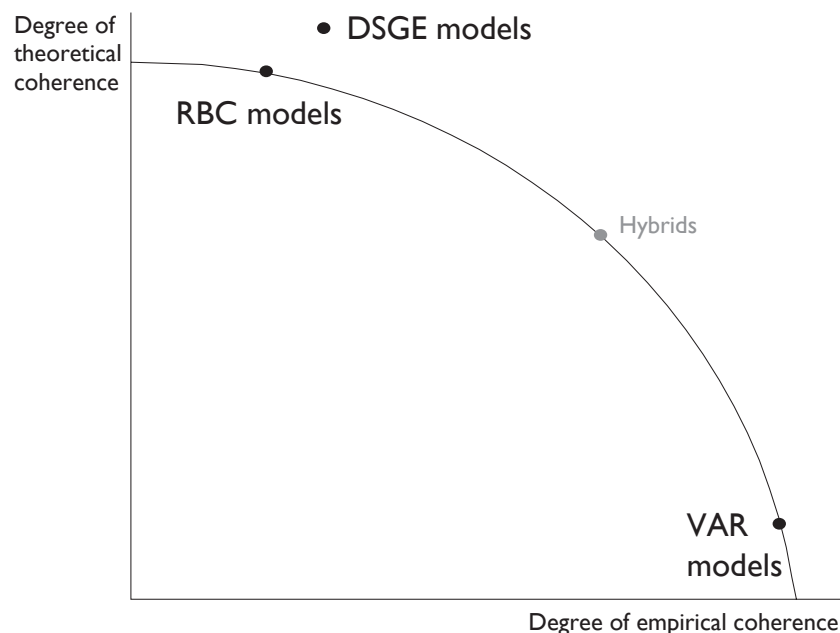
<sup>5</sup>Evidence for these features may be traced back to Friedman and Schwartz (1963), whilst Christiano *et al.* (1999) provide more recent empirical results.

<sup>6</sup>For example, (Blanchard, 2009) suggests that changes in the nominal interest rate would appear to affect real asset prices by more than what is generated by the model. In addition, the data would suggest that consumers appear to purchase additional goods, which increases demand leading to an increase in output and employment, following a decline in interest rates.

<sup>7</sup>The term *New-Keynesian* came into popular usage in the 1980s. It sought to describe the incorporation of the rational expectations framework with microeconomic foundations for imperfect competition and other rigidities (notably for prices and wages).

<sup>8</sup>This is also a common feature of many of the later Real Business Cycle models.

<sup>9</sup>A number of methods for solving forward-looking non-linear difference equations have since been developed. These include projection methods, iterative procedures and perturbation methods. Judd (1998) provides a detailed exposition of these methods.



**Figure 1.1:** Theoretical and Empirical Trade-off

ence equations have been developed by Schmitt-Grohé and Uribe (2004).<sup>10</sup>

The earlier models, such as those that are contained in Kydland and Prescott (1982), Prescott (1986), and King and Rebelo (1999), all contained calibrated values for the parameters. After simulating the model, these researches were then able to compare the moments and correlation structure of the data that was generated by the model, with those of the actual data.

When comparing the characteristics of traditional RBC models to those of reduced-form VARs, Pagan (2003) noted that there is usually a trade-off between models that are designed to match theory closely and those that seek to explain historical economic activity. Over time, this frontier has shifted outwards, largely as a result of theoretical, statistical and computational advances. Figure (1.1) contains a diagram that is similar to that which is contained in Pagan (2003), where it is noted that calibrated RBC models provide a greater degree of theoretical coherence, whilst estimated VAR models have traditionally provided more empirical coherence. During the development of these techniques, there were also a number of hybrid models, which would have included the structural VARs of Sims (1980*b*).

To improve upon the empirical coherence of theoretical models, Ireland (2004*a*) provided a framework that may be used to estimate the parameters in the theoretically coherent models from actual data. This framework makes use of a state-space setup that also allows for the inclusion of a number of unobserved variables; where the Kalman filter is used to evaluate the likelihood

<sup>10</sup>This method considers the use of second-order approximations for the model solution.

function of a linear representation of the model. This procedure has been used to estimate the parameters in various models, using Bayesian, maximum likelihood, or general method of moments techniques.<sup>11</sup>

Together, these contributions are encapsulated in the framework for Dynamic Stochastic General Equilibrium (DSGE) models, which has elevated this type of theoretical modelling to the forefront of current macroeconomic research; where a great deal of convergence in both vision and methodology exists (Blanchard, 2009). Indeed, Benassy (2011) attributes the success of this line of research being due to the rigorous micro-foundations that allow for the description of behaviour that may be invariant to policy changes, in response to the Lucas critique. In addition, he notes that these models also allow for a highly desirable integration of growth and cycle theory in a unified framework that may be integrated with the results from observed data.

With this growing consensus, there has been a large growth in the scale of DSGE models, where current versions incorporate open-economy features, additional frictions (such as those that pertain to the financial sector), and a larger set of shocks. These models are frequently termed ‘medium-scale DSGE models’ and are described in the work of Smets and Wouters (2007), Adolfson *et al.* (2008*a,b*), Galí (2010), Gertler and Kiyotaki (2010), Christiano *et al.* (2010), amongst others. An example of an application of these models to South African data is contained in Steinbach (2013).

The results from these studies suggest that modern DSGE models are able to provide an adequate description of business cycle dynamics, which has provided valuable insights into the effects of various economic shocks on an economy (c.f. Smets and Wouters 2007 and Christiano *et al.* 2010). As a result, they are used by central banks and other policy-making institutions to aid policy decisions. With this in mind, Tovar (2009) has suggested that DSGE models provide a popular framework in most central banks for policy analysis and forecasting purposes.

More recently, researchers have started to examine the forecasting performance of these models. In one such investigation, Smets and Wouters (2007) showed that a DSGE model can generate forecasts that have a lower root-mean-squared-error (RMSE) than a similar Bayesian Vector Autoregression (BVAR) models.<sup>12</sup> In addition, Edge *et al.* (2010) has suggested that the out-of-sample forecasting performance of the Federal Reserve Board’s DSGE model for the U.S. economy is in many cases superior to that of their large-scale

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<sup>11</sup>See, Canova (2007) for a comprehensive treatment of the use of these estimation techniques for DSGE models.

<sup>12</sup>They suggest that the superior performance of the DSGE model is conditional on it containing a sufficient number of shocks and backward-looking features that can generate endogenous persistence (e.g. habits and inflation indexation). In addition, they also maintain that the forecasting performance depends upon a suitable identification of the model parameters in the estimation.

macro-econometric model and their own published Greenbook forecasts.<sup>13</sup>

## 1.2 Dynamic Macroeconometric Modelling in South Africa

In South Africa, macroeconometric research has been conducted by the South African Reserve Bank since 1974 (Smal *et al.*, 2007). At a similar point in time, Prof. Geert de Wet also started developing macroeconomic models at the University of Pretoria for the purpose of short and medium-term forecasts.<sup>14</sup> Shortly thereafter, the Bureau of Economic Research published the first results from their macroeconometric model in 1981 (Smit and Pellissier, 1997). These models include both large structural and purely statistical varieties that were closely associated with those of the Cowles Commission.

More recently, various examples of DSGE models have been developed for the South African economy. Early variants include those of Liu and Gupta (2007), which are based on calibrated versions of the Hansen (1985) closed-economy model.<sup>15</sup> After incorporating several open-economy features, the size of these models grew quite rapidly, in line with the framework of Galí and Monacelli (2005) and Justiniano and Preston (2010). These models are described in Ortiz and Sturzenegger (2007), Steinbach *et al.* (2009a), and Alpanda *et al.* (2010a,b, 2011). They include features that have become standard in the small open-economy DSGE literature; such as nominal rigidities in price and wage-setting, indexation of domestic prices and wages to past inflation, habit formation in consumption, incomplete international risk-sharing, partial pass-through of exchange rate movements to domestic inflation, and the use of Taylor rules to describe monetary policy.

Other examples of DSGE models for the South African economy, include Gupta and Steinbach (2013), which describes the use of a DSGE-VAR model to show the relative importance of each individual rigidity when forecasting economic variables. A closed-economy nonlinear DSGE model for the South African economy is also presented in Balcilar *et al.* (2013), where it is suggested that the explicit incorporation of nonlinearities improves upon the forecasting performance of these models. Finally, following the events that transpired

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<sup>13</sup>Edge *et al.* (2010) find that their Estimated Dynamic Optimisation (EDO) model also outperforms pure time-series models such as AR(2), VAR(1), and BVAR(2) models in terms of forecasting performance. They nevertheless noted that DSGE models have been adjusted over the years with hindsight of their failures, and therefore the forecasting performance of EDO and similar models may not be as good in the future. For a forecasting evaluation of the European Central Bank's DSGE model, which is termed the New Area-Wide model (NAWM), see Christoffel *et al.* (2010).

<sup>14</sup>Details of this model are contained in de Wet and Dreyer (1978).

<sup>15</sup>This model was extended in Liu *et al.* (2010), where they use a similar model with error terms that are specified with a VAR structure (following Ireland (2004b)). Thereafter, they extend the model to incorporate sticky prices in Liu *et al.* (2009).

during the Global Financial Crisis that started in 2007, Steinbach (2013) has shown how it is possible to extend a model for the South African economy to incorporate features of the financial sector.

### 1.3 Motivation for this Dissertation

This dissertation seeks to improve upon our understanding of the South African business cycle with the aid of DSGE models that could be used to assist economic policy-making in South Africa. It considers a number of essential research questions such as, “*How to structure a DSGE model for the South African economy?*”

To accomplish this objective, the dissertation contains a detailed description of the theoretical foundations and derivations for these models in part one. Chapter two contains a description of the stylised features of the business cycle, which are to be incorporated in the model. It also includes a critique of backward-looking reduced-form models that have been used for policy purposes to describe business cycle phenomena. Thereafter, chapter three presents a stylised version of early RBC models, which focuses on the household sector of these models, where firms are perfectly competitive and prices are perfectly flexible. This model is then expanded to show how it may incorporate a role for the central bank, albeit with various limitations.

Chapter four provides details of the rigidities that are the central feature of New Keynesian models, following the introduction of monopolistic competitive firms. The importance of this rigidity and how it relates to the relative non-neutrality of monetary policy, is described with the aid of simulated model. In the subsequent sections of this chapter, the model is extended to incorporate a role for money and habits in consumption, which are features that are included in the models that are estimated in the latter part of the dissertation.

Chapter five extends the model to facilitate the incorporation features that are typical of small-open economies. It includes a discussion of the theoretical foundations for two different methods that may be used to ensure that these open-economy features do not allow for persistent trends that may arise from a continued increase in the borrowing of foreign capital. The methods include the use of the assumption of complete asset markets or a debt-elastic interest rate (risk) premia; and the implications of these different methods, when applied to South African data, is expanded upon in part three.

Whilst much of what is discussed in part one may be found in advanced macroeconomic texts, the derivations that are provided in this part include explicit detail. It is hoped that this detail will promote a deeper understanding for these models, whilst also making the literature more accessible to those who would like to conduct further research with these models. In addition, by providing a great degree of detail (and motivation) for this particular model

structure, the description of the models that are used in the latter parts of the dissertation do not burden the reader with an exhaustive narrative.

After describing the theoretical features that are important to the model structure, the second part of this dissertation seeks to address the next essential research question, “*What data should be used in these models?*”, which is the focus of part two in this dissertation.<sup>16</sup> Therefore, chapter six describes the importance of the methods that may be used to decompose the data into components that represent the trend and the cycle for the variables that are eventually incorporated in the model. Thereafter, chapter seven makes use of various univariate and multivariate analyses to look for structural breaks in the data. These results suggest that there were significant structural breaks in the data during the mid 1980s, and as such the starting point for the sample of data that is to be modelled in the latter sections of this dissertation occur after this point in time. Chapter eight then focuses on measures of core inflation, which may be incorporated in the model. This chapter compares the use of trimmed means, dynamic factor models and various wavelet decompositions. The results suggest that the wavelet decomposition provides the most suitable measure of core inflation.

The third part of the dissertation considers topics that relate to the estimation and use of these models.<sup>17</sup> In addition, it considers the essential research question of, “*How do we evaluate the applicability of a DSGE model for a*

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<sup>16</sup>This part relies on work that has been published in:

- DU PLESSIS, S., DU RAND, G. and KOTZÉ, K. forthcoming. Measuring Core Inflation in South Africa. Working paper series, University of Stellenbosch.
- DU PLESSIS, S. and KOTZÉ, K. 2012. Trends and Structural Changes in South African Macroeconomic Volatility, ERSA Working Paper No. 297.
- DU PLESSIS, S. and KOTZÉ, K. 2010. The Great Moderation of the South African Business Cycle. *Economic History of Developing Regions*. 25(1):105-125.

<sup>17</sup>This part makes use of techniques that were utilised in the following published articles. However, to ensure that the data is consistent with what is discussed in part two, the datasets and model structures do differ to those that were used in these papers:

- ALPANDA, S., KOTZÉ, K. and WOGLOM, G. 2011. Forecasting Performance of an Estimated DSGE Model for the South African Economy. *South African Journal of Economics*, 79(1):50-67.
- ALPANDA, S., KOTZÉ, K. and WOGLOM, G. 2010b. Should Central Banks of Small Open Economies Respond to Exchange Rate Fluctuations? The Case of South Africa, ERSA Working Paper No. 174.
- ALPANDA, S., KOTZÉ, K. and WOGLOM, G. 2010a. The Role of the Exchange Rate in a New Keynesian DSGE Model for the South African Economy. *South African Journal of Economics*, 78(2):170-191.



*small-open economy?*". After estimation the model with the aid of Bayesian techniques in chapter nine, using the variables that are described in part two, the model is then evaluated with the aid of impulse response functions. In addition, the posterior parameter estimates are also discussed, with reference to parameter estimates that have been derived for other models for the South African economy.

This chapter also includes a discussion on the role of the exchange rate in these models, which may be included as an observed variable after making use of the debt-elastic interest rate (risk) premium to close off the open economy features in the model. When using the assumptions that underpin complete asset markets, it may be shown that it is not possible to include the exchange rate as an observed variable. The use of a historical variance decomposition is then used to suggest that the exchange rate contains critical information that may be used to describe important open economy features in the model.

Chapter ten contains an optimal policy investigation, which seeks to identify optimal values for the parameters that define the central bank's reaction function. This exercise may be used to determine, "*How do these models assist in the policy-making process?*", where the results suggest that to reduce economic volatility, the central bank should disregard any changes to the exchange rate, when setting the path for the policy interest rate. In addition, the results in this chapter also suggest that there is scope for the central bank to react more aggressively to changes in the rate of inflation.

The penultimate part of this dissertation considers more recent research into the features of DSGE models.<sup>18</sup> It includes an example of a nonlinear model for the South African economy, which makes use of a technique for solving a second-order approximation of the model and a nonlinear filter for the evaluation of the likelihood function. It considers the methods that may be used to forecast with these models to address the essential question "*How important are nonlinearities in a DSGE model for the South Africa economy?*". The results from this model are compared to linear DSGE models and a large suite of Vector Autoregressive (VAR) and Bayesian VAR models. The results suggest that many of the South African macroeconomic variables may include important nonlinear features, which should be incorporated in the model.

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<sup>18</sup>This part relies on work that has been published in:

- BALCILAR, M., GUPTA, R. AND KOTZÉ, K. forthcoming. Forecasting Macroeconomic Data for an Emerging Market with a Nonlinear DSGE Model. *Economic Modelling*.

# Part I

## Theoretical Model Construction

## Chapter 2

# Stylised Business Cycle Features

## 2.1 Introduction

The business cycle constitutes a series of expansionary and contractionary phases in economic activity that may result from various types of economic shocks. The relative importance of these shocks is supported by empirical evidence, where in one of the early macroeconomic models of the business cycle, Adelman and Adelman (1959) suggested that to model cyclical behaviour that is consistent with macroeconomic activity, one would need to incorporate exogenous stochastic shocks to the model equations.<sup>1</sup>

Since the work of Tinbergen (1939), macroeconomists have tried to model the relationship between variables that influence the business cycle with reduced-form models. These models have been used in various studies to describe the empirical relationships between money, inflation and the business cycle. Variants of these models are also able to distinguish between long-run and short-run behavioural relationships, which may be of interest in certain policy investigations. Furthermore, as they have been used for an extremely long period of time, they may be used as a benchmark, against which more recent models may be compared.

Much of the discussion in this chapter (which relates to the business cycle), will incorporate details that are encountered when investigating the conduct of monetary policy, as these models have been extensively used and developed within this area. In addition, exogenous changes to interest rates (which are imposed by the central bank) may influence the economic activity of a particular country or region; since it has been suggested that they affect asset prices, expected returns, consumer spending, investment decisions, and a number of other important economic measures.<sup>2</sup>

## 2.2 Historical Findings

One of the most extensive investigations into the conduct of monetary policy is provided by Friedman and Schwartz (1963), who argued that changes in the stance of monetary policy result in output fluctuations (which influences the business cycle).<sup>3</sup> Their evidence goes on to suggest that a tighter monetary

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<sup>1</sup>When evaluating the results of their model, they compare the model's description of cyclical behaviour to that of the business cycle dating procedure of Burns and Mitchell (1947).

<sup>2</sup>Reviews of suggested transmission mechanisms of monetary policy may be found in Mishkin (1995), Boivin *et al.* (2010), Mukherjee and Bhattacharya (2011), and others.

<sup>3</sup>The manuscript is 860 pages and the empirical work spans the sample period 1867 to 1960, using data on the United States economy. The book is also noted for suggesting that

policy would *cause* (with long and variable lag) a decline in nominal gross domestic product.<sup>4</sup>

However, as the supply of broader monetary aggregates is endogenous, where an economic expansion may result in an increase in money supply, this correlation between monetary supply and output growth may be due to the effect of output growth on money supply (and not the other way around).<sup>5</sup> This argument is in line with the earlier evidence of reverse causation that is provided by Tobin (1970), where he suggests that it may be the case that output growth causes an increase in money supply. It has subsequently been supported by King and Plosser (1984) who show that the correlation between broad money supply and output arises from the endogenous response of the banking sector to economic disturbances that are not the result of monetary policy.

In a more recent study, Friedman and Kuttner (1992) suggest that the relationship between measures of money supply and output, where slowdowns in money lead most business cycle downturns, no longer exists in the data from 1982 onward. Hence, the early research which considered the relationship between changes to monetary policy and its effect on economic output, has provided mixed results. One of the reasons for this could be that these investigations did not always distinguish between the long-run and short-run effects of monetary policy, which is currently considered to be an important aspect of such research (Walsh, 2010).

In addition, these early studies were also hampered by the inadequacies of empirical methods that were used to identify the lag structure for the transmission of monetary policy. For example, much of this research was conducted with the aid of correlation statistics and basic linear regression models.

## 2.3 Reduced-Form Modelling

To summarise the more recent evidence on the short term effects of monetary policy, we may consider the results of various reduced-form models. With the aid of these techniques, one is able to employ formal statistics that include Granger causality tests, impulse response functions, and variance decompositions, to describe the behaviour of variables in a multivariate setting. In

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excessively tight monetary policy that followed the boom of the 1920s turned an otherwise severe recession into the Great Depression of the 1930s.

<sup>4</sup>Whilst a more expansionary monetary policy would result in output growth that is temporarily above the long-term trend.

<sup>5</sup>In the famous debate that was published in the *Journal of Post Keynesian Economics*, Basil Moore (1988, 1989) made a strong argument against Charles Goodhart (1989) that the supply of money is endogenous, as households and firms have access to credit, which would prevent central bankers for exerting complete exogenous control over monetary supply. During an economic expansion, households and firms would have access to additional credit in the presence of constant interest rates, as their credit rating would improve.

addition, this framework would also facilitate investigations into the identification of the lag structure, and the distinguishing features of long-run and short-run behaviour.

Early variants of reduced-form models included the linear simultaneous equation models, which were developed by the Cowles Commission in 1932.<sup>6</sup> This programme sought to develop a synthesis between mathematical measurement and economic theory, by estimating the parameters in a model that embodied a Keynesian variety of aggregate demand and supply curves. Unfortunately, this programme was perceived to be an empirical failure, by the mid 1960s and was abandoned during the 1970s (Heckman, 2000).

Following concerns relating to the “incredible nature” of the identification restrictions that were applied to the Cowles Commission model, Sims (1972, 1980a) advocated the use of loosely specified economic models, such those that utilise a vector autoregressive (VAR) framework. These models formed an integral part of monetary policy research, where a number of different identification schemes were used to investigate the effects of monetary policy shocks. An extensive review of this literature is provided by Christiano *et al.* (1999), who suggest that when using various forms of VAR models that are applied to data from the United States, there is a great degree of consensus that:

“... after a contractionary monetary policy shock, short term interest rates rise, aggregate output, employment, profits and various measures of wages fall, the aggregate price level responds very slowly, and various measures of wages fall, albeit by very modest amounts.”

To quantify the effect of a monetary policy shock on other critical macroeconomic variables Christiano *et al.* (1999) look to estimate the policy rule of the Federal Reserve. Whilst their model has an elaborate VAR structure with a number of important identifying restrictions that allow for more complex interactions, the policy rule essentially takes the form,

$$i_t = \varrho_i i_{t-1} + \varrho_{y,1} y_t + \varrho_{y,2} y_{t-1} + \varrho_\pi \pi_t + \varrho_p p_t + \varrho_m m_{t-1} + \epsilon_t \quad (2.3.1)$$

where  $i_t$  is the Federal Funds rate,  $y_t$  is the real GDP growth rate,  $\pi_t$  is the change in the GDP deflator (i.e. inflation),  $p_t$  is the change in commodity prices,  $m_t$  represents a vector for the change in monetary aggregates, and  $\epsilon_t$  is the monetary policy shock.

This model may be estimated using ordinary least squares (OLS) and the effects of  $\epsilon_t$  on the respective variables ( $i_t, y_t, \pi_t, p_t$  and  $m_t$ ) can be constructed

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<sup>6</sup>For further details on the Cowles Commission and an evaluation of its contribution to econometric modelling, see Christ (1995), Epstein (1987), and Morgan (1990).

from the impulse response functions, which can be derived from the partial derivatives,

$$\begin{aligned} IRF_{i,j} &= \frac{\partial i_{t+j}}{\partial \epsilon_t}, \quad IRF_{y,j} = \frac{\partial y_{t+j}}{\partial \epsilon_t}, \\ IRF_{\pi,j} &= \frac{\partial \pi_{t+j}}{\partial \epsilon_t}, \quad , IRF_{m,j} = \frac{\partial m_{t+j}}{\partial \epsilon_t} \end{aligned}$$

These impulse response functions, which describe the dynamic responses of the above variables to a monetary policy disturbance, are contained in figure (2.1).<sup>7</sup> They suggest that a contractionary monetary policy shock of one standard deviation would cause the Federal Funds rate to increase by 0.75% during the initial period, before it returns to its long-run level after 5 quarters. The shock also results in a decline in output that reaches a trough after 5 quarters, where it declines by 0.5%, before it slowly moves back towards its long-term level. Note that after 15 quarters, output is yet to return to its original level, which would suggest that whilst the response of output to monetary policy is relatively large, these effects are also relatively persistent (but not permanent).

With regards the response of broad based inflation, it takes over 4 quarters to respond to the monetary policy shock. This has lead researchers to suggest that the transmission of monetary policy to inflation occurs largely through a persistent reduction in output growth (Christiano *et al.*, 1999). In addition, it also may support the argument that the response of dynamic economic agents may result in behaviour that arises in the presence of significant price rigidities (Galí, 2008).

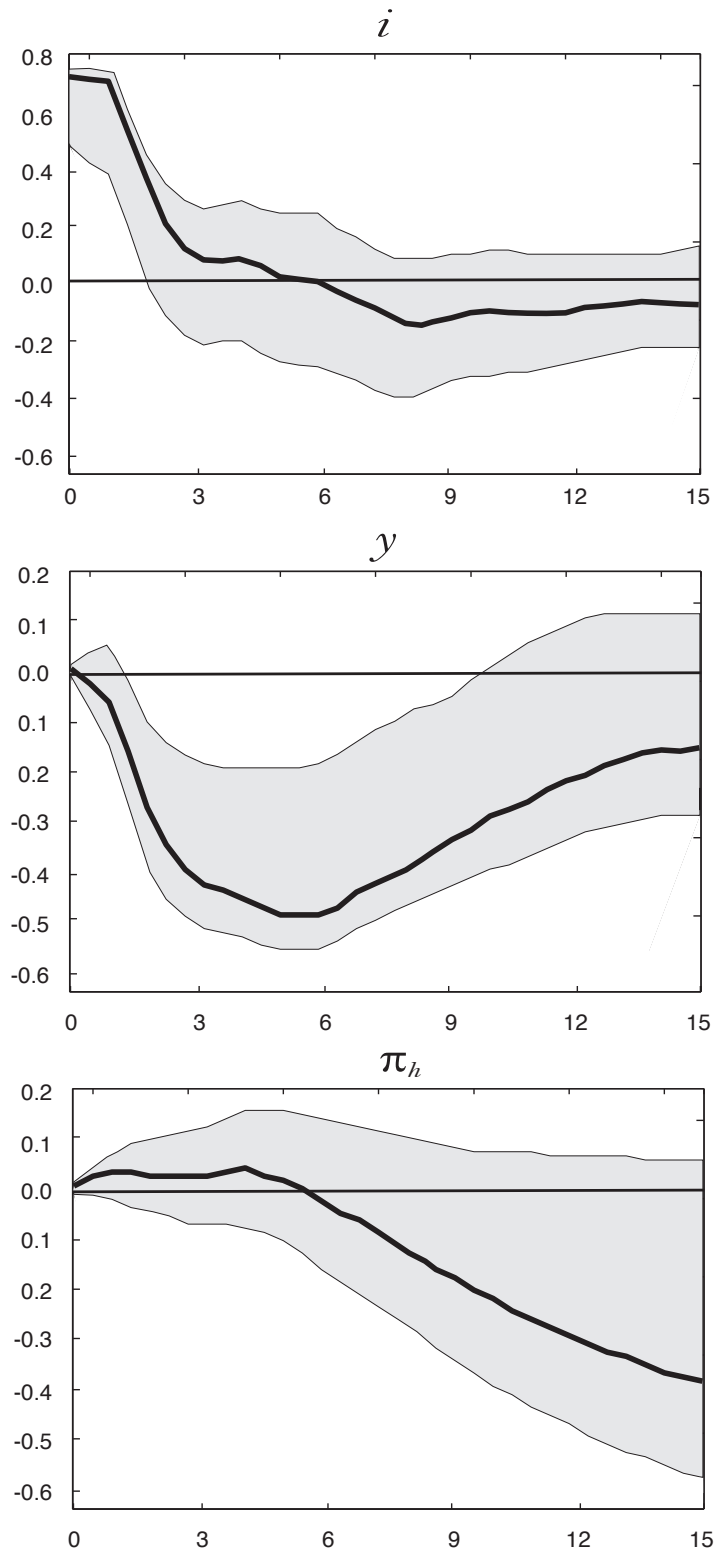
Note that whilst there is broad agreement about the direction of these variables, there is some disagreement about the quantum of the movements that should be provided by the impulse response functions. In addition, there is also some disagreement about the fraction of variation in each variable that is explained by a monetary policy shock. For example more recently, Uhlig (2005) has suggested that monetary policy shocks account for between 5% and 10% of output volatility, which is similar to that of the findings in Altig *et al.* (2011) where they account for 9% of the volatility. These findings would seem to contradict those of Christiano *et al.* (2005), who suggested that monetary policy accounts for between 15% to 38% of output volatility.

## 2.4 South African Evidence

All of the quantitative results that were presented above made use of data from the United States. When considering the effects of a South African monetary policy shock on output, Du Plessis *et al.* (2007) and Cuevas and Topak (2009)

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<sup>7</sup>A confidence interval of 95% is included in these diagrams.



**Figure 2.1:** Impulse Response Functions - Christiano *et al.* (1999)



have suggested that this effect may be smaller than what is reported for the United States. In particular, Du Plessis *et al.* (2007) noted that for a monetary policy shock of one standard deviation, the real interest rose by approximately 3.5%, which resulted in an output decline of less than 0.5% after 5 quarters before it returned to its original level. Furthermore, when considering the effect of a monetary policy shock on South African inflation, the results of the South African Reserve Bank *Core Forecasting Model* suggested that inflation reacted by about 0.35% after 7 quarters, following a 1% change in interest rates (Smal *et al.*, 2007).

More recently, Gumata *et al.* (2013) made use of a large-scale Bayesian VAR model to consider the effects of a monetary policy shock of 1%. The effect of this shock on output and inflation may be seen in figure (2.2), where output declined by less than 0.05% after 4 quarters and inflation declined by about 0.12% after 5 quarters.<sup>8</sup> These results would support previous findings, which suggested that the effect of a monetary policy shock in South Africa may be smaller than what is reported for the United States. However, there is some disagreement on the exact quantum of the response of key macroeconomic variables to such a shock.<sup>9</sup>

## 2.5 Critique of Reduced-Form Evidence

One of the most important critiques of these traditional types of reduced-form models is that they do not allow for the explicit incorporation of expected future values of variables. This would prevent these models from describing behaviour that is consistent with the forward-looking decisions of economic agents, since the identification of the impulse response functions and the timing of these responses would be incorrect. As such, reduced-form models would not be able to provide a suitable description of an inflationary process that may depend upon the economic agent's expectations of future activity.

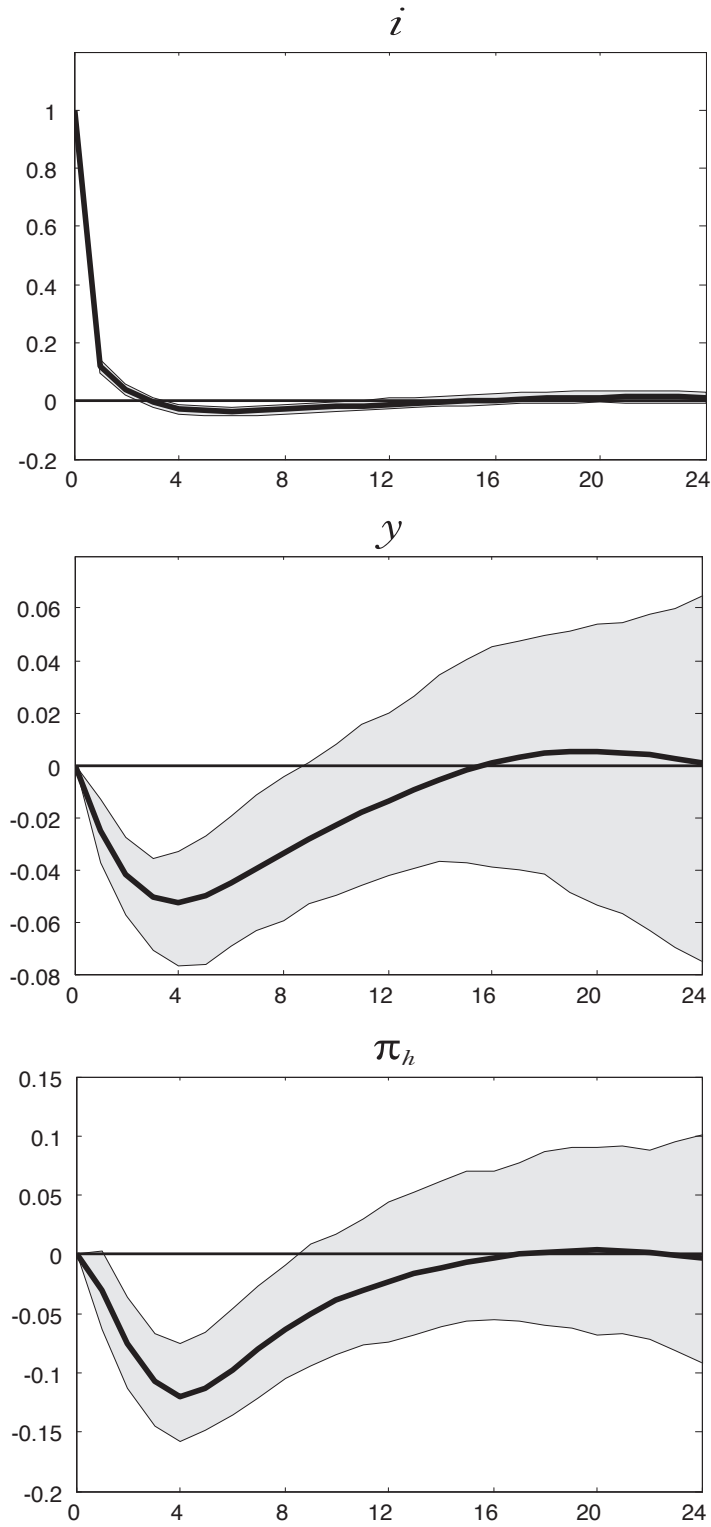
This feature is also important within the context of monetary policy, where central banker's make use of expected future values of key macroeconomic variables before they set interest rates. In particular, when implementing an *inflation forecast targeting* framework, policy-makers are guided by the expected future value of inflation, which may be provided by various forecasting models.<sup>10</sup> After evaluating the behaviour of the South Africa Reserve Bank,

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<sup>8</sup>After comparing various transmission mechanisms in their model that includes 165 variables, Gumata *et al.* (2013) find that the transmission of a South African monetary policy shock largely occurs through output (i.e. the interest rate channel).

<sup>9</sup>The studies that have been cited above make use of different datasets and sample periods. As such, one should expect to find some variation in the effect of shocks on other variables.

<sup>10</sup>See, Svensson (1997) and Svensson (2005) for an articulate description of the inflation forecast targeting framework.



**Figure 2.2:** Impulse Response Functions - South Africa - Gumata *et al.* (2013)

Du Plessis (2002) has suggested that this practice has also been followed by the domestic central bank.

Further critiques are provided by those who consider the applicability of these models. For example, Heckman (2000) has noted that it is often difficult to interpret the results of unrestricted VAR-based models, particularly when they make use of a relatively large number of variables in the policy function. In such cases, it is often difficult to determine which of the variables in the model is responsible for generating the results. He also added support to the critique of Leamer (1983), who argued that in many of these models, the choice of variables would appear to be somewhat arbitrary, which may detract from their relevance.

In addition, Rudebusch (1998) has noted that a number of important empirical results from VAR models would appear to be unreasonable. For instance, a number of models (including Christiano *et al.* (1999)) have suggested that prices would initially rise after a rise in interest rates, which gives rise to a *price puzzle*. Whilst the inclusion of commodity prices in the model would appear to minimise this effect, it is nevertheless apparent in many results. Sims (1992) suggested that this result may be attributed to the fact that VAR models are not able to include variables that may be used to predict future inflation. Hence, when these omitted variables intimated an increase in inflation, the central bank would increase interest rates. As such, an increase in interest rates may be associated with an increase in observed inflation over the short-term.

This argument is of importance within the context of VAR models, as an increase in interest rates that is due to the omitted variables would be incorporated in the residual. This would imply that the error term may contain information that is not consistent with a definition of unexplained monetary policy *shocks* (i.e. they are incorrectly identified), which would result in biased impulse response functions (Stock and Watson, 2001).

After investigating the residuals from a number of VAR models, which are used to characterise monetary policy shocks, Rudebusch (1998) noted that the shocks from these models did not correspond with the policy actions of the central bank that had occurred during the period under investigation. Hence, none of the specifications that he considered were able to provide a suitable description of past monetary policy actions.

In addition, Rudebusch (1998) has also noted that the residuals from the various VAR models that he specified were not highly correlated, which would suggest that the results of these models are not consistent. This finding is supported by Stock and Watson (2001), where after implementing modest changes to the specification of the monetary policy rule, they noted substantial changes in the impulse response functions. They also found that there is widespread instability in the parameters of policy rules that were specified with VAR models in their earlier research (Stock and Watson, 1996).

## 2.6 Conclusion

This stylised representation of temporary economic fluctuations suggest that monetary policy affects the business cycle over a number of periods. However, the use of reduced-form models are severely limited in the sense that they are not able to account for the forward-looking decisions of economic agents and may provide results that are largely influenced by shocks that have been poorly identified. These mis-specification errors would appear to provide a relatively poor description of the effects of a monetary policy shock on inflation, which may limit their usefulness when used for policy investigations.

In an effort to provide a consistent description of the business cycle and the effects of monetary policy, Kydland and Prescott (1982), Galí (2002) and many others, sought to make use of an alternative modelling framework that incorporated explicit microeconomic foundations. This lead to a new direction in the empirical research that was conducted on business cycles and these efforts have been described in the following two chapters.

## Chapter 3

# The Real Business Cycle Model

### 3.1 Introduction

The foundations for many modern structural macroeconometric models for the business cycle, arise out of the Real Business Cycle (RBC) literature, which places emphasis on the effect of real shocks on the business cycle.<sup>1</sup> These models integrate aspects that affect economic growth and business cycle fluctuations within a single framework, where agents maximise their intertemporal utility, subject to the shocks that have already been realised, as well as those that are expected in the near future.

The literature builds upon the work of Kydland and Prescott (1982), who made use of a dynamic general equilibrium model to describe the behaviour of economic agents that form rational expectations about the future. Their results suggest that these models describe certain features of the macroeconomic data particularly well.<sup>2</sup> Additional key references of models from the original RBC literature include, Hansen (1985), Prescott (1986), King *et al.* (1988a), King *et al.* (1988b), Cooley and Prescott (1995), King and Rebelo (1999), and King *et al.* (2002).

RBC techniques have been used extensively in the literature, and as a result they serve as an important benchmark, against which subsequent developments are currently assessed. In addition to their use for describing and forecasting economic behaviour, they have been used for general policy analysis and for investigations into optimal fiscal and monetary policy.<sup>3</sup> This is an important aspect of macroeconomic modelling; for as Lucas (1980) notes,

*“One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost.”*

In this chapter we introduce a simple economic model that has a number of features that are characteristic of the original RBC model of Kydland and Prescott (1982) to show the effect of a technology shock on output in section 3.2. This model seeks to describe the role of a number of representative agents

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<sup>1</sup>An example of a *real* shock would include a change to the nature of the production process, which may be reported as shock to technology with the use of the specification of Solow (1956, 1957).

<sup>2</sup>The work of Kydland and Prescott (1982) builds on prior work by Lucas and Prescott (1977). The fit of the Kydland and Prescott (1982) model is assessed by comparing correlations of the model variables with correlations of actual economic data.

<sup>3</sup>The parameters in these models may be policy invariant, which would infer that these models satisfy the Lucas (1976) critique, when used to consider the effects of a change to economic policy.

for households and firms. The agents will then interact in the goods and labour markets to derive some form of steady state (or equilibria), which may be described by a linearised closed-form solution.

Important extensions to early real business cycle models, incorporate a role for central banks within this modelling framework, to allow for an analysis into the role of monetary policy. Early variants of these models included the work by Cooley and Hansen (1989), which maintained the standard RBC features of perfect competition and fully flexible prices and wages. An example of these types of models is contained in section 3.3, where we will see that this framework suggests that monetary policy does not have a significant impact on real variables (such as output and employment).<sup>4</sup>

## 3.2 The Effects of Technology Shocks

In the following model, we describe the foundations of the RBC model with capital accumulation, in an extremely small model that seeks to focus our attention on the effects of a technology shock. As such, this model is able to capture most of the essential characteristics of the model of Kydland and Prescott (1982).<sup>5</sup>

### 3.2.1 Households

In this model, it is assumed that households maximise consumption,  $C_t$  and leisure,  $1 - N_t$ , where  $N_t$  represents labour input. In this case we make use of a simple logarithmic utility function,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [\log C_t + \psi \log(1 - N_t)] \quad (3.2.1)$$

where the subjective time preference factor,  $\beta$ , is used to infer that the household has a preference for consuming sooner rather than later. It may then be assumed that in this closed-economy model, the national income identity assumes that income,  $Y_t$ , is either consumed,  $C_t$ , or invested,  $X_t$ . This provides us with the expression,

$$Y_t = C_t + X_t \quad (3.2.2)$$

To describe the evolution of capital, we assume that the capital stock,  $K_t$ , depreciates by  $\delta$ ; whilst investment,  $X_t$  would replenish the capital stock. Hence,

$$K_{t+1} = (1 - \delta)K_t + X_t \quad (3.2.3)$$

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<sup>4</sup>This finding is in contrast with a large body of evidence, including Friedman and Schwartz (1963) and Christiano *et al.* (1999).

<sup>5</sup>This model is notably smaller than that which is presented in Kydland and Prescott (1982), to facilitate an easier explanation.

which could be used to describe investment as  $X_t = K_{t+1} - (1 - \delta)K_t$ .

Income in this model is derived from the total factor payments (as there is no profit or dividend distribution that is transferred back to the household). This would imply that income is derived from wages  $W_t$  and interest (or cost of capital)  $r_t$ , where,

$$Y_t = W_t N_t + r_t K_t \quad (3.2.4)$$

Therefore, in any given period, the consumer faces a tradeoff between consuming and investing their income, as suggested by (3.2.2). If they should choose to invest then they will increase the capital stock, as suggested by (3.2.3). This would allow them to increase their income, as suggested by (3.2.4), which may allow them to consume more in future.

Combing (3.2.4), (3.2.2) and (3.2.3) would also allow us to write the budget constraint as,

$$\begin{aligned} C_t + X_t &= W_t N_t + r_t K_t \\ \therefore C_t + K_{t+1} &= W_t N_t + r_t K_t + (1 - \delta)K_t \end{aligned}$$

This budget constraint could be thought of as an accounting identity, with total expenditures on the left-hand-side and revenues (which include the liquidation value of capital stock) on the right-hand-side.

To derive the maximum amount of consumption, labour and capital stock, we can establish the Lagrangian,

$$\begin{aligned} \mathcal{L}_{C_t, N_t, K_{t+1}} &= E_0 \sum_{t=0}^{\infty} \beta^t \{ [\log C_t + \psi \log(1 - N_t)] + \dots \\ &\dots \lambda_t [W_t N_t + r_t K_t + (1 - \delta)K_t - C_t - K_{t+1}] \} \end{aligned}$$

with the accompanying partial derivatives,

$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0 \quad (3.2.5)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = \frac{\psi}{1 - N_t} + \lambda_t W_t = 0 \quad (3.2.6)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \beta r_{t+1} \lambda_{t+1} + \beta(1 - \delta)\lambda_{t+1} = 0 \quad (3.2.7)$$

Summarizing expression (3.2.5) and (3.2.6),

$$\lambda_t = \frac{1}{C_t} \quad \text{and} \quad W_t = \frac{\psi/(1 - N_t)}{\lambda_t}$$

which leaves us with the labour supply equation that links labour positively to wages and negatively to consumption (i.e. the wealthier the representative



agent, the more leisure they make use of, if they were encountered a significant decrease in their marginal utility of consumption).

$$\psi \frac{C_t}{1 - N_t} = W_t \quad (3.2.8)$$

Similarly, using (3.2.7) and (3.2.5),<sup>6</sup>

$$\lambda_t = \beta E_t r_{t+1} \lambda_{t+1} + \beta E_t (1 - \delta) \lambda_{t+1} \quad \text{where } \lambda_t = \frac{1}{C_t} \quad \text{and } \lambda_{t+1} = \frac{1}{C_{t+1}}$$

This provides us with the Euler equation in consumption, which captures the intertemporal tradeoff between consuming during the present or future period,

$$\begin{aligned} \frac{1}{C_t} &= \beta E_t r_{t+1} \frac{1}{C_{t+1}} + \beta E_t (1 - \delta) \frac{1}{C_{t+1}} \\ &= \beta E_t \left[ \frac{1}{C_{t+1}} (1 + r_{t+1} - \delta) \right] \end{aligned} \quad (3.2.9)$$

Note that this expression suggests that the decision to consume either now or in the future is dependent upon the interest rate (rate of the return on capital) and the rate of depreciation.

### 3.2.2 Firms

If we assume that the representative firm faces a neoclassical production function, with constant rate of substitution, which is defined as,

$$Y_t = K_t^\alpha (e^{A_t} N_t)^{1-\alpha} \quad (3.2.10)$$

where  $\alpha$  is the capital elasticity in the production function, with  $0 < \alpha < 1$ . In this case,  $A_t$  captures labour augmenting technology which evolves according to an autoregressive process,

$$A_t = \rho A_{t-1} + \epsilon_t \quad (3.2.11)$$

where  $\rho$  is a parameter that captures the persistence of a shock to the technological process, and the properties of the stochastic shock are,  $\epsilon_t \sim \mathcal{N}(0, \sigma)$ . The firm would then seek to maximise profits,  $\Pi_t$ , for a given price level and wage rate, such that,

$$\Pi_t = Y_t - W_t N_t - r_t K_t \quad (3.2.12)$$

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<sup>6</sup>Note, we make use of the expectations operator to establish that the expectations for the values of variables in period  $t + 1$  are formed in period  $t$ .

where they now need to pay interest on borrowed capital,  $r_t K_t$ . Hence, to maximise profits firms would need to minimise real costs that depend upon the respective values of  $N_t$  and  $K_t$ . Therefore, the problem of the firm, which is subject to the constraint of the production function, may be summarised by the Lagrangian,

$$\mathcal{L}_{\Pi,t} = Y_t - W_t N_t - r_t K_t + \lambda_t [K_t^\alpha (e^{A_t} N_t)^{1-\alpha} - Y_t]$$

This expression allows one to derive the partial derivatives,<sup>7</sup>

$$\begin{aligned} \frac{\partial \mathcal{L}_{\Pi,t}}{\partial Y_t} &= 1 - \lambda_t = 0 \\ \frac{\partial \mathcal{L}_{\Pi,t}}{\partial N_t} &= -W_t + \lambda_t [(1 - \alpha) K_t^\alpha (e^{A_t} N_t)^{1-\alpha} N_t^{-1}] = 0 \\ \frac{\partial \mathcal{L}_{\Pi,t}}{\partial K_t} &= -r_t + \lambda_t [\alpha K_t^{\alpha-1} (e^{A_t} N_t)^{1-\alpha}] = 0 \end{aligned}$$

Now, since  $\lambda_t = 1$ , the other expressions will simplify to,

$$W_t = (1 - \alpha) K_t^\alpha (e^{A_t} N_t)^{1-\alpha} N_t^{-1} \quad (3.2.13)$$

$$r_t = \alpha K_t^{\alpha-1} (e^{A_t} N_t)^{1-\alpha} \quad (3.2.14)$$

These expression could be combined to derive the capital to labour ratio (which describes the relationship between payments to factors of production), so by dividing (3.2.14) by (3.2.13), we get,

$$\begin{aligned} \frac{r_t}{W_t} &= \frac{\alpha K_t^{\alpha-1} (e^{A_t} N_t)^{1-\alpha}}{(1 - \alpha) K_t^\alpha (e^{A_t} N_t)^{1-\alpha} N_t^{-1}} \\ &= \frac{\alpha K_t^{-1}}{(1 - \alpha) N_t^{-1}} \\ &= \frac{\alpha}{(1 - \alpha)} \frac{N_t}{K_t} \end{aligned}$$

Hence,

$$r_t K_t = \frac{\alpha}{(1 - \alpha)} N_t W_t \quad (3.2.15)$$

This relationship suggests that the capital to labour ratio is dependant upon the elasticity of capital, as well as the respective interest and wage rate.

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<sup>7</sup>Note that we may express the derivative of  $f = a^x$  as  $f'(x) = x(a)^{x-1}$  or  $f'(x) = x(a)^x(a)^{-1}$ .

### 3.2.3 Choice of parameter values

This model makes use of six parameters for,  $\alpha, \beta, \delta, \psi, \rho$ , and  $\sigma$ , which may be used to simulate the effect of technology shock. Calibration techniques may then be used to mimic features of an actual economy. These techniques may also be used to consider the implications of changes to economic policy as described in Lucas (1980).<sup>8</sup> In addition, when defending the use of these techniques, Prescott (1986) notes,

*“The models constructed within this theoretical framework are necessarily highly abstract. Consequently, they are necessarily false, and statistical hypothesis testing will reject them. This does not imply, however, that nothing can be learned from such quantitative theoretical exercises.”*

The values for these parameters are largely calibrated to those that are contained in Kydland and Prescott (1982). The parameter  $\alpha$  represents the percentage of total output that goes towards the costs of capital used in production, and  $(1 - \alpha)$  represents labour’s share of output. Using microeconomic evidence for the United States, Kydland and Prescott (1982) suggested that for their sample period, this parameter should have a value of 0.36.

Similarly, they suggest that the subjective time discount factor is assumed to take a value of 0.99 and the rate of depreciation is given as 10% per annum. To characterise the results from non-separable utility from leisure at different points in time, we make use of a Frisch elasticity of labour supply of 1.75, which is within the range proposed by King and Rebelo (1999). The persistence of a technology shock is purported to be quite high, where a value of 0.95 is used in this empirical exercise and the standard deviation of the shock to technology would induce an initial increase of 1% in technology.

### 3.2.4 Model results

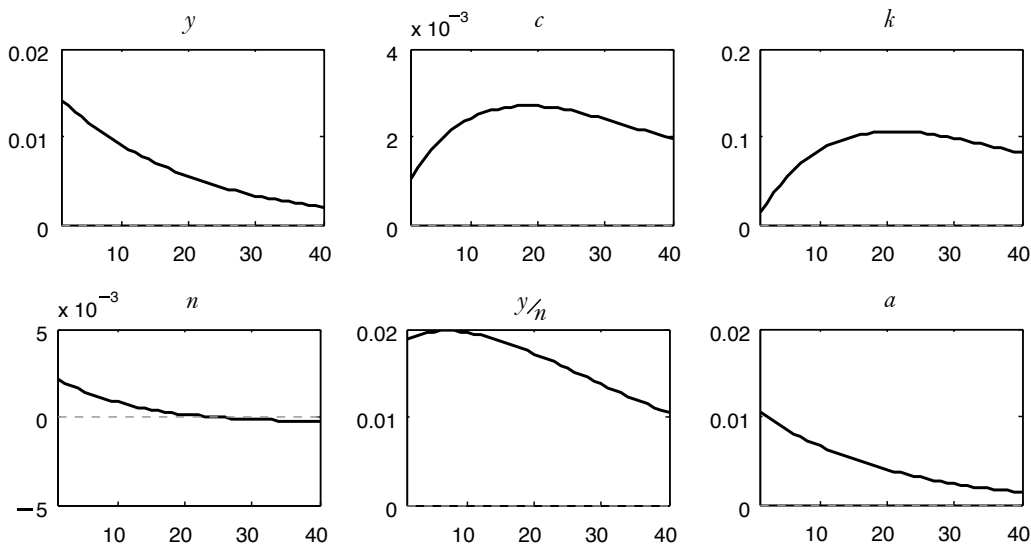
The impulse response functions for this model are provided in figure (3.1). They suggest that a shock to technology has resulted in an initial increase in technology of 1%. Thereafter, the value of  $a_t$  returns towards its steady-state over an extended period of time. Note that the shock to technology has also resulted in a relatively large and persistent effect on output, which initially increased by over 1%. The remaining impulse response functions suggest that most of the other variables are pro-cyclical, as consumption, capital, employment and productivity (output per worker) all move with output.

These results provide empirical support for the argument of Kydland and Prescott (1982), who suggested that shocks to real variables may influence the

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<sup>8</sup>A more detailed discussion of this practice is contained in Kydland and Prescott (1991, 1996).

business cycle, despite the fact that this particular model does not incorporate a role for monetary policy. This finding was extremely important at the time, as Friedman (1968) had suggested that the dominant cause of business cycle fluctuations was monetary policy.



**Figure 3.1:** Impulse Response Functions - RBC model with technology shock

When considering the data adherence of this model we can compare the statistical properties of the model generated variables with the moments of observed data. Since the focus of this evaluation is on the degree to which the model fits the business cycle, we focus primarily on the second moments (or volatility) of the respective data. In addition, researchers also usually compare the degree to which the respective variables are correlated with output, as well as their first order autocorrelation, which gives an indication of persistence.

To extract the business cycle component from a particular variable, one would usually remove the long-term trend from the data. Deviations from this trend are then referred to as the cyclical component. The most widely used technique for extracting the (stochastic) trend from economic data is to make use of the filter that was developed by Hodrick and Prescott (1980). Tables (3.1) and (3.2) summarise the results that are provided in Kydland and Prescott (1982), where the moments for the data relate to the post war sample for the United States between 1950Q1 and 1979Q2.<sup>9</sup>

The autocorrelation coefficients in table (3.1) suggest that the level of persistence in output data is replicated by model. In addition, the standard deviations of the variables in the model are largely consistent with the data, where table (3.2) suggests that the volatility of consumption is much less than the

<sup>9</sup>In this example, the autocorrelations from the filtered data are compared to the simulated data that is provided by the model.

	AC (1)	AC (2)	AC (3)	AC (4)	AC (5)	AC (6)
Model	0.71	0.45	0.28	0.19	0.02	-0.13
Data	0.84	0.57	0.27	-0.01	-0.20	-0.3

**Table 3.1:** Autocorrelation of Output - RBC model with technology shock

volatility that in investment, whilst the volatility in productivity is less than the volatility in employment. Furthermore, when considering the correlation of the model variables with output, we note once again, that almost all the variables are strongly pro-cyclical, as consumption, investment, employment, productivity and inventories all move with output.

	Standard Deviation		Correlation with $Y_t$	
	Model	Data	Model	Data
$Y_t$	1.8	1.8	-	-
$C_t$	0.63	1.3	0.94	0.74
$X_t$	6.45	5.1	0.8	0.71
$N_t$	1.05	2	0.93	0.85
Productivity	0.9	1	0.9	0.1
Inventories	0.89	1.7	-0.15	0.51

**Table 3.2:** Standard Deviation and Correlation with Output

### 3.2.5 Criticism of Kydland and Prescott (1982)

Whilst the methodological contribution of this paper is extremely impressive, there are a number of characteristics that are not replicated by the data. For example, most RBC models perform poorly when seeking to replicate the volatility of interest rate data. In addition, real interest rates are usually too pro-cyclical relative to the data, where they lead output in the model, whereas actual data suggests that interest rates should lag output (King and Watson, 1996).

Several authors have also criticized RBC models on the grounds that they don't seem particularly realistic (Summers, 1986). For example, to generate fluctuations that resemble those in the United States, one needs large, high frequency variation in the technology shock. Within the confines of this broad RBC structure, no other shock (e.g. government spending, preferences, monetary policy, etc.) can be the main driving force behind the data, although

adding other shocks could improve the overall correlations. Hence, there is a limited role for stabilisation policy through either monetary or fiscal policy.<sup>10</sup>

In addition, Summers (1986) notes that the use of a technology shock that is normally distributed is unappealing in the sense that there should be an equal amount of positive and negative technology shocks. There are also difficulties that surround the definition of negative technology shocks, which would be required to induce a recession or trough in the cycle.

Owing to these critiques, much of business cycle research since the 1980s has been involved in modifying the basic model to allow for other shocks to matter in a way that they can't in this model (e.g. incorporate meaningful monetary policy shocks that may affect the values of real variables). In addition, researchers also sought to generate better and more realistic mechanisms that would allow for smaller shocks (as opposed to large technology shocks) to produce observed business cycle behaviour.

### 3.3 Monetary Policy in a Flexible-Price Model

The influential work of Cooley and Hansen (1989) and others allowed for the incorporation of money into an RBC model. This model makes use of the popular Cash-In-Advance specification, which is discussed in Lucas and Stokey (1987), where agents are required to carry over money from the previous period to make their purchases. This allowed for early investigations into the effects changes in monetary stocks and prices, which are important facets of monetary policy (and were particularly relevant during this period of time).

Modern monetary policy is largely conducted with the aid of policy rules, such as those discussed in Woodford (2003). The essential features of a RBC model that may be used to investigate the effects of monetary policy, where there is perfect competition and flexible prices, is discussed below. In this model, it is not necessary to include the Cash-In-Advance specification and to simplify the description of the model, we abstract from this additional feature.<sup>11</sup>

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<sup>10</sup>The use of a logarithmic utility and Cobb-Douglas production functions may also give rise to low variations in hours worked, since the income effect cancels out the substitution effect. In an attempt to address this feature of the model, Hansen (1985) developed a specification that incorporated an indivisible labour function.

<sup>11</sup>Galí (2008) shows that the inclusion of money in the utility function does not affect the quantitative implications of the model, with respect to the effects of a shock on the real variables that have been included in this specification.

### 3.3.1 Households

Consider the household of a representative economic agent that seeks to maximise the following utility function,

$$\max_{C_t, N_t} U = E_0 \sum_{t=0}^{\infty} \beta^t [C_t, N_t] \quad (3.3.1)$$

where  $C_t$  and  $N_t$  refer to consumption and labour at time  $t$ . Total utility is represented by  $U$ , the subjective discount factor is  $\beta$ , and  $E_0$  is the expectations operator that is conditional on current information. As in most economic problems, it is assumed that utility increases with additional amounts of consumption (and less units of labour), but at decreasing rates. As such the marginal utility of consumption is positive and non-increasing, whilst the marginal disutility of labour is positive and non-decreasing.<sup>12</sup>

This maximisation problem is subject to the following budget constraint,

$$P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + H_t \quad (3.3.2)$$

where  $P_t$  is the price of consumable goods,  $B_t$  is the quantity of bonds, and  $Q_t$  represents the difference between the principle amount invested and the maturity value.<sup>13</sup> Wages are reflected by  $W_t$  and the nominal lump-sum transfers are represented by  $H_t$  (which could include profit distributions).<sup>14</sup>

In addition to the above constraint, we also assume that the household is subject to a solvency constraint that prevents it from engaging in Ponzi-type

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<sup>12</sup>Here we make use of a non-separable utility function. In the following section we make use of separable utility function, which features more prominently in applied research.

<sup>13</sup>Note that we could avoid the use of  $Q_t$  to rather express the budget constraint in terms of,  $B_t = (1+i_t)B_{t-1}$ , where  $i_t$  is the real interest rate. This would imply that  $Q_t = (1+i_t)^{-1}$ , such that the budget constraint could be written as,  $P_t C_t + B_t = W_t N_t + B_{t-1}(1+i_t) + H_t$ , which could be expressed as,  $P_t C_t + (B_t - B_{t-1}) = W_t N_t + (i_t)B_{t-1} + H_t$ . Alternatively, we could make use of the timing convention,  $P_t C_t + \Delta B_{t+1} = W_t N_t + (i_t)B_t + H_t$ . In this case, the left-hand side represents household expenditure, which incorporates consumption expenditure and savings. Similarly, the right-hand side represents household income, which incorporates wages from labour, interest on savings and lump-sum transfers that include the repatriation of profits back to households. The reason for making use of the expression  $Q_t$  in the above budget constraint, instead of  $i_t$ , is that we are going to log-linearise this model around the steady-state. In this case, taking the logarithmic transformation of the interest rate would be inappropriate, as interest rates are already expressed in percentage terms. To avoid the use of exponential terms in this expression, (i.e.  $\exp(1+i_t)$ ), we rather make use of  $Q_t$ . Therefore, when log-linearising  $Q_t$  we will derive an expression for the interest rate,

$$\log Q_t = \log \left( \frac{1}{\exp(1+i_t)} \right) = -(1+i_t)$$

No matter which budget constraint you are more familiar with, you should appreciate that they are essentially describing the same phenomena.

<sup>14</sup>Introducing prices for consumer goods allows us to consider the evolution of consumer price inflation.

schemes, where,

$$\lim_{T \rightarrow \infty} = E_t(B_T) \geq 0 \quad (3.3.3)$$

As such, households are not able to borrow indefinitely, but they are allowed to save (until the end of time). For the above utility function, budget constraint and the Ponzi constraint, the first order conditions could be found by setting up the Lagrangian,

$$\begin{aligned} \mathcal{L}_{C_t, N_t, B_t} = E_0 \sum_{t=0}^{\infty} \beta^t \{ [U_t^C, U_t^N] + \dots \\ \dots \lambda_t [W_t N_t + B_{t-1} + H_t - P_t C_t - Q_t B_t] \} \end{aligned}$$

with the accompanying partial derivatives,

$$\frac{\partial \mathcal{L}}{\partial C_t} = U_t^C - \lambda_t P_t = 0 \quad (3.3.4)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = U_t^N + \lambda_t W_t = 0 \quad (3.3.5)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -Q_t \lambda_t + \beta E_t \lambda_{t+1} = 0 \quad (3.3.6)$$

Note that (3.3.6) is possibly not as straightforward as the previous results, since we need to differentiate  $B_{t-1}$  with respect to  $B_t$ . which requires the use of  $\lambda_{t+1}$ . This expression would also need to be multiplied through by the subjective time preference rate  $\beta^1$ , since it relates to future behaviour.

Hence, for the expressions (3.3.4) and (3.3.5),

$$\begin{aligned} U_t^C = \lambda_t P_t \quad \text{and} \quad \lambda_t = -\frac{U_t^N}{W_t} \\ U_t^C = -\frac{U_t^N}{W_t} P_t \quad \text{s.t.} \quad \frac{U_t^C}{U_t^N} = -\frac{P_t}{W_t} \end{aligned}$$

This would leave us with the expression,

$$-\frac{U_t^N}{U_t^C} = \frac{W_t}{P_t} \quad (3.3.7)$$

where the left-hand-side represents the marginal rate of substitution between consumption and labour and the right-hand-side is the relative price of leisure.

Similarly, using (3.3.6) and (3.3.4),

$$Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \quad \text{where} \quad \lambda_t = \frac{U_t^C}{P_t} \quad \text{and} \quad \lambda_{t+1} = \frac{U_{t+1}^C}{P_{t+1}}$$



Substituting in for  $\lambda_t$  and  $\lambda_{t+1}$ ,

$$Q_t = \beta E_t \left[ \left( \frac{U_{t+1}^C}{P_{t+1}} \right) \frac{P_t}{U_t^C} \right]$$

and rearranging,

$$Q_t = \beta E_t \left[ \left( \frac{U_{t+1}^C}{U_t^C} \right) \frac{P_t}{P_{t+1}} \right] \quad (3.3.8)$$

where the right-hand-side represents the marginal rate of substitution between  $C_t$  and  $C_{t+1}$  to the relative price of consumption in period  $t$ , which is a function of interest rates (as provided on the left-hand-side).

### 3.3.2 Incorporating separable preferences

To derive a more meaningful result, we may invoke the use of a utility function with separable preferences for consumption and labour. The most popular specification was originally applied in King *et al.* (1988a), where they incorporate  $\sigma$ , which represents the inverse of the intertemporal elasticity of substitution in consumption. Similarly,  $\gamma$ , represents the inverse of the Frisch elasticity of labour supply.<sup>15</sup>

$$\max_{C_t, N_t} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] \quad (3.3.9)$$

Using the utility function in (3.3.9), the budget constraint in (3.3.2), and the Ponzi constraint in (3.3.3), the first order conditions could be found by setting up the Lagrangian,

$$\begin{aligned} \mathcal{L}_{C_t, N_t, B_t} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] + \dots \right. \\ \left. \dots \lambda_t [W_t N_t + B_{t-1} + H_t - P_t C_t - Q_t B_t] \right\} \end{aligned}$$

where we can derive the partial derivatives,

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} - \lambda_t P_t = 0 \quad (3.3.10)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -N_t^\gamma + \lambda_t W_t = 0 \quad (3.3.11)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -Q_t \lambda_t + \beta E_t \lambda_{t+1} = 0 \quad (3.3.12)$$

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<sup>15</sup>The specification of this utility function is consistent with the neoclassical growth model, where balanced growth occurs along the optimal steady-state. A full description and derivation of the implications of this utility function is provided in King *et al.* (2002).

Hence, for the expressions (3.3.10) and (3.3.11),

$$\begin{aligned} \frac{1}{C_t^\sigma} &= \lambda_t P_t \quad \text{and} \quad \lambda_t = \frac{N_t^\gamma}{W_t} \\ \frac{1}{C_t^\sigma} &= \frac{N_t^\gamma}{W_t} P_t \quad \text{s.t.} \quad \frac{1}{C_t^\sigma N_t^\gamma} = \frac{P_t}{W_t} \end{aligned}$$

This would leave us with,

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\gamma \quad (3.3.13)$$

Similarly, using (3.3.12) and the (3.3.10),

$$Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \quad \text{where} \quad \frac{1}{\lambda_t} = \frac{P_t}{C_t^{-\sigma}} \quad \text{and} \quad \lambda_{t+1} = \frac{C_{t+1}^{-\sigma}}{P_{t+1}}$$

Substituting in for  $\lambda_t$  and  $\lambda_{t+1}$ ,

$$Q_t = \beta E_t \left[ \left( \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right) \cdot \frac{P_t}{C_t^{-\sigma}} \right]$$

and rearranging,

$$Q_t = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \quad (3.3.14)$$

Hence, the use of the relative price of leisure (3.3.13) and the intertemporal rate of consumption (3.3.14), would summarise the optimal activities of households in this economic model. We may then derive log-linear expressions for these solutions as described in the appendix.<sup>16</sup> Hence, for (3.3.13),

$$w_t - p_t = \sigma c_t + \gamma n_t \quad (3.3.15)$$

where the lowercase letters,  $w_t$ ,  $p_t$ ,  $c_t$  and  $n_t$  refer to the log-linear counterparts of  $W_t$ ,  $P_t$ ,  $C_t$  and  $N_t$ , respectively. This expression could suggest that the quantity of labour is a function of the real wage and the level of consumption.

The log-linear expression for (3.3.14) could also be derived for the relationship that described intertemporal consumption for deviations from steady state,<sup>17</sup>

$$c_t \approx E_t[c_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t[\pi_{t+1}]) \quad (3.3.16)$$

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<sup>16</sup>See, §(B.3.6).

<sup>17</sup>See, §(B.3.5).

### 3.3.3 Firms

Assume that the representative firm has a type of Cobb-Douglas production function that is given by,

$$Y_t = A_t N_t^{1-\alpha} \quad (3.3.17)$$

where  $Y_t$  represents output and  $A_t$  represents the level of technology. The parameter  $\alpha$  is then used to describe the elasticity of labour. It is further assumed that technology shocks are fairly persistent, and as such, we make use of an AR(1) process to describe these features of the shock,

$$a_t = \rho^a a_{t-1} + \epsilon_t^a$$

where the distribution of the stochastic shock takes the form,  $\epsilon_t^a \sim [0, 1]$ . Each firm would then seek to maximise profits,  $\Pi_t$ , for a given price level and wage rate, such that,

$$\max_{\Pi_t} \quad \Pi_t = P_t Y_t - W_t N_t$$

To maximise this function, subject to the constraint in (3.3.17), formulate the Lagrangian as,

$$\mathcal{L}_{\Pi,t} = P_t Y_t - W_t N_t + \lambda_t (A_t N_t^{1-\alpha} - Y_t)$$

This expression allows one to derive the partial derivatives,

$$\frac{\partial \mathcal{L}_{\Pi,t}}{\partial Y_t} = P_t - \lambda_t = 0 \quad (3.3.18)$$

$$\frac{\partial \mathcal{L}_{\Pi,t}}{\partial N_t} = -W_t + \lambda_t (1 - \alpha) A_t N_t^{-\alpha} = 0 \quad (3.3.19)$$

Making use of (3.3.18) we can derive the expression,

$$\lambda_t = P_t$$

which can be substituted into (3.3.19), such that,

$$\begin{aligned} W_t &= P_t (1 - \alpha) A_t N_t^{-\alpha} \\ \therefore \frac{W_t}{P_t} &= (1 - \alpha) A_t N_t^{-\alpha} \end{aligned} \quad (3.3.20)$$

This would imply that the firm hires labour and makes use of the level of available technology, up to the point where marginal product equals the real wage. This expression can then be expressed in log-linear terms,<sup>18</sup>

$$w_t - p_t = a_t - \alpha n_t \quad (3.3.21)$$

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<sup>18</sup>See, §(B.3.7).

### 3.3.4 Monetary policy rules & shocks

One of the most insightful papers in the monetary policy literature is that of Taylor (1993). He suggested that during the Greenspan period, the stance of monetary policy could be explained by the output gap and a deviation in inflation from a 2% target.<sup>19</sup> The specification of the monetary policy reaction function has been modified slightly in more recent models and it may be argued that the central bank currently conducts policy by changing the nominal interest rate according to the following rule,

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) [\varrho_\pi \pi_t + \varrho_y \tilde{y}_t] + \epsilon_t^i \quad (3.3.22)$$

In this specification, the term  $\varrho_i$  is termed the smoothing parameter and usually takes on a value of 0.75, whilst  $\varrho_\pi$  and  $\varrho_y$  refer to the reaction of central bank to inflation and deviations from the output gap (which usually take values of approximately 1.5 and 0.5 for the United States). The monetary policy shock is then described by  $\epsilon_t^i$  that results from a sudden rise in the central bank interest rate. It is assumed that the distribution of this stochastic shock would take a form, where  $\epsilon_t^i \sim [0, 1]$ .<sup>20</sup>

Where we would look to incorporate an expression for an inflation *forecast* targeting central bank, then we could rewrite the above monetary policy rule as,

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) \{ \varrho_\pi E_t [\pi_{t+1}] + \varrho_y \tilde{y}_t \} + \epsilon_t^i$$

Note that the relationship between real and nominal interest rates is given by the expression,  $r_t = i_t - E_t [\pi_{t+1}]$ . In the RBC model, it is assumed that prices are perfectly flexible, and as such they will adjust to any shock to nominal interest rates, to leave real interest rates unchanged. This would imply that real interest rates are unaffected by changes to monetary policy, which is an important consideration that forms the basis of the New Keynesian framework.

### 3.3.5 Steady-state & technology shocks

The underlying presumption of these models is that the economic system is stable in that there are forces that will draw it to a point of general equilibria.

<sup>19</sup>This rule also takes on a normative perspective, as Taylor (1993) argued that it could be used as a general guiding principle to describe the degree to which central banks *should* change interest rates in response to changes in the inflation and the output gap. Indeed, Taylor (2009) has suggested that one of the contributing factors to the recent Global Financial Crisis, of 2007/8 was that the Federal Reserve Bank strayed too far from this rule.

<sup>20</sup>Since these expressions are linear, no further manipulation is necessary. Whilst the monetary policy rule that is provided in (Galí, 2008, ch. 2), differs slightly to that which is provided here, it captures many of the same features.

However, this system is also subject to various stochastic shocks which will temporarily move the system away from these points of equilibria. After describing the behaviour of household, firm and central bank one is able to derive the relationships that describe the steady-state and the subsequent effect of shocks.

To identify the steady-state, we would need to set the log-linear expressions off against one another. In this model the goods market will clear when all output is consumed, as we have not included aggregate demand components like investment, government purchases, or net exports.<sup>21</sup> Therefore,

$$y_t = c_t \quad (3.3.23)$$

Making use of the Cobb-Douglas aggregate demand function in (3.3.17), we may describe output in log-linear terms,<sup>22</sup>

$$y_t = a_t + (1 - \alpha)n_t \quad (3.3.24)$$

The equilibrium levels of employment and output could be derived by combining the log-linear expressions (3.3.15) and (3.3.21), with (3.3.23) and (3.3.24).

$$\begin{aligned} w_t - p_t &= w_t - p_t \\ \therefore \sigma c_t + \gamma n_t &= a_t - \alpha n_t \\ \therefore \sigma [a_t + (1 - \alpha)n_t] + \gamma n_t &= a_t - \alpha n_t \\ \therefore \sigma(1 - \alpha)n_t + \gamma n_t + \alpha n_t &= a_t - \sigma a_t \\ \therefore [\sigma(1 - \alpha) + \gamma + \alpha] n_t &= (1 - \sigma)a_t \\ \therefore n_t &= \frac{1 - \sigma}{\sigma(1 - \alpha) + \gamma + \alpha} a_t \end{aligned} \quad (3.3.25)$$

This expression may be summarised to show that the equilibrium levels of employment is a function of technology,

$$n_t = \psi^{na} a_t \quad (3.3.26)$$

where,  $\psi^{na} = \frac{1 - \sigma}{\sigma(1 - \alpha) + \gamma + \alpha}$ .

Then by substituting this expression into (3.3.24), it may be shown that for output,

$$\begin{aligned} y_t &= a_t + (1 - \alpha)n_t \\ &= a_t + (1 - \alpha) \frac{(1 - \sigma)}{\sigma(1 - \alpha) + \gamma + \alpha} a_t \\ &= \frac{[\sigma(1 - \alpha) + \gamma + \alpha] + [(1 - \alpha)(1 - \sigma)]}{\sigma(1 - \alpha) + \gamma + \alpha} a_t \\ &= \frac{1 + \gamma}{\sigma(1 - \alpha) + \gamma + \alpha} a_t \end{aligned}$$

<sup>21</sup>See, §(B.3.2) for the log-linearisation.

<sup>22</sup>See, §(B.3.1).

Once again, this could be summarised to show that the equilibrium level of output is also a function of technology,

$$y_t = \psi^{ya} a_t \quad (3.3.27)$$

where,  $\psi^{ya} = \frac{1+\gamma}{\sigma(1-\alpha)+\gamma+\alpha}$ .

We may then derive an expression for the implied real interest rate,  $r_t$ , where  $r_t = i_t - E_t[\pi_{t+1}]$ , using (3.3.16) and (3.3.23).

$$\begin{aligned} y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \rho - E_t[\pi_{t+1}]) \\ \therefore \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}]) &= E_t[y_{t+1}] - y_t + \frac{1}{\sigma}\rho \\ \therefore i_t - E_t[\pi_{t+1}] &= \sigma(E_t[y_{t+1}] - y_t) + \rho \\ \therefore r_t &= \sigma E_t[\Delta y_{t+1}] + \rho \end{aligned}$$

Using the expression for  $y_t$  in (3.3.27), we may write the above expression as,

$$r_t = \rho + \sigma \psi^{ya} E_t[\Delta a_{t+1}] \quad (3.3.28)$$

Where the real interest rate is also a function of technology.

Then finally, the equilibrium real wage  $\omega_t = w_t - p_t$  is derived from combining (3.3.21) and (3.3.25).

$$\begin{aligned} \omega_t &= a_t - \alpha n_t \\ &= a_t - \alpha \left[ \frac{(1-\sigma)}{\sigma(1-\alpha)+\gamma+\alpha} a_t \right] \\ &= \left[ \frac{\sigma(1-\alpha)+\gamma+\alpha}{\sigma(1-\alpha)+\gamma+\alpha} \right] a_t \dots \\ &\quad \dots - \left[ \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)+\gamma+\alpha} a_t \right] \\ &= \left[ \frac{\{\sigma(1-\alpha)+\gamma+\alpha\} - \{\alpha(1-\sigma)\}}{\sigma(1-\alpha)+\gamma+\alpha} \right] a_t \\ &= \left[ \frac{\sigma+\gamma}{\sigma(1-\alpha)+\gamma+\alpha} \right] a_t \end{aligned}$$

which may be summarised as,

$$\omega_t = \psi^{wa} a_t \quad (3.3.29)$$

where,  $\psi^{wa} = \frac{\sigma+\gamma}{\sigma(1-\alpha)+\gamma+\alpha}$ .

In this simple model, output, employment and the real wage rate rise with productivity, when  $\psi^{ya}, \psi^{na}, \psi^{wa} > 0$ . Hence, in this example technology is

the only real driving force, although it would be relatively straightforward to introduce other real driving forces like variations in government purchases. In addition, note that the equilibrium dynamics of employment, output, and the real interest rate are all largely driven by the technology shock. Hence, these equilibrium values are not affected by monetary policy, which is implemented through changes to the nominal interest rate. In other words, monetary policy is neutral with respect to these real variables.

### 3.4 Conclusion

This chapter introduced a simple economic model that can be used to describe the behaviour of representative agents for the households and firms. This framework may be used to investigate the effect of a technology shock on the business cycle, where after characterising the model of Kydland and Prescott (1982), we note that a shock to technology could induce behaviour that is consistent with certain features of the business cycle.

The model is then extended to incorporate a role for the central bank. This model makes use of a number of log-linearised expressions for which we are able to derive a solution to a simple forward-looking rational expectations problem. After deriving analytical solutions for the dynamic equilibria, we are then able to show that deviations from the steady state for output, inflation and real interest rates are largely driven by technology shocks in this model. In addition, it was noted that when prices are perfectly flexible, any changes to the nominal interest rate will result in a similar (immediate) change in inflation, to leave the real interest rate unaffected. This would imply that monetary policy is neutral, which is not consistent with the characterisation of the business cycle in the previous chapter. In addition, in this case the nominal interest rate and rate of inflation are not determined by any of the real variables.

In the following chapter, we consider the introduction of price rigidities that allow for cases where changes in nominal interest rates influence the real variables in the model.

## Chapter 4

# New Keynesian Modelling



## 4.1 Introduction

The canonical New Keynesian model introduces a number of frictions, particularly with regards to the way in which prices adjust. This exposition assumes imperfect competition in the goods market, where firms produce differentiated goods and reset a proportion of the prices of these goods during each period, using the staggered price setting mechanism of Calvo (1983).<sup>1</sup>

An important feature of the New Keynesian model is that it allows for cases where changes to the short-term nominal interest rate are not matched by a simultaneous changes to expected inflation. This would imply that a change in monetary policy may result in temporary variations in the real interest rate, which would effect other real variables (such as output).<sup>2</sup>

These models are largely based on three broad relations. The *first* describes aggregate demand, where output is determined by demand, and demand depends in turn on anticipations of both past as well as future output, as well as *ex ante* real interest rates. The *second* contains an expression for the Phillips-curve (for the supply side of the model), in which inflation depends on both past and expected future inflation, as well as the marginal costs of firms. Then *lastly*, a monetary policy rule is used to describe monetary policy, which is conducted by adjusting the nominal interest rate in response (largely) to changes in output and inflation. These relationships allow for the explicit incorporation of the economic-agents expectations of future activity, which could play an important role when describing current economic activity.<sup>3</sup>

Once the model has been derived, we are able to illustrate the importance of these price rigidities with the aid of models that have been simulated with various calibrated parameter values. These results show how it is possible to structure a model that allows for the non-neutrality of monetary policy.

The use of additional extensions to the household sector are then considered in subsequent sections of this chapter. The first of these extensions incorporates a description for money in the model, by making provision for

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<sup>1</sup>Early variants of these models were termed real business cycle (RBC) models with monopolistic competition, as in Galí (2002); whilst more recently, these models are also referred to as sticky-price models.

<sup>2</sup>In the case of South Africa, Gupta and Steinbach (2013) show that the addition of price rigidities, where firms are constrained by the amount of times (or frequency) with which they change prices, improves the explanatory power of these models.

<sup>3</sup>The seminal papers that lead to the development of this literature include Clarida *et al.* (1999) and Ireland (2004*b*). Excellent textbook references include Woodford (2003), Galí (2008), and Walsh (2010).

it in the utility function.<sup>4</sup> Thereafter, the subsequent extension illustrates the importance incorporating habits in consumption, where the agent derives utility from the level of consumption, relative to the past level of consumption.

## 4.2 Households

In this case, households once again seek to maximise consumption and minimise labour, but they now consume a selection of  $j$  goods, which spans over the interval  $[0, 1]$ . Hence, we now define the consumption index,  $C_t$ , by making use of an integral for  $j$  goods,

$$C_t = \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad (4.2.1)$$

where,  $C_{j,t}$  is the quantity of good  $j$  consumed by each household and  $\epsilon$  is the marginal rate of substitution between goods. In this example we follow Galí (2008) where the behaviour of the household is used to show how we can derive an expression that relates the price index for aggregate goods to the price of all the different  $j$  goods in the economy.<sup>5</sup> In the final part of this analysis, we summarise these expressions for the representative household, to arrive at the behavioural equations that may be incorporated in the final model.

### 4.2.1 Deriving a price index for goods

Turning our attention to the budget constraint, we would now also need to make allowance for the range of  $j$  goods. Hence, we include the integral,  $\int_0^1 P_{j,t} C_{j,t} dj$ , to describe consumption expenditures in the expression,

$$\int_0^1 P_{j,t} C_{j,t} dj + Q_t B_t = W_t N_t + B_{t-1} + H_t \quad (4.2.2)$$

Once again, we are also going to assume the solvency constraint for no-Ponzi conditions,  $\lim_{T \rightarrow \infty} E_t(B_T) \geq 0$ . To simplify notation and focus on the relevant variables, we assume  $Z_t = Q_t B_t - W_t N_t - B_{t-1} - H_t$ .

This allows us to set up the Lagrangian for the maximisation of consumption,

$$\mathcal{L}_{C_{j,t}} = \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda_t \left( \int_0^1 P_{j,t} C_{j,t} dj - Z_t \right) = 0 \quad (4.2.3)$$

---

<sup>4</sup>As an alternative, one may wish to include money by constructing a *Cash-in-Advance* structure that is described in Walsh (2010), amongst others.

<sup>5</sup>Alternatively, one could make use of separate roles for intermediate and final goods producers, which may be used to derive equivalent expressions.

Note that the integrals in the above expression are with respect to  $j$  differentiated goods. To derive the first order condition, we need to find the partial derivative with respect to  $C_{j,t}$ , which would be set equal to zero.

$$\begin{aligned}
 & \frac{\epsilon}{\epsilon-1} \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon-1}{\epsilon} C_{j,t}^{\frac{\epsilon-1}{\epsilon}-1} = \lambda_t [P_{j,t}] \\
 \therefore & \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}-\frac{\epsilon-1}{\epsilon}} C_{j,t}^{\frac{\epsilon-1}{\epsilon}-\frac{\epsilon}{\epsilon}} = \lambda_t [P_{j,t}] \\
 \therefore & \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{1}{\epsilon-1}} C_{j,t}^{-\frac{1}{\epsilon}} = \lambda_t [P_{j,t}] \\
 \therefore & \left( \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{1}{\epsilon}} C_{j,t}^{-\frac{1}{\epsilon}} = \lambda_t [P_{j,t}] \\
 \therefore & C_t^{\frac{1}{\epsilon}} C_{j,t}^{-\frac{1}{\epsilon}} = \lambda_t [P_{j,t}] \tag{4.2.4}
 \end{aligned}$$

where we are able to move from the third to the final line by making use of the fact that the original consumption index in (4.2.1) is given as

$$C_t = \left[ \int_0^1 C_{j,t}^{1-\frac{1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

Using the expression for the maximisation of  $j$  goods in (4.2.4), we could similarly write for  $k$  goods,

$$\begin{aligned}
 C_t^{\frac{1}{\epsilon}} C_{k,t}^{-\frac{1}{\epsilon}} &= \lambda_t [P_{k,t}] \\
 \text{where } \lambda_t &= \frac{C_t^{\frac{1}{\epsilon}} C_{k,t}^{-\frac{1}{\epsilon}}}{P_{k,t}}
 \end{aligned}$$

Substituting this expression for  $\lambda_t$  into (4.2.4) we have,<sup>6</sup>

$$\begin{aligned}
 C_t^{\frac{1}{\epsilon}} C_{j,t}^{-\frac{1}{\epsilon}} &= \frac{C_t^{\frac{1}{\epsilon}} C_{k,t}^{-\frac{1}{\epsilon}}}{P_{k,t}} [P_{j,t}] \\
 \therefore C_{j,t}^{-\frac{1}{\epsilon}} &= \frac{C_{k,t}^{-\frac{1}{\epsilon}}}{P_{k,t}} [P_{j,t}] \\
 \therefore C_{j,t}^{-\frac{1}{\epsilon}} &= \frac{P_{j,t}}{P_{k,t}} [C_{k,t}^{-\frac{1}{\epsilon}}] \\
 \therefore \left( C_{j,t}^{-\frac{1}{\epsilon}} \right)^{-\epsilon} &= \left( \frac{P_{j,t}}{P_{k,t}} [C_{k,t}^{-\frac{1}{\epsilon}}] \right)^{-\epsilon} \\
 \therefore C_{j,t} &= \left[ \frac{P_{j,t}}{P_{k,t}} \right]^{-\epsilon} C_{k,t}
 \end{aligned}$$

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<sup>6</sup>Note, these variables are eventually all going to be log-linearised so there should be no problem with raising the expression by an exponent.

Now, at the point where consumption for  $k$  goods is maximised, it will be the case that the budget constraint is effectively spent,  $\int_0^1 P_{k,t} C_{k,t} dj - Z_t = 0$ . Similarly, for the aggregate consumption index,  $C_t$ , we would have,  $P_t C_t - Z_t = 0$ . Hence at the point of optimal consumption it would be the case that,  $\int_0^1 P_{k,t} C_{k,t} dj = P_t C_t$ . This allows us to substitute  $P_t C_t$  into expression (4.2.5), such that,

$$C_{j,t} = \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon} C_t \quad (4.2.5)$$

This expression suggests that the demand for each good depends negatively on its relative price and positively on aggregate consumption. Since  $\epsilon$  is the marginal rate of substitution between goods, as  $\epsilon \rightarrow 1$  the goods are perfect substitutes as a small change in  $P_{j,t}$  will result in large changes in the consumption of that good. This would be the case of perfect competition.<sup>7</sup>

Now since it is given, that at the optimal consumption point,

$$\int_0^1 P_{j,t} C_{j,t} dj = P_t C_t \quad (4.2.6)$$

Taking the aggregate measures to the left-hand-side and substituting in for  $C_{j,t}$  we have,

$$P_t C_t = \int_0^1 P_{j,t} \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon} C_t dj \quad (4.2.7)$$

We can then take out of the integral the variables that are not indexed by  $j$  on the right hand side (since we are primarily interested in relating  $P_t$  to  $P_{j,t}$ ). Hence,

$$\begin{aligned} P_t C_t &= \int_0^1 P_{j,t} \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon} C_t dj \\ \therefore P_t C_t &= C_t P_t^\epsilon \int_0^1 P_{j,t}^{1-\epsilon} dj \\ \therefore P_t &= P_t^\epsilon \int_0^1 P_{j,t}^{1-\epsilon} dj \\ \therefore P_t^{1-\epsilon} &= \int_0^1 P_{j,t}^{1-\epsilon} dj \end{aligned}$$

Therefore, the aggregate price index could be expressed as,<sup>8</sup>

$$P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \quad (4.2.8)$$

<sup>7</sup>The marginal rate of substitution is sometimes referred to as being the elasticity of consumption demand.

<sup>8</sup>This is the expression that we wanted to derive. It will be used later on when we discuss the firms price setting behaviour.

## 4.2.2 Deriving optimal levels of consumption

Now to return to the problem of deriving optimal levels of consumption and labour for the household. Using the argument in equation (4.2.6) for the point of optimal consumption, we may substitute  $P_t C_t$  into the budget constraint in (4.2.2), such that,

$$P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + H_t \quad (4.2.9)$$

This is equivalent to the expression for the budget constraint that we had in the RBC model that we previously encountered. Therefore, when combined with the utility function with separable preferences, we have,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

We are then able to derive the optimal level of consumption/savings and labour supply as before,

$$\begin{aligned} \frac{W_t}{P_t} &= C_t^\sigma N_t^\gamma \\ Q_t &= \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \end{aligned} \quad (4.2.10)$$

From these expressions we may derive the log-linear expressions,

$$w_t - p_t = \sigma c_t + \gamma n_t \quad (4.2.11)$$

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}]) \quad (4.2.12)$$

This feature of the model is not surprising since the household is effectively a price taker and has little influence over the way in which prices are set in the model economy.

## 4.3 Price Setting Behaviour

Following Calvo (1983), the parameter  $\theta$  is used to describe the proportion of goods for which the current price,  $P_t$ , is equal to that of the previous period (i.e.  $P_{t-1}$ ).<sup>9</sup> Hence, as  $\theta \rightarrow 0$  we have perfectly flexible prices. Similarly, this expression implies that each firm has the probability,  $1 - \theta$ , of being able to change the price of the goods that are produced. In the notation that follows, we use  $P_t^*$  for those prices that have been changed. Therefore, given the aggregate price index that was derived in (4.2.8),

$$P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$$

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<sup>9</sup>These producers face sticky-prices.

We may now include expressions that allow us to distinguish between the  $j$  goods for those that have the old price,  $P_{t-1}$ , and those that have the new price,  $P_t^*$ .

$$P_t = [\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

which is equivalent to,

$$P_t^{1-\epsilon} = \theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}$$

After dividing through by  $(P_{t-1})^{1-\epsilon}$ , we are able to get a measure for the change in prices on the left-hand side. We summarise this measure using the notation,  $\Phi_t = \frac{P_t}{P_{t-1}}$ , and similarly so for  $\Phi_t^* = \frac{P_t^*}{P_{t-1}}$ . Hence we are able to write this expression as,

$$\begin{aligned} \left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} &= \frac{\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}}{(P_{t-1})^{1-\epsilon}} \\ \therefore \Phi_t^{1-\epsilon} &= \theta + (1-\theta)[\Phi_t^*]^{1-\epsilon} \end{aligned}$$

For which the log-linear expression is,<sup>10</sup>

$$(1-\epsilon)(\Phi_t - 1) \approx (1-\theta)(1-\epsilon)(\Phi_t^* - 1)$$

With the use of the expression for inflation,  $\pi_t = (P_t - P_{t-1})/P_{t-1}$  we may then derive,

$$\begin{aligned} \pi_t &= \frac{P_t}{P_{t-1}} - \frac{P_{t-1}}{P_{t-1}} \\ &= \Phi_t - 1 \end{aligned}$$

Therefore,  $\Phi_t = (1 + \pi_t)$  and  $\Phi_t^* = (1 + \pi_t^*) = (p_t^* - p_{t-1})$ . This allows us to use the above log-linear expression to describe the inflationary process as,

$$\begin{aligned} (1-\epsilon)\pi_t &= (1-\theta)(1-\epsilon)\pi_t^* \\ \therefore \pi_t &= (1-\theta)\pi_t^* \\ &= (1-\theta)(p_t^* - p_{t-1}) \end{aligned} \tag{4.3.1}$$

This expression suggests that inflation arises from those firms that are able to set new prices in the economy.

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<sup>10</sup>See, §(B.3.9).

## 4.4 The Behaviour of the Firm

Once again, it is assumed that all firms use the same technology, as represented by the production function,

$$Y_{j,t} = A_t N_{j,t}^{1-\alpha}$$

We have changed the notation slightly in this case, as the firms now produce an array of  $j$  goods. Hence output in the current period is given as  $Y_{j,t}$ , which is derived from the labour that were employed in the production of  $j$  goods. In this case, technology is the same across all firms.

Since firms seek to maximise profit, they will change the price of goods (i.e. choose  $P_t^*$ ) to maximise the difference between their revenue (i.e. the price that the firm would receive multiplied by the number of goods that are produced/consumed) and their production costs (i.e. the average cost of producing a good multiplied by the number of goods that are produced/consumed).

When setting the price for the current period, firms would also like to take into account future profitability and as such their optimisation problem would take on a forward looking perspective, where future profitability is considered to be slightly less important than current profitability. In this case, we make use of the household's subjective time discount factor,  $Q_{t,t+k}$ , since the household effectively owns the firms. When setting prices they also need to take into account the probability of being stuck with the current price,  $k$  periods ahead, which may be summarised by  $\theta^k$ .

Hence, when choosing to set prices in a particular period, the objective function of a firm is to set its price,  $P_t^*$ , that would maximise current and future profitability. This could be derived from the revenue function,

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_{t+k} Y_{t+k|t})]$$

where  $\Psi_{t+k}(\cdot)$  is used to summarise the cost function, and  $Y_{t+k|t}$  denotes output in period  $t+k$  for a firm that last reset prices in period  $t$ .

This objective function is subject to the constraint from consumer demand, which describes the relationship between output, consumption, prices, and the marginal rate of substitution in consumption,<sup>11</sup>

$$\begin{aligned} Y_{t+k|t} &= \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \\ &= (P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k} \end{aligned} \quad (4.4.1)$$

---

<sup>11</sup>Note that in this case we are primarily interested in the goods,  $Y_{t+k|t}$ , that are produced at the new price,  $P_t^*$ . Note that this expression is very similar to (4.2.5), which describes the substitution effects between two goods.

After inserting the constraint into the objective function we can solve this expression as if it were an unconstrained optimisation problem. The Lagrangian would then take the form,

$$\begin{aligned}\mathcal{L}_{P_t^*} &= E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} (P_t^* \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\} - \Psi_{t+k} \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\})] \\ &= E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} (\{(P_t^*)^{1-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\} - \Psi_{t+k} \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\})] \\ &= 0\end{aligned}$$

with the accompanying partial derivative,

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_t^*} &= E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} (1 - \epsilon \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\} \dots \\ &\quad - \psi_{t+k|t} \{-\epsilon (P_t^*)^{-\epsilon-1} P_{t+k}^{\epsilon} C_{t+k}\})] = 0 \\ &= E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} (1 - \epsilon \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\} \dots \\ &\quad + \epsilon \psi_{t+k|t} (P_t^*)^{-1} \{(P_t^*)^{-\epsilon} P_{t+k}^{\epsilon} C_{t+k}\})] = 0 \\ &= E_t \sum_{k=0}^{\infty} \theta^k \left[ Q_{t,t+k} \left( \{Y_{t+k|t}\} - \frac{\epsilon}{\epsilon-1} \psi_{t+k|t} (P_t^*)^{-1} \{Y_{t+k|t}\} \right) \right] = 0 \\ &= E_t \sum_{k=0}^{\infty} \theta^k \left[ Q_{t,t+k} Y_{t+k|t} \left( P_t^* - \frac{\epsilon}{\epsilon-1} \psi_{t+k|t} \right) \right] = 0\end{aligned}\tag{4.4.2}$$

where  $\psi_{t+k|t}$  is used to reflect the nominal marginal costs in period  $t+k$  for a firm that last set its price in period  $t$ , where  $\psi_{t+k|t} = \Psi'_{t+k}(Y_{t+k|t})$ . This expression could then be summarised to provide a description for  $P_t^*$ ,<sup>12</sup>

$$E_t \sum_{k=0}^{\infty} \theta^k [P_t^*] = \frac{\epsilon}{\epsilon-1} \frac{E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} Y_{t+k|t} (\psi_{t+k|t})]}{E_t \sum_{k=0}^{\infty} \theta^k [Q_{t,t+k} Y_{t+k|t}]}\tag{4.4.3}$$

where  $P_t^*$  is a function of the marginal rate of substitution and the marginal costs,  $\psi_{t+k}$ . As previously noted, the marginal costs would be influenced by the wages and technology, which are identical for all firms. Hence, when marginal costs increase, this expression would suggest that the optimal price will also increase. In the limiting case, where we tend towards fully-flexible prices (i.e. when  $\theta \rightarrow 0$ ); this condition would represent the familiar optimal price-setting condition where there are no price rigidities,

$$P_t^* = \frac{\epsilon}{\epsilon-1} \psi_{t|t}$$

<sup>12</sup>We are able to remove  $P_t^*$  from the brackets as it is not indexed by  $k$ .



This allows us to interpret  $\frac{\epsilon}{\epsilon-1}$  as the desired markup, in the absence of constraints on the price adjustment mechanism. Note also that in the steady-state, where  $Q_{t,t+k}$  and  $Y_{t+k}$  take on constant values, then one may define the relationship,  $\bar{\psi} = \frac{\epsilon-1}{\epsilon} \bar{P}^*$ .

#### 4.4.1 Log-linearisation

After specifying the way in which firms set prices, we are able to change the prices of certain goods, we are then able to log-linearise the optimal condition that is given by (4.4.2), around the steady-state of zero inflation. Hence, it would be the case that at the steady-state,  $\frac{P_t^*}{P_{t-1}} = \frac{P_t^*}{P_t} = \frac{P_t}{P_{t+k}} = \frac{P_{t+k}}{P_{t-1}} = 1$ . In addition, it is also usually more convenient to define prices in terms of inflation, and as such we divide through by  $P_{t-1}$ , such that the first order condition may be expressed as,

$$E_t \sum_{k=0}^{\infty} \theta^k \left[ Q_{t,t+k} Y_{t+k|t} \left( \frac{P_t^*}{P_{t-1}} - \frac{\epsilon}{\epsilon-1} \frac{\psi'_{t+k|t}}{P_{t-1}} \right) \right] = 0$$

Multiplying the last terms by  $P_{t+k}/P_{t+k}$  and rearranging, where real marginal costs are equivalent to nominal marginal costs when they are divided through by the relevant price index,  $MC_{t+k|t}^r = \psi_{t+k|t}/P_{t+k}$ .<sup>13</sup> Furthermore, after denoting  $\Phi_{t,t+k} = P_{t+k}/P_t$ , we are then able to write the above condition as,

$$E_t \sum_{k=0}^{\infty} \theta^k \left[ Q_{t,t+k} Y_{t+k|t} \left( \frac{P_t^*}{P_{t-1}} - \frac{\epsilon}{\epsilon-1} MC_{t+k|t}^r \Phi_{t-1,t+k} \right) \right] = 0$$

After substituting the Euler equation (4.2.10), for  $Q_t = \beta^k [C_{t+k}^{-\sigma} C_t^{\sigma} P_t P_{t+k}^{-1}]$ , and after making use of,  $\Phi_t^* = P_t/P_{t-1}$ , we have an expression,

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ C_{t,t+k}^{-\sigma} C_t^{\sigma} P_t P_{t,t+k}^{-1} Y_{t+k|t} \left( \Phi_t^* - \frac{\epsilon}{\epsilon-1} MC_{t+k|t}^r \Phi_{t-1,t+k} \right) \right] = 0$$

For which we have the log-linear expression,<sup>14</sup>

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\pi_t^*] &= E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{mc}_{t+k|t} + \pi_{t-1,t+k}] \\ E_t \sum_{k=0}^{\infty} (\theta\beta)^k [p_t^* - p_{t-1}] &= E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}] \end{aligned}$$

<sup>13</sup>This would relate to the real marginal costs that arise in period  $t+k$ , for the firm that last set prices in period  $t$ .

<sup>14</sup>See, §(B.3.10)

To remove the infinite summation operator from the term on the left-hand side, we need to make use of the following property of discounted sums, which is described in §(A.1.1).

Hence, we can then rewrite the previous expression as,

$$\begin{aligned} \frac{p_t^* - p_{t-1}}{1 - \theta\beta} &= E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}] \\ \therefore p_t^* - p_{t-1} &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}] \quad (4.4.4) \end{aligned}$$

where  $\widehat{mc}_{t+k|t}$  represents the log deviation of marginal costs from its steady-state, such that  $\widehat{mc}_{t+k|t} = mc_{t+k|t} - \bar{mc}$ . The use of  $\mu = -\bar{mc}$ , allows us to derive the expression,

$$\begin{aligned} p_t^* &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [mc_{t+k|t} - \bar{mc} + p_{t+k}] \\ \therefore p_t^* &= \mu + (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [mc_{t+k|t} + p_{t+k}] \end{aligned}$$

As we already have a relationship between the new prices charged by firms and marginal costs in (4.4.3), which at the steady-state is  $\bar{\psi} = \frac{\epsilon-1}{\epsilon}$ . Then after expressing  $\bar{\psi} = (\epsilon - 1/\epsilon)\bar{P}^*$ , in terms of real marginal costs and given that at the steady-state,  $P_t^*/P_{t+k} = 1$ , we are then able to derive  $\bar{mc} = \log \frac{\epsilon-1}{\epsilon}$ . For values of  $\frac{\epsilon}{\epsilon-1}$  that are close to one, this is approximately equal to  $(\frac{\epsilon}{\epsilon-1} - 1)$ , which may be interpreted as the net markup.

Where we made use of the expression  $MC_{t=k|t}^r = \psi_{t+k|t}/P_{t+k}$ , then we are able to describe nominal marginal costs in log-linear terms as  $mc_{t+k|t} + p_{t+k}$ . Hence, firms that are able to reset prices in period  $t$ , will choose a price that corresponds to the desired markup over a weighted average of current and expected nominal marginal costs, where the weights are influenced by the probability of being able to charge a new price during each horizon and the subjective discount factor.

#### 4.4.2 Goods Market

As we had before, the goods market will clear when the economy is at a point of equilibrium. In the case of differentiated goods, we would have,

$$Y_{j,t} = C_{j,t}$$

Therefore, since it is given that

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

it would imply that in the case of all goods,

$$Y_t = C_t \quad (4.4.5)$$

Using this condition, we could substitute into the log-linear expression for the household's consumption Euler equation,<sup>15</sup>

$$y_t = E_t [y_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}])$$

### 4.4.3 Labour Market

As in the case of other markets, labour can also be differentiated, where market clearing conditions in the labour market require that the broad index is an aggregate of differentiated labour supply,

$$N_t = \left[ \int_0^1 N_{j,t} dj \right]$$

We can then use the production function of the firm to substitute in for  $N_{j,t}$

$$\begin{aligned} Y_{j,t} &= A_t N_{j,t}^{1-\alpha} \\ \therefore N_{j,t} &= \left( \frac{Y_{j,t}}{A_t} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

This would leave us with the expression for the broad employment index,

$$N_t = \int_0^1 \left( \frac{Y_{j,t}}{A_t} \right)^{\frac{1}{1-\alpha}} dj \quad (4.4.6)$$

To express this condition in terms of  $Y_t$ , we could make use of the households consumption index with the goods markets, where,

$$\begin{aligned} C_{j,t} &= \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon} C_t \\ \text{s.t. } Y_{j,t} &= \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon} Y_t \end{aligned}$$

We could then substitute in for  $Y_{j,t}$  to derive,

$$N_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} dj$$

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<sup>15</sup>Note that one could have made use of the consumption Euler expression if you also include the market clearing condition,  $c_t = y_t$ .

where the expression  $\int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} dj$  is a measure of relative price dispersion. We can then log-linearise this expression to derive.

$$(1 - \alpha)n_t = y_t - a_t + d_t$$

where  $d_t$  is used to summarise for  $(1 - \alpha) \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} dj$ .

It can be shown that in the neighbourhood of a zero inflation steady-state,  $d_t \sim 0$ , when taking a first-order Taylor series approximation to linearise this function. Therefore, after rearranging, this allows us to write the expression for the labour market,

$$y_t = a_t + (1 - \alpha)n_t \quad (4.4.7)$$

#### 4.4.4 Marginal Costs

For the production function,  $Y_{j,t} = A_t N_{j,t}^{1-\alpha}$ , we can derive the marginal productivity of labour,

$$\frac{\partial Y_{j,t}}{\partial N_{j,t}} = A_t (1 - \alpha) N_{j,t}^{-\alpha} \quad (4.4.8)$$

After log-linearising this expression we would be left with,

$$mpn_t = a_t - \alpha n_t \quad (4.4.9)$$

After including the employment index (4.4.6), we may then define the total cost function for the firm, which is dependant on wages and the level of employment,

$$W_t N_t = W_t \int_0^1 \left( \frac{Y_{j,t}}{A_t} \right)^{\frac{1}{1-\alpha}} dj$$

To calculate the marginal costs for an additional amount of output, we would then take the derivative,

$$\begin{aligned} \frac{\partial W_t N_t}{\partial Y_{j,t}} &= W_t \frac{1}{(1 - \alpha)} \{A_t\}^{\frac{-1}{1-\alpha}} \int_0^1 \{Y_{j,t}\}^{\frac{1}{1-\alpha}-1} dj \\ &= W_t \frac{1}{(1 - \alpha)} \{A_t\}^{\frac{-1}{1-\alpha}} \{A_t\}^{\frac{1-\alpha}{1-\alpha}} \{A_t\}^{-1} \int_0^1 \{Y_{j,t}\}^{\frac{\alpha}{1-\alpha}} dj \\ &= W_t \frac{1}{(1 - \alpha)} \{A_t\}^{\frac{-\alpha}{1-\alpha}} \{A_t\}^{-1} \int_0^1 \{Y_{j,t}\}^{\frac{\alpha}{1-\alpha}} dj \\ &= W_t \frac{1}{(1 - \alpha)} \{A_t\}^{-1} \int_0^1 \left\{ \frac{Y_{j,t}}{A_t} \right\}^{\frac{\alpha}{1-\alpha}} dj \\ &= W_t \frac{1}{(1 - \alpha)} \frac{1}{A_t} N_t^\alpha \\ &= W_t (A_t (1 - \alpha) N_{j,t}^{-\alpha})^{-1} \end{aligned} \quad (4.4.10)$$

Note that in this case we are once again looking to differentiate with respect to  $Y_{j,t}$ , whilst the integral is only with respect to  $j$ . To move to the second last line we substituted in for the employment index (4.4.6), once again. Given this expression, we can see how it is related to the marginal production function in (4.4.8), which is used to describe the individual firms marginal costs in terms of the economy's average costs. Hence, the effective marginal costs of the firm are positively influenced by the wage rate and negatively influenced by the marginal productivity of labour.

We can then divide through by prices, to define the relationship between real marginal costs and the marginal productivity of labour,

$$\begin{aligned} MC_t &= \frac{W_t}{P_t} (A_t(1-\alpha)N_{j,t}^{-\alpha})^{-1} \\ &= \frac{W_t}{P_t} (MPN_t)^{-1} \end{aligned}$$

After log-linearising this function, we are able to derive the expression,

$$mc_t = w_t - p_t - mpn_t$$

for which we could use (4.4.9) to expand the expression,

$$mc_t = w_t - p_t - (a_t - \alpha n_t)$$

Now that we have an expression for the aggregate production function in (4.4.7), which may be expressed in terms of  $n_t$ , we have,

$$\begin{aligned} mc_t &= w_t - p_t - \left( a_t - \alpha \left[ \frac{y_t - a_t}{1 - \alpha} \right] \right) \\ &= w_t - p_t - \left[ \frac{a_t(1 - \alpha) - \alpha y_t + \alpha a_t}{1 - \alpha} \right] \\ &= w_t - p_t - \frac{1}{1 - \alpha} [a_t - \alpha y_t] \end{aligned} \quad (4.4.11)$$

We may then define real marginal cost in period  $t + k$ , given the costs in period  $t$ ,

$$\begin{aligned} mc_{t+k|t} &= w_{t+k} - p_{t+k} - mpn_{t+k|t} \\ &= w_{t+k} - p_{t+k} - \frac{1}{1 - \alpha} [a_{t+k} - \alpha y_{t+k|t}] \end{aligned} \quad (4.4.12)$$

We are then able to combine (4.4.12) with (4.4.11) to derive an expression for the difference in marginal costs,

$$\begin{aligned} mc_{t+k|t} - mc_{t+k} &= \left\{ w_{t+k} - p_{t+k} - \frac{1}{1 - \alpha} [a_{t+k} - \alpha y_{t+k|t}] \right\} \dots \\ &\quad - \left\{ w_{t+k} - p_{t+k} - \frac{1}{1 - \alpha} [a_{t+k} - \alpha y_{t+k}] \right\} \\ &= \frac{1}{1 - \alpha} [\alpha y_{t+k|t} - \alpha y_{t+k}] \end{aligned}$$

This relationship could be expressed in terms of prices after making use of the demand schedule  $Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon} Y_{t+k}$ , which may be log-linearised as,

$$y_{t+k|t} = -\epsilon (p_t^* - p_{t+k}) + y_{t+k}$$

Hence,

$$\begin{aligned} mc_{t+k|t} - mc_{t+k} &= \frac{\alpha}{1-\alpha} [-\epsilon (p_t^* - p_{t+k}) + y_{t+k} - y_{t+k}] \\ &= -\frac{\alpha\epsilon}{1-\alpha} [p_t^* - p_{t+k}] \end{aligned} \quad (4.4.13)$$

When there are constant returns to scale, (i.e.  $\alpha = 0$ ), then the marginal costs are constant across all firms, as  $mc_{t+k|t} = mc_{t+k}$ .

To derive an expression for inflation, which will provide us with the New Keynesian Phillips Curve, we return to the expression that describes the optimal price setting behaviour of the firm, in (4.4.4), which is given as,

$$p_t^* - p_{t-1} = (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}]$$

Now after making use of (4.4.13), we may derive,

$$\begin{aligned} p_t^* - p_{t-1} &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ \widehat{mc}_{t+k} - \frac{\alpha\epsilon}{1-\alpha} [p_t^* - p_{t+k}] + p_{t+k} - p_{t-1} \right] \\ \therefore p_t^* - p_{t-1} + \frac{\alpha\epsilon}{1-\alpha} [p_t^*] &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ \widehat{mc}_{t+k} + \frac{(1-\alpha) + \alpha\epsilon}{1-\alpha} [p_{t+k}] - p_{t-1} \right] \\ \therefore \frac{(1-\alpha) + \alpha\epsilon}{1-\alpha} [p_t^*] - p_{t-1} &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ \widehat{mc}_{t+k} + \frac{(1-\alpha) + \alpha\epsilon}{1-\alpha} [p_{t+k}] - p_{t-1} \right] \\ \therefore p_t^* - p_{t-1} &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\Theta \widehat{mc}_{t+k} + p_{t+k} - p_{t-1}] \end{aligned}$$

where we use,  $\Theta = \frac{1-\alpha}{(1-\alpha) + \alpha\epsilon}$  and also add  $\left[ \frac{(1-\alpha) + \alpha\epsilon}{1-\alpha} p_{t-1} - p_{t-1} \right]$  to both sides to derive the final expression. To further simplify this expression we could make use of the following property of forward looking difference equations.

After writing out this expression as,

$$\begin{aligned} p_t^* - p_{t-1} &= (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\Theta \widehat{mc}_{t+k}] \dots \\ &\quad + (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [p_{t+k} - p_{t-1}] \end{aligned}$$

We note that the last term on the right-hand side could be expanded as,

$$\begin{aligned}
 &\Rightarrow (1 - \theta\beta) E_t [(\theta\beta)^0 (p_t - p_{t-1}) + (\theta\beta)^1 (p_{t+1} - p_{t-1}) + (\theta\beta)^2 (p_{t+2} - p_{t-1}) + \dots] \\
 &\Rightarrow E_t [(\theta\beta)^0 (p_t - p_{t-1}) + (\theta\beta)^1 (p_{t+1} - p_{t-1}) + (\theta\beta)^2 (p_{t+2} - p_{t-1}) + \dots] \dots \\
 &\quad - E_t [(\theta\beta)^1 (p_t - p_{t-1}) + (\theta\beta)^2 (p_{t+1} - p_{t-1}) + (\theta\beta)^3 (p_{t+2} - p_{t-1}) + \dots] \\
 &\Rightarrow E_t [(\theta\beta)^0 (\pi_t) + (\theta\beta)^1 (\pi_{t+1}) + (\theta\beta)^2 (\pi_{t+2}) + \dots] \\
 &\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\pi_{t+k}]
 \end{aligned}$$

This allows us to summarise this using the property of discounted sums, such that,

$$p_t^* - p_{t-1} = (1 - \theta\beta) E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\Theta \widehat{mc}_{t+k}] + E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\pi_{t+k}]$$

Then finally, we could write the above with more compact notation, using the property of difference equations that is described in §(A.1.2).

This would allow for the above expression to be written as,

$$p_t^* - p_{t-1} = (1 - \theta\beta) [\Theta \widehat{mc}_t] + \pi_t + (\theta\beta) [p_{t+1}^* - p_t]$$

And after making use of the relationship that is described in (4.3.1),

$$\begin{aligned}
 \frac{\pi_t}{1-\theta} &= (1 - \theta\beta) [\Theta \widehat{mc}_t] + \pi_t + (\theta\beta) \frac{E_t [\pi_{t+k}]}{1 - \theta} \\
 \therefore \frac{\pi_t - \pi_t(1-\theta)}{1-\theta} &= (1 - \theta\beta) [\Theta \widehat{mc}_t] + \frac{(\theta\beta) E_t [\pi_{t+k}]}{1 - \theta} \\
 \therefore \theta \pi_t &= (1 - \theta) (1 - \theta\beta) [\Theta \widehat{mc}_t] + \frac{(1 - \theta) (\theta\beta) E_t [\pi_{t+k}]}{1 - \theta} \\
 \therefore \pi_t &= \frac{(1 - \theta) (1 - \theta\beta)}{\theta} [\Theta \widehat{mc}_t] + \beta E_t [\pi_{t+k}]
 \end{aligned}$$

This can be summarised as,

$$\pi_t = \lambda \widehat{mc}_t + \beta E_t [\pi_{t+k}] \quad (4.4.14)$$

where,  $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta} \Theta$ .

This expression would suggest that the level of current inflation is positively dependent on marginal costs. Hence, inflation would largely be due to the purposeful price-setting decisions of firms, which adjust prices in light of current and anticipated inflationary conditions. Since it is forward looking (i.e.  $\pi_t$  is influenced by  $\pi_{t+1}$ ), it will effectively also be influenced by the expected future marginal costs as well. The discount factor  $\beta$  will influence the degree to which more distant expected increases in marginal costs will impact on current inflation. Furthermore, since  $\frac{\partial \lambda}{\partial \theta} < 0$ , it would suggest that  $\theta$  increases,  $\lambda$

decreases. This would imply that with more stickiness, the pricing mechanism would take longer to adjust to economic shocks. Hence, stimulus programs of government would be more influential (with a greater impact on output).

It is also worth noting that in the New Keynesian model, inflation results from the price setting decisions of firms, where they adjust prices after considering current and future cost conditions. This differs to the characterisation of inflation in the Real Business Cycle model which resulted from changes in the aggregate price level following some form of technology shock.

To complete the model we would like to relate the evolution of inflation to output on the supply side of the model. Using the expressions for the households optimality condition that was derived in (4.2.11) earlier,  $w_t - p_t = \sigma c_t + \gamma n_t$ , and the aggregate production function in (4.4.7) to substitute in for  $a_t = y_t - (1 - \alpha)n_t$ , we may show that,

$$mc_t = (\sigma c_t + \gamma n_t) - (y_t - n_t)$$

Since it is assumed that the goods market is in equilibrium,  $y_t = c_t$ , we may show that,

$$\begin{aligned} mc_t &= (\sigma y_t + \gamma n_t) - (y_t - n_t) \\ &= (\sigma y_t - y_t) + (1 + \gamma)n_t \end{aligned}$$

Using the aggregate production function in (4.4.7) once again to substitute in for  $n_t = \frac{y_t - a_t}{1 - \alpha}$ .

$$\begin{aligned} mc_t &= (\sigma y_t - y_t) + (1 + \gamma) \frac{y_t - a_t}{1 - \alpha} \\ &= \sigma y_t - \frac{1 - \alpha}{1 - \alpha} y_t + \frac{1 + \gamma}{1 - \alpha} y_t - \frac{1 + \gamma}{1 - \alpha} a_t \\ &= \left( \sigma + \frac{\gamma + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \gamma}{1 - \alpha} a_t \end{aligned} \tag{4.4.15}$$

We could make use of a similar expression for the marginal costs under fully flexible prices. Under such conditions (in the absence of frictions), the marginal costs are assumed to be constant over time ( $mc$ ), and the level of output would be at its natural level, which we term  $y_t^n$ . Hence,

$$mc = \left( \sigma + \frac{\gamma + \alpha}{1 - \alpha} \right) y_t^n - \frac{1 + \gamma}{1 - \alpha} a_t \tag{4.4.16}$$

Subtracting equations (4.4.16) from (4.4.15) would provide an expression for the deviation of output from its natural level.

$$\widetilde{mc}_t = \left( \sigma + \frac{\gamma + \alpha}{1 - \alpha} \right) [y_t - y_t^n]$$



This deviation is termed the output gap in the literature and we could use,  $\tilde{y}_t = y_t - y_t^n$ , to describe this variable. We could also then substitute the marginal costs into the previous expression for the New Keynesian Phillips curve that we derived in (4.4.14). The advantage of doing so is that we can express the inflationary process in terms of one of the key variables in the model, the output gap. Hence,

$$\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1} \quad (4.4.17)$$

where  $\kappa = \lambda \left( \sigma + \frac{\gamma + \alpha}{1 - \alpha} \right)$ .

#### 4.4.5 Real Interest Rates & Dynamic Output

Using the households expression for the optimal level of consumption in (4.2.12) and assuming that the goods market clears,

$$\begin{aligned} c_t &= E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}]) \\ \text{s.t. } y_t &= E_t [y_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}]) \end{aligned}$$

With the definition of the real interest rate,  $r_t = i_t - E_t [\pi_{t+1}]$ , the above expression would take the form,  $y_t = E_t [y_{t+1}] - \frac{1}{\sigma} (r_t - \rho)$ , which could be utilised to define the natural output as a function of the natural real interest rate,

$$y_t^n = E_t [y_{t+1}^n] - \frac{1}{\sigma} (r_t^n - \rho) \quad (4.4.18)$$

The evolution of the output gap may then be derived as,

$$\begin{aligned} \tilde{y}_t &= y_t - y_t^n \\ &= \left\{ E_t [y_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}]) \right\} - \left\{ E_t [y_{t+1}^n] - \frac{1}{\sigma} (r_t^n - \rho) \right\} \\ &= E_t [\tilde{y}_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}] - r_t^n) \end{aligned}$$

Using (4.4.16) we can then solve for the natural level of output,

$$\begin{aligned} y_t^n &= \left( \frac{1 - \alpha}{\sigma(1 - \alpha) + \gamma + \alpha} \right) \left( \frac{1 + \gamma}{1 - \alpha} \right) a_t + \left( \frac{1 - \alpha}{\sigma(1 - \alpha)\gamma + \alpha} \right) mc \\ &= \left( \frac{1 + \gamma}{\sigma(1 - \alpha) + \gamma + \alpha} \right) a_t + \left( \frac{(1 - \alpha)mc}{\sigma(1 - \alpha)\gamma + \alpha} \right) \\ &= \psi_{ya}^n a_t + \vartheta_y^n \end{aligned}$$

where,  $\psi_{ya}^n = \frac{1 + \gamma}{\sigma(1 - \alpha) + \gamma + \alpha}$  and  $\vartheta_y^n = \frac{(1 - \alpha)mc}{\sigma(1 - \alpha)\gamma + \alpha}$ . The first difference of this expression could then be written as,  $\Delta E_t [y_{t+1}^n] = \{\psi_{ya}^n \Delta a_{t+1}\}$ . To define the

natural rate of interest, we make use of our expression for the natural rate of interest (4.4.18),

$$\begin{aligned} r_t^n &= \sigma \left( E_t [y_{t+1}^n] - y_t^n \right) + \rho \\ &= \sigma \left( \psi_{ya}^n E_t \{ \Delta a_{t+1} \} \right) + \rho \end{aligned} \quad (4.4.19)$$

At this point we been able to explain the evolution of prices in terms of future inflation and output. And we have been able to explain output in terms of future output and interest rates. If we could then present an expression for interest rates in terms of either output or inflation (or a combination of the two), we would then be able to close the model.

## 4.5 Monetary Policy Rules

The specification of the monetary policy rule within the context of a New Keynesian model would be equivalent to those that were considered in the previous chapter, for the RBC model. Hence, where the central bank changes nominal interest rates according to the changes in inflation and the output gap,

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) \{ \varrho_\pi \pi_t + \varrho_y \tilde{y}_t \} + \varepsilon_t^i \quad (4.5.1)$$

where  $\varepsilon_t^i$  describes the monetary policy shock that may be subject to certain degree of persistence (that may be modelled as an first-order autoregressive process).

### 4.5.1 Essential behavioural equations

The following expressions may be used to describe the dynamic behaviour of the agents,

- *The New Keynesian Philips Curve:*

$$\pi_t = \beta \pi_{t+1} + \kappa \tilde{y}_t;$$

- *The Dynamic IS equation:*

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - \pi_{t+1} - r_t^n) + \tilde{y}_{t+1}$$

- *Monetary Policy Rule:*

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) \{ \varrho_\pi \pi_t + \varrho_y \tilde{y}_t \} + \varepsilon_t^i$$

- *Production Function:*

$$y_t = a_t + (1 - \alpha)n_t;$$

- *Natural Interest Rate:*

$$r_t^n = \sigma \psi_{n,ya}(a_{t+1} - a_t)$$

- *Real Interest Rate:*

$$r_t = i_t - E[\pi_{t+1}]$$

- *Steady-State of Output:*

$$y_t^n = \phi * a_t$$

- *Output Gap:*

$$\tilde{y}_t = y_t - y_t^n$$

- *Persistence in Monetary Policy Shock:*

$$\varepsilon_t^i = \rho^i \varepsilon_{t-1}^i - \epsilon_t^i$$

- *Persistence in Technology Shock:*

$$a_t = \rho^a a_{t-1} - \epsilon_t^a$$

where the first three expressions summarise the essential behavioural equations of the respective agents in the model.

### 4.5.2 Choice of parameter values

In this model we have seventeen parameters for,

$$\sigma, \phi, \varrho_i, \varrho_{pi}, \varrho_y, \theta, \rho_i, \rho_a, \beta, \zeta, \eta, \alpha, \epsilon, \Omega, \psi_{ya}^n, \lambda \text{ and } \kappa.$$

Values for these parameters may be calibrated to simulate the effect of the respective technology and monetary policy shocks,  $\epsilon_t^a$  and  $\epsilon_t^i$ . The values for these parameters are taken from Galí (2008),

The size of the monetary policy shock is then set to 25 basis points per quarter (i.e. approximately 1% per year) and the technology shock is assumed to be 1%. To describe the effects of a model that exhibits near perfect price flexibility, we simulate a second model and set the Calvo parameter,  $\theta$ , to 0.01. The results from both of these simulations are provided in figure (4.1).

Parameter	Value	Description
$\sigma$	1	log utility
$\phi$	1	unitary Frisch elasticity
$\varrho_i$	0.75	smoothing in Taylor Rule
$\varrho_p^i$	1.5	inflation feedback Taylor Rule
$\varrho_y$	0.5/4	output feedback Taylor Rule
$\theta$	2/3	Calvo parameter
$\rho_i$	0.5	autocorrelation monetary policy shock
$\rho_a$	0.9	autocorrelation technology shock
$\beta$	0.99	discount factor
$\zeta$	0.8	habits in consumption
$\alpha$	1/3	capital share
$\epsilon$	6	demand elasticity
$\Omega$	$(1 - \alpha)/(1 - \alpha + \alpha * \epsilon)$	
$\psi_{ya}^n$	$(1 + \phi)/(\sigma * (1 - \alpha) + \phi + \alpha)$	
$\lambda$	$(1 - \theta) * (1 - \beta * \theta)/\theta * \Omega$	
$\kappa$	$\lambda * (\sigma + (\phi + \alpha)/(1 - \alpha))$	

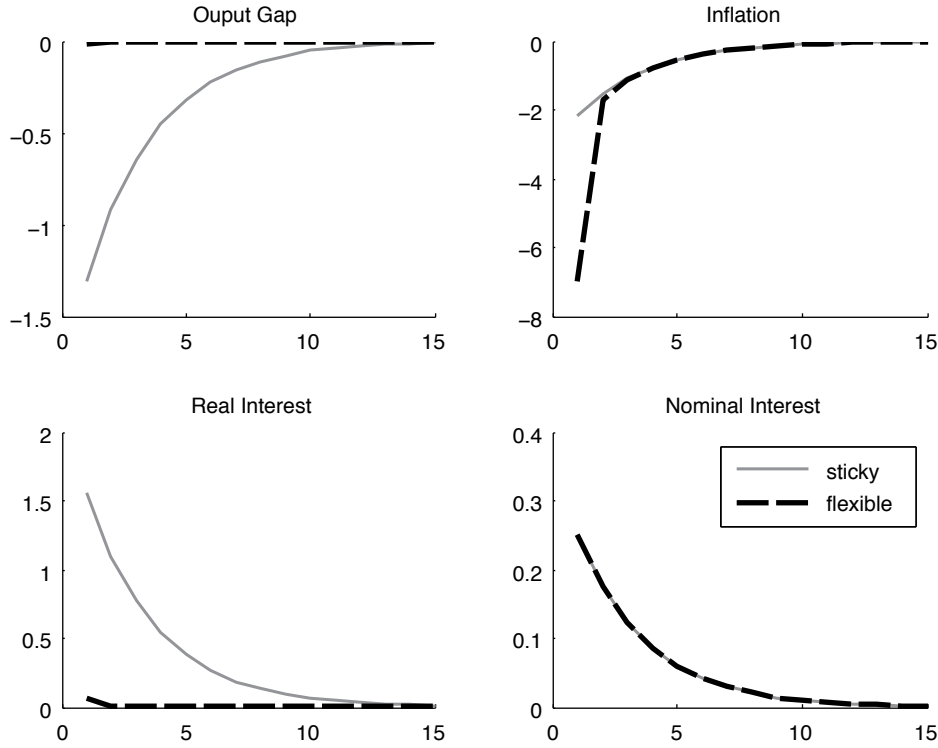
**Table 4.1:** Calibrated Parameter Values - New Keynesian Model

### 4.5.3 Model results

The impulse response function in figure (4.1) suggests that a monetary policy shock of 25 basis points (in a quarter), has a similar effect on nominal interest rates in both the sticky and flexible price models. However, the effect of this shock on inflation is very different as prices are allowed to adjust in the model where prices are nearly perfectly flexible, which would leave real interest rates (almost) unchanged. In the model that incorporates price rigidities, inflation does not adjust to the same degree during the initial periods, and as such, the monetary policy shock has a relatively large effect on real interest rates. This would explain the effects of this shock on output, where monetary policy has no effect on output when the model is specified with near perfect price flexibility, whilst it has a large effect on output when the model incorporates sticky prices.

## 4.6 Money in the Utility Function

To introduce money into the model, we define it as “anything that serves as a medium of exchange, unit of account, or store of value.” Given this definition, the agents that have described in the previous model would not willingly hold money, at the steady-state, as it does not pay interest. Any rational agent



**Figure 4.1:** Monetary Policy Shock - New Keynesian Model

would prefer to use money to *save* in capital goods or bonds, which would pay a rate of return.

However, one should acknowledge that money has the ability to alleviate some of the problems that are incurred in a system of exchange, such as one that is based on barter. Therefore, if we propose that the representative agent expresses a desire for holding money, based on the fact that it makes conducting exchange *easier*, we could allow for the instance where agents derive utility from holding money. This could be included in the model, in a highly tractable manner, to allow for the possibility that the economic agents have a demand for real money balances.<sup>16</sup>

For example, consider the case where utility is separable,

$$\max_{C_t, N_t, M_t/P_t} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-v}}{1-v} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] \quad (4.6.1)$$

where,  $\mathcal{X}_t = M_t/P_t$ , is the demand for real monetary balances and the parameter  $v$  is used to describe the preference for holding money. The budget

<sup>16</sup>Fernández-Villaverde (2010) notes that whilst the incorporation of money in the utility function is not particularly elegant, as it may miss important channels through which money matters and would not be invariant to policy changes, it is currently one of the most efficient ways of justifying the existence of money within the context of a model. For a more comprehensive discussion of this topic see, Wallace (2001).

constraint for this problem could then be given as,

$$P_t C_t + Q_t B_t + \mathcal{X}_t = W_t N_t + B_{t-1} + \mathcal{X}_{t-1} + H_t$$

By letting  $\mathcal{D}_t = B_{t-1} + \mathcal{X}_{t-1}$ , which seeks to describe total financial wealth at the beginning of the period, and assuming that the price of the consumed good is 1, we can write the budget constraint as,

$$C_t + Q_t \mathcal{D}_{t+1} + (1 - Q_t) \mathcal{X}_t = W_t N_t + \mathcal{D}_t + H_t \quad (4.6.2)$$

Using the utility function in (4.6.1), the budget constraint in (4.6.2), and the Ponzi constraint, the first order conditions could be found by setting up the Lagrangian,

$$\begin{aligned} \mathcal{L}_{C_t, N_t, \mathcal{D}_{t+1}, \mathcal{X}_t} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\mathcal{X}_t^{1-v}}{1-v} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] + \dots \right. \\ \left. \dots \lambda [W_t N_t + \mathcal{D}_t + H_t - C_t - Q_t \mathcal{D}_{t+1} - (1 - Q_t) \mathcal{X}_t] \right\} \end{aligned}$$

where we can derive the partial derivatives,

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} - \lambda = 0 \quad (4.6.3)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -N_t^\gamma + \lambda W_t = 0 \quad (4.6.4)$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{D}_{t+1}} = -Q_t \lambda_t + \beta E_t \lambda_{t+1} = 0 \quad (4.6.5)$$

$$\frac{\partial \mathcal{L}}{\partial \mathcal{X}_t} = \mathcal{X}_t^{-v} - \lambda [1 - Q_t] = 0 \quad (4.6.6)$$

Hence, as expressions (4.6.3) and (4.6.4) are as before, the combination of these conditions would leave us with,

$$W_t = C_t^\sigma N_t^\gamma$$

Similarly, using (4.6.5) and (4.6.3), we would be left with,

$$Q_t = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right]$$

These two expressions give us the same log-linear expressions,<sup>17</sup>

$$\begin{aligned} w_t &= \sigma c_t + \gamma n_t \\ c_t &= E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - \rho) \end{aligned}$$

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<sup>17</sup>Note that we assumed  $P_t = 1$ , which allows us to omit  $p_t$  for the following log-linear expression.

For the final condition, note that after substituting (4.6.3) into (4.6.6), and where  $\mathcal{X}_t = M_t/P_t$ , we are able to derive the expression,

$$\begin{aligned}\mathcal{X}_t^{-v} &= C_t^{-\sigma} [1 - Q_t] \\ \left(\frac{M_t}{P_t}\right)^{-v} &= C_t^{-\sigma} [1 - Q_t]\end{aligned}$$

Taking the log-linear transformation of this expression, would leave us with,<sup>18</sup>

$$m_t - p_t = \frac{\sigma}{v} c_t - \frac{1}{v} i_t \quad (4.6.7)$$

Note that after including an additional endogenous variable in the model,  $M_t$ , we have included a further equation that describes its evolution. This expression is rather intuitive, as it suggests that the demand for nominal money increases with the price level, such that changes to real monetary balances may be explained by changes in consumption and the interest rate. In addition, where the rate of consumption and the interest rate remain unchanged, an increase in prices (i.e. positive inflation) may cause the agent to increase their holding of money to pay for consumption goods that are going to become more expensive.

The relationship between the demand for real monetary balances and consumption is also intuitive as it describes instances where relatively wealthy agents, enjoy higher levels of consumption and greater monetary balances. Hence, with this expression money takes the functional form of a *normal* good. Furthermore, since the nominal interest rate is the opportunity cost of holding money, this relationship suggests that any increase in the real interest rate would be accompanied by a decrease in nominal monetary holdings.

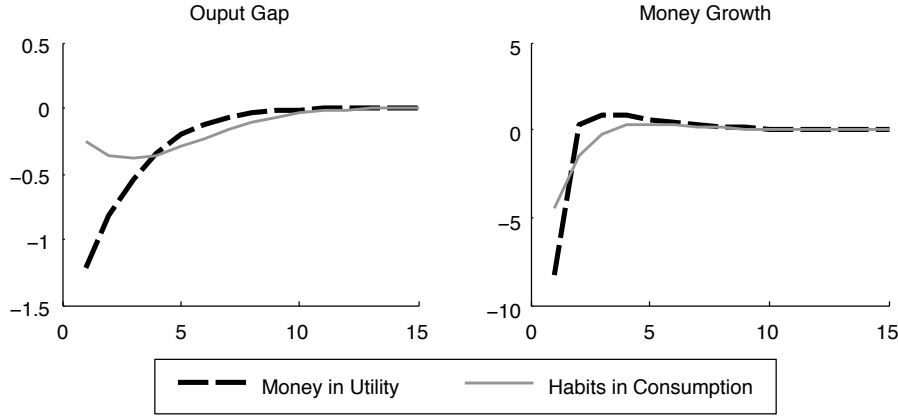
When incorporating this feature in a model that is specified in terms of inflation we may take the first difference of the expression in (4.6.7), such that we report on changes in money growth. The results in figure (4.2) suggest that to implement a monetary policy shock that will result in a rise in nominal interest rates, the central bank must reduce monetary supply.

## 4.7 Habits in Consumption

In each of the models that have been considered, it has been assumed that utility was derived from the level of consumption. As an alternative theoretical postulate, one could consider the case where the agent becomes accustomed to a certain level of consumption, and to derive greater utility, the new level of consumption must be higher than the previous level.<sup>19</sup> To simplify the

<sup>18</sup>See, §(B.3.8).

<sup>19</sup>In this section we consider the effects of including habits in consumption, to ascertain whether the inclusion of this feature would improve the model's characterisation of observed economic behaviour.

**Figure 4.2:** Monetary Policy Shock - MIU & Habits

exposition, we make use of a separable utility function without real balances for money,

$$\max_{C_t, N_t} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - \zeta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] \quad (4.7.1)$$

where the parameter  $\zeta$  refers to the degree of habits in consumption. The accompanying budget constraint may then be given as,

$$P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + H_t \quad (4.7.2)$$

These expressions allow us to establish the Lagrangian,

$$\begin{aligned} \mathcal{L}_{C_t, N_t} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{(C_t - \zeta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] + \dots \right. \\ \left. \dots \lambda [W_t N_t + B_{t-1} + H_t - P_t C_t - Q_t B_t] \right\} \end{aligned}$$

where we can derive the partial derivatives,

$$\frac{\partial \mathcal{L}}{\partial C_t} = (C_t - \zeta C_{t-1})^{-\sigma} - \zeta \beta E_t (C_{t+1} - \zeta C_t)^{-\sigma} - \lambda P_t = 0 \quad (4.7.3)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = -N_t^\gamma + \lambda W_t = 0 \quad (4.7.4)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -Q_t \lambda_t + \beta E_t \lambda_{t+1} = 0 \quad (4.7.5)$$

after simplifying for  $\frac{N_t^\gamma}{W_t} = \lambda$ , and substituting,

$$\mathcal{Z}_t = (C_t - \zeta C_{t-1})^{-\sigma} - \zeta \beta E_t (C_{t+1} - \zeta C_t)^{-\sigma}$$



we can then derive the expression,

$$\begin{aligned} \mathcal{Z}_t &= \lambda P_t & \text{and} & & \mathcal{Z}_t &= \frac{N_t^\gamma P_t}{W_t} \\ \text{s.t. } \frac{\mathcal{Z}_t}{N_t^\gamma} &= \frac{P_t}{W_t} & \text{and} & & \frac{W_t}{P_t} &= \frac{1}{\mathcal{Z}_t} \cdot N_t^\gamma \end{aligned}$$

If there are no habits in consumption, such that  $\zeta = 0$ , then we have,  $\mathcal{Z}_t = C_t^{-\sigma}$ , and,

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\gamma$$

However, in cases where  $\zeta \neq 0$  this expression can get quite cumbersome to work with. One simplification that is often applied is due to Abel (1990), where additional utility is derived from deviations from *aggregate* consumption. This behaviour has been used to describe cases where the rational economic agent would derive additional utility when consuming more than his/her neighbour. However, in a model with a single representative agent, the *aggregate* consumption is usually interpreted as the *average* consumption level.<sup>20</sup> Hence, the utility function could be expressed as,

$$\max_{C_t, N_t} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t - \zeta \tilde{C})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right] \quad (4.7.6)$$

where,  $\tilde{C}$  is average consumption. After setting up the Lagrangian, the partial derivative with respect to consumption would be,

$$\frac{\partial \mathcal{L}}{\partial C_t} = (C_t - \zeta \tilde{C})^{-\sigma} - \lambda P_t = 0 \quad (4.7.7)$$

which would allow us to derive,

$$\frac{W_t}{P_t} = (C_t - \zeta \tilde{C})^\sigma N_t^\gamma \quad (4.7.8)$$

Note that this expression is not all that different to what was derived in the previous chapter. In this case, changes in consumption would have a much smaller initial effect on the real wage, particularly when  $\zeta$  is large.

When including this feature in a model, we usually observe *humped-shaped* impulse response functions in consumption; following the imposition of an identified shock (i.e. monetary policy shock). An example of this is provided in figure (4.2), where  $\zeta$  is calibrated to a value of 0.7. This would affect most

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<sup>20</sup>Some researchers have also used the steady-state level of consumption as a proxy for the aggregate consumption.

of the other variables in the model, including output and inflation, which also take on impulse response functions that are more *humped-shaped*.

Since most of the stylised facts in chapter two suggest that the full effects of most shocks only impact after a relatively short period of time, most of the impulse response functions should be characterised with a *humped-shaped* pattern. Generating such behaviour without the inclusion of habits in consumption is a significant challenge, and as such, most modern models include this feature.

## 4.8 Conclusion

Whilst it is desirable to incorporate nominal and real rigidities in a dynamic stochastic general equilibrium model, the manner in which one embarks on this task requires careful manipulation of the model equations. In this chapter we have constructed a closed-economy model that is consistent with the framework of the New Keynesian theorists. It seeks to describe the behaviour of the representative agents with the aid of a linear New Keynesian Phillips Curve for the evolution of prices that takes the form,  $\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}$ , where  $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta} (\sigma + \frac{\gamma+\alpha}{1-\alpha})$ . Demand is then described with the aid of a New Keynesian IS Curve, which may be expressed as,  $\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \frac{1}{\sigma} (i_t - r_t^n - E_t [\pi_{t+1}])$  and monetary policy is described with the aid of a Taylor rule,  $i_t = \rho_i i_{t-1} + (1 - \rho_i) [\varrho_\pi \pi_t + \varrho_y \tilde{y}_t] + \epsilon_t^i$ .<sup>21</sup>

Following the introduction of sticky-prices, the effect of a nominal interest rate shock may result in a temporary change in the real interest rate, as inflation would not be able to react instantaneously to this disturbance. This change to the real interest rate would subsequently affect the other real variables in the model, including the measure of the output gap. In this case, monetary policy is no longer neutral, since it has ability to affect real and nominal variables.

This description of the affect of a monetary policy shock would appear to be consistent with the stylised features of the business cycle that were presented in chapter two. Furthermore, with the inclusion of habits we are also able to produce the hump-shaped nature of the stylised impulse response functions.

After developing a model that is able to replicate important features of the business cycle, we are then able to consider the inclusion of open-economy features that also have the ability to influence the business cycle of a small open economy in the next chapter.

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<sup>21</sup>Note that in terms of these expressions, inflation is described by the discounted stream of current and future output gaps. And the output gaps are influenced by the discounted stream of current and future real interest rates.

## Chapter 5

# Small Open-Economy Models

## 5.1 Introduction

The introduction of the small open-economy New Keynesian model is described in Galí and Monacelli (2005) and Monacelli (2003). It incorporates expressions for exchange rates, terms of trade, exports, imports, and international financial market interaction. Many influential papers have made use of this framework, and perhaps one of the most significant has been Justiniano and Preston (2010), where they show that the central banks of most small open economies should not react to exchange rate movements.

Two variants of these models have been applied to South African macroeconomic data. The first type of small open-economy models, which assume complete asset markets and perfect risk sharing is provided in Steinbach *et al.* (2009a), and later variants, which allow for incomplete asset markets and a debt-elastic interest rate (risk) premium is provided in Alpanda *et al.* (2010a, 2010b, 2011).

These models provide for a relatively rich characterisation of the data and their out-of-sample properties would appear to be superior to those of most other types of reduced form models. In addition, when using this framework one is able to analyse a number of interesting policy questions, such as whether or not it would be optimal for the central bank to react to exchange rate fluctuations, when setting the nominal interest rate.

## 5.2 Households

Households would once again seek to maximise the consumption of goods, but in this case they may choose to consume either domestic or foreign goods, where foreign goods may come from one of a number of countries.

### 5.2.1 Consumption of locally produced goods

To describe the consumption of domestically produced home goods,  $C_{H,t}$ , we could use an expression that is similar to those that were derived for the closed economy. Therefore, where the household is able to consume  $j$  different goods, within the continuum  $j \in [0, 1]$ ; in the presence of constant elasticity of substitution, which is described with the aid of the parameter,  $\epsilon$ ,

$$C_{H,t} = \left[ \int_0^1 C_{H,j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad (5.2.1)$$

We are then able to derive the demand function, which is similar to what was derived earlier, where the demand for the differentiated goods is dependent

upon relative prices and the degree of substitution between goods,

$$C_{H,j,t} = \frac{P_{H,j,t}^{-\epsilon}}{P_{H,t}} C_{H,t} \quad (5.2.2)$$

Thereafter, we were able to derive the price index,

$$P_{H,t} = \left( \int_0^1 P_{H,j,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \quad (5.2.3)$$

Then finally, we can combine expression (5.2.2) with the price index in (5.2.3) and the consumption index in (5.2.1) to derive the relationship between the measure of broad consumption expenditure and the consumption expenditure on disaggregated items,

$$\int_0^1 P_{H,j,t} C_{H,j,t} dj = P_{H,t} C_{H,t} \quad (5.2.4)$$

## 5.2.2 Consumption of foreign goods

To incorporate the consumption of foreign goods into the model, we assume that domestic consumers purchase  $C_{F,t}$  goods using the interval  $i \in [0, 1]$  for the different countries. This allows us to construct an index for all imported (foreign) goods, using the familiar expression,

$$C_{F,t} = \left[ \int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}} \quad (5.2.5)$$

where  $\gamma$  represents the substitutability of goods between the  $i$  foreign countries, and  $C_{i,t}$  is the index of goods that are imported from country  $i$ , for domestic consumption in the home economy. We could then proceed as before to derive the demand function,

$$C_{i,t} = \frac{P_{i,t}^{-\gamma}}{P_{F,t}} C_{F,t} \quad (5.2.6)$$

and the price index for foreign goods,

$$P_{F,t} = \left( \int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \quad (5.2.7)$$

This would allow us to describe the consumption of goods from all  $i$  countries as,

$$\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t} \quad (5.2.8)$$

### 5.2.3 Consumption of different imported goods

We could then allow for the case where each of the  $i$  country's produces  $j$  different goods, where  $C_{i,j,t}$  describes the domestic consumption of good  $j$ , that was imported from country  $i$ . This would involve making use of a similar integral,

$$C_{i,t} = \left[ \int_0^1 C_{i,j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \quad (5.2.9)$$

Note that we can use  $\epsilon$  in this expression as it describes the substitution effects in the home country, with respect to its demand for the  $j$  different goods. Using this equality, we may then derive the demand function,

$$C_{i,j,t} = \frac{P_{i,j,t}^{-\epsilon}}{P_{i,t}} C_{i,t} \quad (5.2.10)$$

and the price index is as expected,

$$P_{i,t} = \left( \int_0^1 P_{i,j,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \quad (5.2.11)$$

This would allow us to describe the consumption of  $j$  goods in country  $i$  as,

$$\int_0^1 P_{i,j,t} C_{i,j,t} dj = P_{i,t} C_{i,t} \quad (5.2.12)$$

### 5.2.4 Aggregate consumption

Since the broad consumption index,  $C_t$ , would comprise of the consumption of locally produced goods,  $C_{H,t}$ , and foreign imported goods,  $C_{F,t}$ , we may then express the broad consumption index as,

$$C_t = \left[ (1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (5.2.13)$$

In this case,  $\alpha$  refers to the degree of openness, such that  $(1-\alpha)$  would indicate the degree of home bias. Similarly,  $\eta$  would refer to the measure of substitutability between domestic and foreign goods from the viewpoint of the domestic consumer. Furthermore, the optimal allocation of expenditures between domestic and imported goods is given by,

$$C_{H,t} = (1-\alpha) \frac{P_{H,t}^{-\eta}}{P_t} C_t \quad ; \quad C_{F,t} = \alpha \frac{P_{F,t}^{-\eta}}{P_t} C_t \quad (5.2.14)$$

Similarly, the broad consumer price index could be described as,

$$P_t = \left[ (1-\alpha) (P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (5.2.15)$$

One is then able to log-linearise the price index in (5.2.15), to provide,

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t} \quad (5.2.16)$$

### 5.2.5 Household's budget constraint

This information could be used to derive the budget constraint for the household, which could then be given as,

$$\int_0^1 P_{H,j,t} C_{H,j,t} dj + \int_0^1 \int_0^1 P_{i,j,t} C_{i,j,t} dj di + \dots$$

$$E_t [Q_{t|t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \quad (5.2.17)$$

where  $P_{H,j,t}$  is the price of good  $j$  that has been locally produced and  $P_{i,j,t}$  is the price of good  $j$  that has been produced by country  $i$ . The dividends and interest earned on investment in the following period are denoted  $D_{t+1}$ , the stochastic discount factor is  $Q_{t|t+1}$ , nominal wages are represented by  $W_t$  and lump sum taxes or transfers are denoted  $T_t$ .

Given that  $P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t$ , the budget constraint can then be expressed as,

$$P_t C_t + E_t [Q_{t|t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \quad (5.2.18)$$

### 5.2.6 Deriving optimal levels of consumption

Now to return to the problem of deriving optimal levels of consumption and labour for the household. Using the budget constraint in (5.4.5), and the utility function with separable preferences,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

we are then able to derive the optimal level of consumption/savings and labour supply as before,

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\gamma$$

$$Q_t = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \quad (5.2.19)$$

From these expressions we may derive the log-linear expressions,

$$w_t - p_t = \sigma c_t + \gamma n_t \quad (5.2.20)$$

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - \rho - E_t [\pi_{t+1}]) \quad (5.2.21)$$

## 5.3 The Real Exchange Rate and the Terms of Trade

### 5.3.1 The terms of trade

The price of country  $i$ 's goods in terms of home goods may be expressed by the bilateral terms of trade,

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$$

The effective terms of trade would then be given by,

$$\begin{aligned} S_t &= \frac{P_{F,t}}{P_{H,t}} \\ &= \left( \int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \end{aligned} \quad (5.3.1)$$

This expression could be log-linearised around the steady state,  $S_{i,t} = 1$ , with the aid of a first order Taylor series approximation, such that,

$$s_t = \int_0^1 s_{i,t} di \quad (5.3.2)$$

where  $s_t = \log S_t = p_{F,t} - p_{H,t}$ . Making use of the log-linearised expression in (5.2.16), we can substitute in for  $p_{F,t} - p_{H,t}$ , such that,

$$\begin{aligned} p_t &= (1 - \alpha)p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} + \alpha(p_{F,t} - p_{H,t}) \\ &= p_{H,t} + \alpha s_t \end{aligned} \quad (5.3.3)$$

This would imply that where inflation is given as,  $\pi_t = p_{t+1} - p_t$ , domestic inflation and the terms of trade would be given as,  $\pi_{H,t} = p_{H,t+1} - p_{H,t}$ , and,  $\Delta s_t = s_{t+1} - s_t$ , respectively. We may then express inflation as,

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \quad (5.3.4)$$

where the gap between the two measures of inflation is proportional to the change in the terms of trade and the degree of openness.

### 5.3.2 Law of one price

If we are to assume that the law of one price holds for individual goods at all times, then  $P_{i,j,t} = \mathcal{E}_{i,t} P_{i,j,t}^i$ , where  $\mathcal{E}_{i,t}$  is the nominal exchange rate for country  $i$  and  $P_{i,j,t}^i$  is the price of good  $j$  that was imported from country  $i$  (expressed in the currency of country  $i$ ). Hence, making use of the expression



in (5.2.11), we have the index  $P_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i$ . Substituting this expression into the definition of  $P_{F,t}$  and log-linearising around the steady state,<sup>1</sup>

$$\begin{aligned} p_{F,t} &= \int_0^1 (d_{i,t} + p_{i,t}^i) di \\ &= d_t + p_t^w \end{aligned} \quad (5.3.5)$$

where  $p_{i,t}^i = \int_0^1 p_{i,j,t}^i dj$ , is the domestic price index for country  $i$ , expressed in terms of its own currency. The effective nominal exchange rate is given as,  $d_t = \int_0^1 d_{i,t} di$ , and the world price index is given by,  $p_t^w = \int_0^1 p_{i,t}^i di$ .

Combining the expression in (5.3.5) with  $s_t = p_{F,t} - p_{H,t}$ , we get,

$$\begin{aligned} s_t + p_{H,t} &= d_t + p_t^w \\ s_t &= d_t + p_t^w - p_{H,t} \end{aligned} \quad (5.3.6)$$

To derive an expression for the relationship between the terms of trade and the real exchange rate, note that after the nominal exchange rate has been deflated by the ratios of the two countries consumer price indices (expressed in their own currencies) the bilateral real exchange rate may be defined by,

$$\mathcal{Q}_{i,t} = \mathcal{E}_{i,t} \frac{P_t^i}{P_t} \quad (5.3.7)$$

Where the logarithm of the bilateral real exchange rate,  $q_{i,t} = \log \mathcal{Q}_{i,t}$ , and real effective exchange rate is,  $q_t = \int_0^1 q_{i,t} di$ , we may then derive,

$$\begin{aligned} q_t &= \int_0^1 (d_{i,t} + p_{i,t}^i - p_t) di \\ &= d_t + p_t^w - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1 - \alpha)s_t \end{aligned} \quad (5.3.8)$$

where the first line is given by the log-linearisation of  $\mathcal{Q}_{i,t} = \mathcal{E}_{i,t} \frac{P_t^i}{P_t}$ . We are then able to move from the first to the second line by using (5.3.5). Thereafter, substituting in for  $d_t + p_t^w$ , using (5.3.6); and using (5.3.3), to substitute in for  $p_{H,t} - p_t$ , we are then able to derive the final condition. This expression relates the terms of trade to the exchange rate and the degree of openness, which would make it extremely useful.

## 5.4 Closing Small open-economy Models

This model allows for the economic agent from the small open-economy to hold an international asset. However, these agents are unable to affect the price of

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<sup>1</sup>Note that the superscript indicates that these prices are denominated in the foreign country.

this asset. Given this condition, if the real rate of return of the international asset exceeds the subjective discount (or real domestic interest rate), then there would be no reason to hold the domestic asset at any point in the future. This would imply that any shock that affects domestic interest rates would have long-run effects, which may induce random walk behaviour.<sup>2</sup>

In addition, since these models allow for the small open-economy to run a current account deficit, against which they would need to borrow. However, we need to ensure that this borrowing is limited by some mechanism, or else the representative agents from the small open-economy would continue to borrow without any restraint, to the point where this borrowing could only be explained by an explosive process.<sup>3</sup>

The imposition of constraints that prevent the small open-economy from engaging in unbounded lending is termed *closing-off the model*. These conditions allow for researchers to derive stable-steady conditions around which we are able to find the model solution. To ensure that the model provides a dynamic characterisation that is consistent with the phenomena of a general equilibrium model, Schmitt-Grohe and Uribe (2003) shows how to employ various techniques to induce stationarity for describing the open-economy dynamics. This section includes a description of the conditions for complete asset markets with international risk sharing, which is followed by the case of incomplete asset markets with a debt-elastic interest rate (risk) premium.

### 5.4.1 Complete Asset Markets with International Risk-Sharing

Under the assumptions of complete markets for international securities, Galí and Monacelli (2005) include the stochastic discount factor for the purchase of goods from country  $i$  (in their prices). Under such conditions, they account for the change in prices  $P_t^i/P_{t+1}^i$  in each country (as measured in their local currency), and the change in the nominal exchange rate,  $\mathcal{E}_t^i/\mathcal{E}_{t+1}^i$ . Hence, they use of the expression,

$$Q_t^i = \beta E_t \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) \right] \quad (5.4.1)$$

where they make use of the expression for the households stochastic discount factor,  $Q_t$  in (5.2.19), and combining it with (5.4.1) together with the

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<sup>2</sup>It is worth noting that these conditions for closing off the open-economy features in the model are not purely micro-founded.

<sup>3</sup>One is not able to model an explosive process, within the present framework.

definition of the real exchange rate in (5.3.7),

$$\begin{aligned}
 Q_t &= Q_t^i \\
 \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] &= \beta E_t \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) \right] \\
 \left( \frac{C_{t-1}}{C_t} \right)^\sigma \left( \frac{P_{t-1}}{P_t} \right) &= \left( \frac{C_{t-1}^i}{C_t^i} \right)^\sigma \left( \frac{P_{t-1}^i}{P_t^i} \right) \left( \frac{\mathcal{E}_{t-1}^i}{\mathcal{E}_t^i} \right) \\
 (\dot{C}_t)^\sigma \dot{P}_t &= (\dot{C}_t^i)^\sigma \dot{P}_t^i \dot{\mathcal{E}}_t^i \\
 (\dot{C}_t)^\sigma &= (\dot{C}_t^i)^\sigma \dot{\mathcal{E}}_t^i \frac{\dot{P}_t^i}{\dot{P}_t} \\
 C_t &= \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}}
 \end{aligned} \tag{5.4.2}$$

where the notation is such that  $\dot{P}_t = P_{t-1}/P_t$ , and similarly so for the other variables. They are able to move to the last equality by making use of the constant  $\vartheta_i$ , which will depend on initial conditions regarding relative net asset positions.<sup>4</sup> Without loss of generality symmetric initial conditions are assumed, such that  $\vartheta_i = \vartheta = 1$  for all  $i$ .<sup>5</sup>

Taking the logarithm of both sides and integrating over the consumption goods from  $i$  countries to derive the world consumption index,  $c_t^w = \int_0^1 c_{i,t}^i di$ , would provide us with,

$$c_t = c_t^w + \frac{1}{\sigma} q_t \tag{5.4.3}$$

Making use of the relationship between  $q_t$  and  $s_t$  in (5.3.8) would then yield,

$$c_t = c_t^w + \left( \frac{1 - \alpha}{\sigma} \right) s_t \tag{5.4.4}$$

This relationship combines world consumption and the terms of trade to describe consumption in a particular country. As noted in Galí (2008), it may be shown that the terms of trade are a function of current and anticipated real interest rate differentials, where if we allow for the household to invest in both domestic and foreign bonds, the households budget constraint may be written as,

$$P_t C_t + E_t [Q_{t|t+1} B_{t+1}] + E_t [Q_{t|t+1}^* B_{t+1}^*] \leq B_t + \mathcal{E} B_t^* + W_t N_t + T_t \tag{5.4.5}$$

<sup>4</sup>The relative net asset position at the beginning of the period would be given by  $C_{t-1}^\sigma P_{t-1} = (C_{t-1}^i)^\sigma P_{t-1}^i \mathcal{E}_{t-1}^i$ .

<sup>5</sup>Equivalently, we could assume zero net holdings.

The optimality conditions with respect to these assets would be given by:

$$1 = \beta E_t \left\{ Q_{t|t+1}^{-1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad (5.4.6)$$

$$1 = \beta E_t \left\{ Q_{t|t+1}^{\star-1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad (5.4.7)$$

After dividing (5.4.6) by (5.4.7) we obtain:

$$\frac{Q_{t|t+1}^{\star}}{Q_{t|t+1}} = E_t \left\{ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}$$

Log-linearising this expression would provide the familiar uncovered interest rate parity condition, where the domestic nominal interest rate is equal to the foreign interest rate plus the expected rate of currency depreciation.

$$i_t = i_t^{\star} + E_t \Delta e_{t+1} \quad (5.4.8)$$

This expression could then be used to relate the interest rate differential to the terms of trade or real exchange rate, using any of the above conditions.

### 5.4.2 Debt-Elastic Interest Rate (Risk) Premium

Following Lubik (2007), the specification for a debt-elastic interest rate assumes that interest rates are a function of net foreign assets. If a country is a net foreign borrower, it pays an interest rate that is higher than the world interest rate. In this way, it will pay a risk premium, when the net foreign assets are above the steady-state value.

We may specify the risk premium as,

$$risk_t = \epsilon_t^{risk} + \chi nfa_t$$

where  $risk_t$  captures time varying risk premium and  $nfa_t$  is the net foreign asset position of a country. The parameter  $\chi$  is an elasticity parameter, as in Schmitt-Grohe and Uribe (2003), and would take on a positive value. Hence, when a country takes on a large net foreign asset holding, it would experience a positive risk premium. The inclusion of the term  $\epsilon_t^{risk}$  follows Justiniano and Preston (2010), which allows for shocks to the risk premium that may be modelled as an autoregressive process.

To allow for the risk premium to influence the amount of interest that is paid by a country, we may specify the condition,

$$E[q_{t+1}] - q_t = (i_t - E[\pi_{t+1}]) - (i_t^{\star} - E[\pi_{t+1}^{\star}]) + risk_t$$

where we assume the foreign (world) interest rate, inflation and the exchange rate are largely influenced by exogenous factors, any increase in the

risk premium will result in an increase in the real interest rate that is applicable to the small open-economy.

To complete this specification, we may allow for deviations from the law of one price, such that (5.3.8) may be written as,

$$q_t = (1 - \alpha)s_t + \psi_t^f$$

where  $\psi_t^f$  represents the deviations from one price. The change in the net foreign asset position of a country would be determined by the level of output and consumption in the small open-economy, as well as its terms of trade and any deviations from the law of one price. This would allow us to make use of the expression,

$$nfa_t - \frac{1}{\beta}nfa_{t-1} = (y_t - c_t) - \alpha(s_t + \psi_t^f)$$

A comparison of the results that are derived from the different methods that are used to close off the open economy features of the model is contained in part three.

## 5.5 Firms in a Small open-economy

If we assume that the firm in the home country produces a differentiated good with a production function that makes use of linear technology, with constant returns to scale,

$$Y_{j,t} = A_t N_{j,t} \quad (5.5.1)$$

To allow for a technology shock that may affect the production function during certain periods of time, we would want to include an exogenous stochastic term,  $\epsilon_t$ . Hence, where  $a_t = \log A_t$ , and it is assumed that the technology shock is relatively persistent, we can make use of the AR(1) process,  $a_t = \rho_a a_{t-1} + \epsilon_t$ , to describe the effects of a technology shock.

In this case the marginal productivity of labour would be  $\frac{\partial Y_{j,t}}{\partial N_{j,t}} = A_t$ , for which the log linear term is,  $mpn_t = a_t$ . Furthermore, since the real marginal costs (expressed in terms of domestic prices) will be common across domestic firms,

$$\begin{aligned} mc_t &= w_t - p_{H,t} - mpn_t \\ &= w_t - p_{H,t} - a_t \end{aligned} \quad (5.5.2)$$

### 5.5.1 Price Setting Behaviour

Once again, we can make use of the sticky-price mechanism of Calvo (1983), where each firm has the probability  $(1 - \theta)$  of being able to change the price

of the goods that are produced. In the notation that follows, we use  $P_t^*$  for those prices that have been changed.

As shown in the derivation of the New Keynesian model, the optimal price setting strategy for the domestic firm could be given as,

$$p_{H,t}^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [mc_{t+k|t} + p_{H,t+k}]$$

where  $p_{H,t}^*$  is the newly set domestic prices and  $\mu = \log \frac{\epsilon}{\epsilon-1}$  is the gross markup in the steady state.

## 5.6 Aggregate Demand and Output Determination

The domestic market for  $j \in [0, 1]$  goods will clear in the home country when all the locally produced goods are consumed. In an open-economy setting, the locally produced goods may be consumed by the local market  $C_{H,j,t}$ , or they may be consumed by one of the  $i \in [0, 1]$  foreign countries, which may be represented as  $\int_0^1 C_{H,j,t}^i di$ . Hence,

$$Y_{j,t} = C_{H,j,t} + \int_0^1 C_{H,j,t}^i di \quad (5.6.1)$$

Now by combining (5.2.2) and (5.2.14), we have,

$$C_{H,j,t} = \left( \frac{P_{H,j,t}}{P_{H,t}} \right)^{-\epsilon} \cdot C_{H,t} \quad (5.6.2)$$

$$= \left( \frac{P_{H,j,t}}{P_{H,t}} \right)^{-\epsilon} \cdot \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \right] \quad (5.6.3)$$

If we assume that there are symmetric preferences across countries, this would imply that the home country preferences for foreign goods is equivalent to the foreign countries preference for local goods. Then, after making use of

(5.2.10), (5.2.6), and (5.2.14), and the expression  $P_{H,t} = \mathcal{E}_{i,t} P_{i,t}^i$ , we have,

$$\begin{aligned}
 C_{H,j,t}^i &= \left( \frac{P_{i,j,t}^i}{P_{i,t}^i} \right)^{-\epsilon} \cdot C_{i,t}^i \\
 &= \left( \frac{P_{i,j,t}^i}{P_{i,t}^i} \right)^{-\epsilon} \cdot \left( \frac{P_{i,t}^i}{P_{F,t}^i} \right)^{-\gamma} \cdot C_{F,t}^i \\
 &= \left( \frac{\mathcal{E}_{i,t} P_{H,j,t}}{\mathcal{E}_{i,t} P_{H,t}} \right)^{-\epsilon} \cdot \left( \frac{P_{i,t}^i}{P_{F,t}^i} \right)^{-\gamma} \cdot \alpha \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i \\
 &= \alpha \left( \frac{P_{H,j,t}}{P_{H,t}} \right)^{-\epsilon} \left( \frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{-\eta} C_t^i
 \end{aligned}$$

This allows us to rewrite the market clearing conditions for the  $j$  different goods in (5.6.2) as,

$$\begin{aligned}
 Y_{j,t} &= \left( \frac{P_{H,j,t}}{P_{H,t}} \right)^{-\epsilon} \cdot \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \dots \right. \\
 &\quad \left. \alpha \int_0^1 \left( \frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right] \quad (5.6.4)
 \end{aligned}$$

In this case, we can make use of the relationship between aggregate domestic output, and the output that is produced for each of the  $j$  goods, which is equivalent to the relationship in the closed economy New Keynesian model,

$$Y_t = \left( \int_0^1 Y_{j,t}^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} ; \quad Y_{j,t} = \left( \frac{P_{H,j,t}}{P_{H,t}} \right)^{-\epsilon} Y_t \quad (5.6.5)$$

We may then express the above market clearing conditions in (5.6.4) in terms of  $Y_t$ .

$$Y_t = \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right]$$

Thereafter, using the fact that  $P_{H,t} = \mathcal{E}_{i,t} P_{i,t}^i$ ,

$$\begin{aligned}
 Y_t &= \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma} \left( \frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{-\eta} C_t^i di \right] \\
 &= \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma-\eta} C_t^i di \right]
 \end{aligned}$$

Making use of the expression  $Q_{i,t} = \mathcal{E}_{i,t} P_{i,t}^i / P_t$  and  $P_{H,t} = \mathcal{E}_{i,t} P_{i,t}^i$ , such that  $Q_{i,t} = P_{H,t} / P_t$ ,

$$\begin{aligned} Y_t &= \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \left[ (1 - \alpha) C_t + \alpha \int_0^1 \left( \frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma-\eta} Q_{i,t}^\eta C_t^i di \right] \\ &= \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \left[ (1 - \alpha) C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{P_{i,t}^i} \frac{P_{F,t}^i}{P_{H,t}} \right)^{\gamma-\eta} Q_{i,t}^\eta C_t^i di \right] \end{aligned}$$

Then lastly, where the bilateral terms of trade for the home country, between itself and country  $i$  is  $S_{i,t} = P_{i,t}^i / P_{H,t}$ , then the bilateral terms of trade for country  $i$ , that imports home country goods is  $S_t^i = P_{H,t} / P_{F,t}^i$ , and the condition that links consumption in country  $i$  with broad consumption in (5.4.2) allows for,<sup>6</sup>

$$Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \left[ (1 - \alpha) + \alpha \int_0^1 (S_t^i S_{i,t})^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di \right] \quad (5.6.6)$$

In the case of the specific preferences (i.e.  $\sigma = \eta = \gamma = 1$ ), the previous condition could be rewritten as,

$$Y_t = C_t S_t^\alpha \quad (5.6.7)$$

which would suggest that output is dependent on the level of consumption in the home country, the terms of trade for this country and its degree of relative openness. However, where we allow for various different preferences we may derive the following first order log-linear approximation of (5.6.6), given that the terms of trade in the steady-state are,  $\int_0^1 s_t^i di = 0$ ,

$$y_t = c_t + \alpha \gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t$$

Furthermore, using the relationship in (5.3.8), it may be shown that,

$$\begin{aligned} y_t &= c_t + \alpha \gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) (1 - \alpha) s_t \\ &= c_t + \left( \frac{\alpha \omega}{\sigma} \right) s_t \end{aligned} \quad (5.6.8)$$

where  $\omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$ . By aggregating over all countries, the market clearing should ensure that,

$$\begin{aligned} y_t^w &= \int_0^1 y_t^i di \\ &= \int_0^1 c_t^i di \\ &= c_t^w \end{aligned} \quad (5.6.9)$$

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<sup>6</sup>After manipulating this expression,  $C_t^i = C_t Q_{i,t}^{-\frac{1}{\sigma}}$ , where  $\vartheta = 1$ .



where  $y_t^w$  and  $c_t^w$  are the indices for world output and consumption. Combining (5.6.9) with (5.6.8) and (5.4.4) provides,

$$\begin{aligned} y_t &= c_t^w + \left( \frac{1-\alpha}{\sigma} \right) s_t + \left( \frac{\alpha\omega}{\sigma} \right) s_t \\ y_t &= y_t^w + \frac{1}{\sigma_\alpha} s_t \end{aligned} \quad (5.6.10)$$

where  $\sigma_\alpha = \frac{\sigma}{1+\alpha(\omega-1)} \geq 0$ .

And then finally, after combining (5.6.8) with the consumption Euler equation in (5.2.21), we are left with,

$$\begin{aligned} y_t - \frac{\alpha\omega}{\sigma} s_t &= E_t\{y_{t+1}\} - E_t\left\{\frac{\alpha\omega}{\sigma} s_{t+1}\right\} - \frac{1}{\sigma} [i_t - E_t\{\pi_{t+1}\} - \rho] \\ y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma} [i_t - E_t\{\pi_{t+1}\} - \rho] - \frac{\alpha\omega}{\sigma} E_t\{\Delta s_{t+1}\} \end{aligned}$$

Now, given the expression in (5.3.4) and making use of (5.6.10), where  $\Delta s_t = \sigma_\alpha(\Delta y_t - \Delta y_t^w)$ , it may be shown that,

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma} [i_t - E_t\{\pi_{H,t+1}\} - \alpha E_t\{\Delta s_{t+1}\} - \rho] - \frac{\alpha\omega}{\sigma} E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma} [i_t - E_t\{\pi_{H,t+1}\} - \rho] - \frac{\alpha\Theta}{\sigma} E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha} [i_t - E_t\{\pi_{H,t+1}\} - \rho] + \alpha\Theta E_t\{\Delta y_{t+1}^w\} \end{aligned}$$

where  $\Theta = (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = \omega - 1$ . Note that expected future output / income on world markets will affect current output levels in the home country, and the quantum of the change is dependent on the degree of openness in the home country (where we assume that  $\Theta > 0$ ).

### 5.6.1 Trade Balance

If we denote net exports in terms of domestic output, expressed as a fraction of the steady state output,  $\bar{Y}$ , as

$$nx_t = \left( \frac{1}{\bar{Y}} \right) \left( Y_t - \frac{P_t}{P_{H,t}} C_t \right) \quad (5.6.11)$$

Then in the case where,  $\sigma = \eta = \gamma = 1$  we would have  $P_{H,t}Y_t = P_tC_t$ , since there is a preference for the consumption of home goods, they will all be consumed locally. However, to allow for a more general setting, taking a first order approximation would leave us with  $nx_t = y_t - c_t - \alpha s_t$ , which could be combined with (5.6.8) to provide,

$$nx_t = \alpha \left( \frac{\omega}{\sigma} - 1 \right) s_t \quad (5.6.12)$$

### 5.6.2 Aggregate output and employment

The aggregate measure of output may once again be given as,

$$Y_t = \left( \int_0^1 Y_{j,t}^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5.6.13)$$

This measure may be combined with the aggregate production function in (5.5.1), where  $N_{j,t} = Y_{j,t}/A_t$ , and the functional form of the aggregate demand functions in (5.2.2), where market clearing conditions ensure that  $Y_t = C_t$ ,

$$\begin{aligned} N_t &= \int_0^1 N_{j,t} dj \\ &= \frac{1}{A_t} \int_0^1 Y_{j,t} dj \\ &= \frac{Y_t}{A_t} \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} dj \end{aligned}$$

In this case, the expression,  $d_t = \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} dj$  is of second order and would not affect the result when taking a first order approximation. Hence, we would be left with the result,

$$y_t = a_t + n_t \quad (5.6.14)$$

### 5.6.3 Marginal costs and inflation dynamics

As shown in the description of the New Keynesian model, the optimal price setting condition for firms may be combined with the condition that describes the evolution of inflation (as a function of newly set prices) to provide an expression for domestic inflation, in the New Keynesian Phillips curve,

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda \underline{mc}_t \quad (5.6.15)$$

where  $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ .

The marginal costs in the open-economy may be derived from the production function of the firms (5.5.2), where we can also make use of the household's labour supply equation (5.2.19) and the expression for the terms of trade (5.3.3),

$$\begin{aligned} mc_t &= -v + (w_t - p_{H,t}) - a_t \\ &= -v + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -v + \sigma c_t + \varphi n_t + \alpha s_t - a_t \end{aligned}$$

After making use of (5.6.14), (5.4.4) and (5.6.9),

$$\begin{aligned} mc_t &= -v + \sigma \left( c_t^w + \left[ \frac{1-\alpha}{\sigma} \right] s_t \right) + \varphi(y_t - n_t) + \alpha s_t - a_t \\ &= -v + \sigma y_t^w + \varphi y_t + s_t - (1 + \varphi)a_t \end{aligned} \quad (5.6.16)$$

Substituting in for  $s_t$ , with the use of (5.6.10), allows for domestic output and productivity to be rewritten as,

$$mc_t = -v + (\sigma_\alpha + \varphi)y_t + (\sigma - \sigma_\alpha)y_t^w - (1 + \varphi)\alpha \quad (5.6.17)$$

Here we can see that in the open-economy, a change in domestic output effects marginal costs through its impact on employment (captured through  $\varphi$ ) and the terms of trade (captured by  $\sigma_\alpha$ ), which is a function of openness and the substitutability between domestic and foreign goods. Similarly, world output affects marginal costs through its effects on consumption (and hence the effect on the real wage is captured by  $\sigma$ ) and the terms of trade (captured by  $\sigma_\alpha$ ).

It may be worth considering that under flexible prices, where  $mc_t = -\mu$ , the natural level of output in the open-economy would be given by,

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_w y_t^w \quad (5.6.18)$$

where  $\Gamma_0 = \frac{-\mu}{\sigma_\alpha + \varphi}$ ,  $\Gamma_a = \frac{1+\varphi}{\sigma_\alpha + \varphi} \geq 0$ , and  $\Gamma_w = -\frac{\alpha\Theta\sigma_\alpha}{\sigma_\alpha + \varphi}$ .

#### 5.6.4 Characterising output and inflation dynamics

Where the output gap is represented as  $\tilde{y}_t = y_t - y_t^n$  and it is given that world output levels are not affected by domestic conditions, it may be assumed that domestic real marginal costs and the output gap are related through the expression,

$$\underline{mc}_t = (\sigma_\alpha + \varphi)\tilde{y}_t \quad (5.6.19)$$

Combining this expression with the New Keynesian Phillips curve in (5.6.15) allows for the more convenient expression in terms of output,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \tilde{y}_t \quad (5.6.20)$$

where  $\kappa_\alpha = \lambda(\sigma_\alpha + \varphi)$ . Note that the degree of openness,  $\alpha$ , affects domestic inflation through  $\sigma_\alpha$ . As such, where an increase in openness would lower  $\sigma_\alpha$ , the effects of an increase in domestic output (which may cause real depreciation of the currency), may not result in a large increase in marginal costs or inflation.

Using (5.6.11), we may once again express the New Keynesian IS curve in terms of the output gap,

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \quad (5.6.21)$$

where the natural interest rate is given as,

$$r_t^n = \rho - \sigma_\alpha \Gamma_a (1 - \rho_a) a_t + \frac{\alpha\Theta\sigma_\alpha\varphi}{\sigma_\alpha + \varphi} E_t\{\Delta y_{t+1}^w\}.$$

This relationship would suggest that the forward looking New Keynesian IS curve for the small open-economy is similar to what was produced for the closed economy. However, in this case, the degree of openness affects the sensitivity of the output gap to interest rate changes and the natural interest rate would often depend on both the expected world output growth and domestic productivity.

## 5.7 The Interest Rate Rule

We could make use of an interest rate rule as was discussed in the closed economy New Keynesian model, which is given as,

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) [\varrho_\pi \pi_{H,t} + \varrho_y \tilde{y}_t] + \epsilon_t^i \quad (5.7.1)$$

Certain variants of this rule have sought to suggest that the central bank may (or should) respond to exchange rate variations (Ball, 1999). Under these conditions, monetary policy may be implemented with the aid of a generalised Taylor rule that includes a measure for the external value of the currency, where,

$$i_t = \varrho_i i_{t-1} + (1 - \varrho_i) [\varrho_\pi \pi_{H,t} + \varrho_y \tilde{y}_t + \varrho_d d_t] + \epsilon_t^i \quad (5.7.2)$$

In part three we consider whether or not it would be optimal for the central bank to respond to changes in the exchange rate.<sup>7</sup>

## 5.8 Conclusion

The small open-economy model provides a rich characterisation of economic behaviour, particularly for economies such as South Africa. In addition, the out-of-sample properties of these models would appear to suggest that they fit the data particularly well.<sup>8</sup>

After discussing the use of data in this model, we then turn our attention to the use of this model structure in part three, where we also ask a number of policy relevant questions, such as “would it be optimal to include the exchange rates in the reaction function of the central bank?”.

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<sup>7</sup>Justiniano and Preston (2010) and Alpanda *et al.* (2010a) suggest that incorporating the exchange rate in the response function would lead to greater volatility in inflation, output and exchange rate volatility. However, Kumhof *et al.* (2010) would appear to suggest that for different specifications of the model, these results may not hold.

<sup>8</sup>For example, Alpanda *et al.* (2011) and Steinbach *et al.* (2009b) suggest that the forecasting performance of these models compares favourably to other forecasting models that are used by central banks.

# Part II

## Macroeconomic Data

## Chapter 6

# South African Business Cycle Data

## 6.1 Introduction

The seminal contribution of Burns and Mitchell (1947) established a framework for cataloguing the empirical features of business cycles in the United States. Their methodology for isolating the business cycle from the slowly moving trend and the regular seasonal component has been applied in several countries, including South Africa. Whilst this practice for analysing the business cycle has intuitive appeal, it is extremely time consuming, which has lead to the use of alternative methods by many researchers and policy-makers.<sup>1</sup>

When making use of a particular technique to identify the business cycle, Prescott (1986) notes that all of these measure of the business cycle are imprecise; and hence one should always scrutinise the results to determine whether they are representative of business cycle facts. Therefore, different specifications for the respective techniques that may be used to identify the business cycle from measures of economic output are considered in this chapter, to ensure that a consistent measure is used in the modelling in part three.

The variables that are use to model South African business cycle phenomena include: measures of economic output, aggregate prices, consumer prices, nominal wages, labour productivity, nominal interest rates, exchange rates, foreign economic output, foreign prices, and foreign nominal interest rates. Since measures of economic output are only available at a quarterly frequency all the variables are transformed to quarterly measures. These are displayed in figure 6.1 for the period 1975Q1 to 2012Q4, (i.e. the post Bretton-Woods period).<sup>2</sup>

A preliminary inspection of this data would suggest that, with the exception of interest rates, most of these variables exhibit positive trending behaviour. This is important, since the properties of the data should be consistent with what is to be described in the empirical model. Therefore, if we have formulated a model around the cyclical behaviour of macroeconomic data (that ignores the trends that may be present in the data), then we would first need to distinguish between the trend and cycles, before incorporating the variables in the model.

In addition, it is worth noting that most statistical time-series models have been developed for covariance-stationary data (obviously with the exception of the cointegrated VAR model). Yule (1926) showed that when we include

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<sup>1</sup>This method originally considered the variation in approximately 400 different variables to identify periods of relative expansion and relative decline.

<sup>2</sup>In August 1971, Richard Nixon announced the “temporary” suspension of the dollar’s convertibility into gold; and by March 1973 most of the the major currencies began to float against each other.

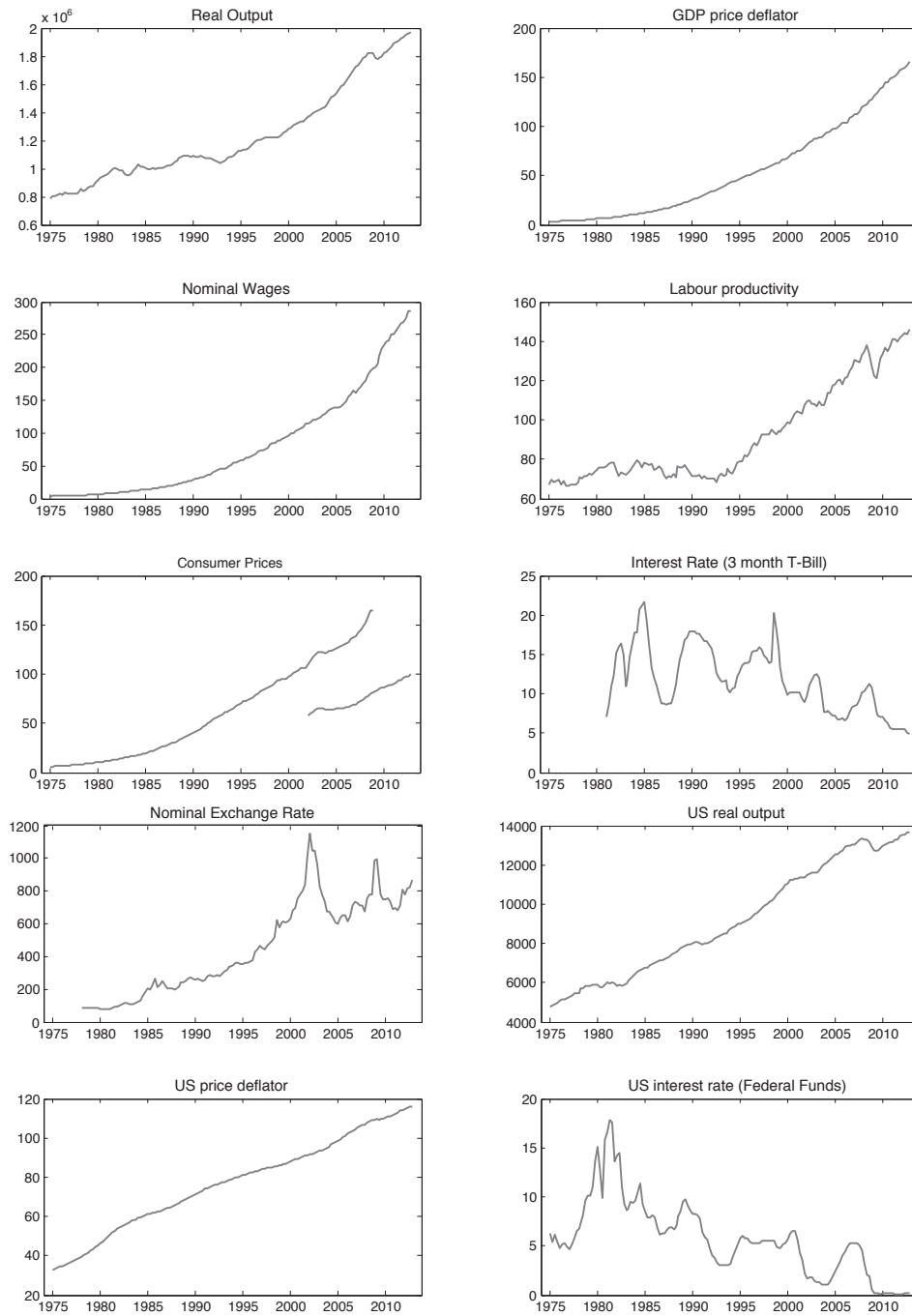


Figure 6.1: Macroeconomic data in levels (1975Q1-2012Q4)



trending variables in such a model, which assumes that the data is stationary, then we run the risk of generating spurious results. Hence, if the model is not able to deal with trends, then we need to ensure that they are removed from the data. At this point, it is worth noting that DSGE model solutions are expressed in terms of temporary departures from steady state values, which would imply that the data should exhibit stationary characteristics.

## 6.2 Decomposing the data into trends & cycles

There are many ways to decompose the data into components that represent the trend and the cycle. Baxter and King (1999) suggest that a business cycle would normally extend from 6 to 40 quarters. In what follows, we focus on several widely used methods that have been used to transform the data for use in many DSGE models.<sup>3</sup>

To ensure that the data is consistent with the structure of the model, most methods employ a logarithmic transformation, as the theoretical models that have been described in parts one and three of this dissertation, make use of log-linear approximations of the structural equations.<sup>4</sup> When making use of the first difference of the logarithm of a variable, the result may be interpreted as a growth rate, which is particularly convenient in many cases.

When making use of a proxy for the foreign economy, we make use of data for the United States economy as it was South Africa's single largest trading partner over the sample period. In certain instances it may have been desirable to make use of trade weighted data, however, whilst the effective nominal exchange rate is calculated by the South African Reserve Bank, the distribution of a trade weighted output and prices variables are not publicly available. In an attempt to remain consistent we only therefore make use of US data for the foreign economy.

### 6.2.1 Output:

The measure of output is provided by seasonally adjusted Gross Domestic Product (GDP) at constant (or chained) prices.<sup>5</sup> This variable represents real GDP, which exhibits strongly trending behaviour.

In several monetary models that rely on variants of the Taylor rule to describe policy makers behaviour, the primary interest is in incorporating a mea-

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<sup>3</sup>Note that there is no guarantee that the data will be covariance stationarity after one has removed the trend to focus on the cycle, and as such it is usually prudent to investigate the properties of the data before it is included in the final model.

<sup>4</sup>In addition, the use of logarithmic transformation also ensures that the data is scaled to a certain degree.

<sup>5</sup>The data was obtained from the South African Reserve Bank (Code: KBP6006D - Gross Domestic Product at Real Prices).

sure of the output gap in the model.<sup>6</sup> To derive a measure of the output gap many economists rely on filter based techniques or model based approaches. The most widely used filtering technique is to apply the filter that was developed by Hodrick and Prescott (1997), which may be used to identify the trend in output. The difference between the trend and the actual data is then termed the output gap, which may represent the cyclical component of output. A popular model-based alternative approach makes use of a structural vector autoregressive (SVAR) model, along the lines proposed by Blanchard and Quah (1989) and Clarida and Gali (1994). The later of these techniques has been applied to South African data by Du Plessis *et al.* (2008).<sup>7</sup>

### 6.2.1.1 The Hodrick-Prescott filter

To apply the Hodrick-Prescott filter, it may be specified as the minimisation of the following expression, where  $\ddot{y}_t$  represents the logarithmic value of output and  $\bar{y}_t$  is the trend of this process.

$$\sum_{t=1}^T (\ddot{y}_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{T-1} [(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1})]^2 \quad (6.2.1)$$

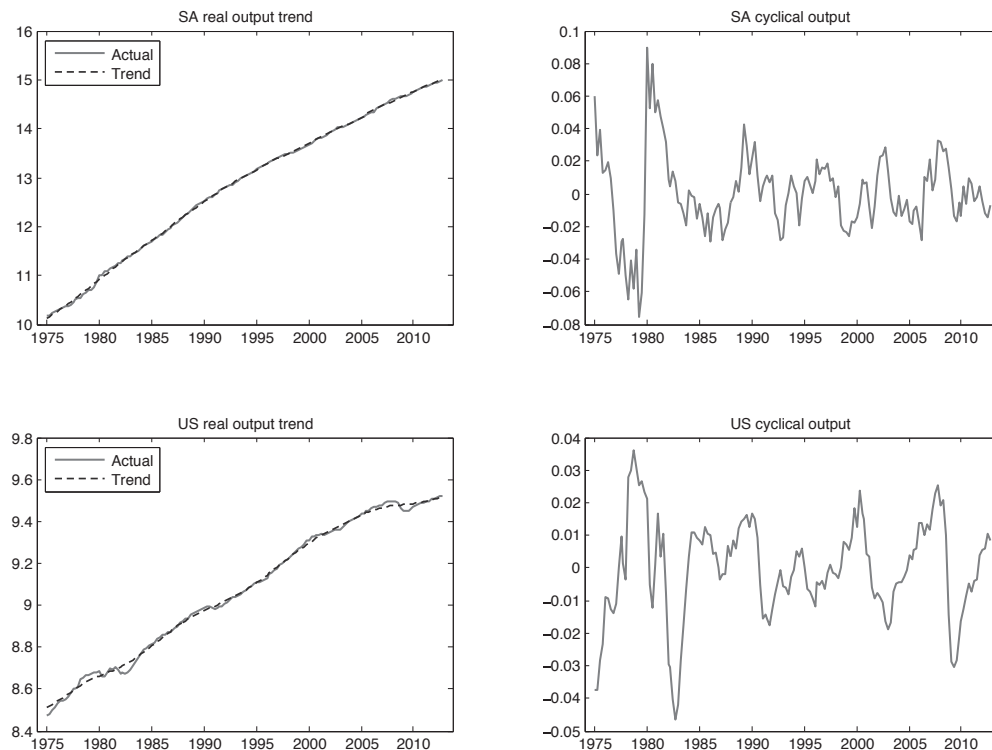
In this case,  $\lambda$  is a constant, which reflects the penalty of incorporating fluctuations in the trend. If  $\lambda$  is a very large number, then the second part of this expression is all important. Hence, as  $\lambda \rightarrow \infty$ , we would minimise the second part of the expression by choosing a values for  $\bar{y}_t$ , where  $(\bar{y}_{t+1} - \bar{y}_t) = (\bar{y}_t - \bar{y}_{t-1})$ . This point would be achieved when the value for all  $\bar{y}_t$  are constant, such that we are left with a linear trend. Alternatively, when  $\lambda$  approximates a very small number (i.e.  $\lambda \rightarrow 0$ ), then the first part of the expression will dominate and the minimisation of the difference between the observed variable and the trend,  $(\ddot{y}_t - \bar{y}_t)$ , would imply that all the values for  $\bar{y}_t$  would follow  $\ddot{y}_t$ .

The actual value that is assigned to  $\lambda$  is dependent upon the frequency of the data. When applied to US data, Hodrick and Prescott (1997) chose a value of 1,600 for quarterly data, as they felt that such a value provided a suitable result for the approximation of the post-war United States business cycle. The use of the Hodrick-Prescott filter for South African and United States output is displayed in figure (6.2).

The practice of adopting this value of  $\lambda$  to other countries may be problematic as the filter may assign parts of the low frequency cyclical fluctuations to the trend (and vice verse). For example, if a particular country experiences shorter cycles than the US, then a portion of the trend component for this country would be assigned to cyclical component. This lead to the development of empirical techniques to derive appropriate values for  $\lambda$ , particularly

<sup>6</sup>See, Taylor (1999), as well as part one of this dissertation.

<sup>7</sup>A review of these practices, when applied to South African macroeconomic data, is provided by Klein (2011).



**Figure 6.2:** Hodrick-Prescott filter for output trend and cycle (1975Q1-2012Q4)

for cases where this filter is applied to the data of countries that may not be equivalent to that of the United States. Two popular approaches include the spectral techniques of Pedersen (2001) and the minimisation routines of Marcet and Ravn (2003).

The spectral technique that was employed in Pedersen (1998), seeks to identify a complete cycle by approximating an ideal high pass filter that cuts off components that are below some pre-specified cut-off frequency, whilst leaving components at higher frequencies unchanged. This technique was applied to South African Gross Domestic Product data in the analysis of Rand and Tarp (2002) and Du Toit (2008). After replicating this technique for our sample, we find that when considering that the duration of the South African cycle may extend over five, six, seven, or eight years, which produce values for  $\lambda$  of 177, 352, 524 and 1339, respectively.

An alternative means of finding an optimal value of  $\lambda$  may be obtained by equating the constraint for the sum of squared second differences in South African to that of the United States. This would suggest that the acceleration and amplitude of the cycle is at least equivalent in the two countries. This methodology was originally developed by Marcet and Ravn (2003) who formulate the minimisation problem,

$$\min_{[g_t]_{t=1}^T} \sum_{t=1}^T (y_t - g_t)^2 \quad (6.2.2)$$

$$\text{subject to } \frac{\sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2}{\sum_{t=1}^T (y_t - g_t)} \leq V \quad (6.2.3)$$

where  $V \geq 0$  is a constant that is specified by the researcher to approximate the acceleration in the trend relative to the variability of the cyclical component. Setting  $V$  constant across countries ensures comparability across countries in the sense that the variability of the acceleration of the trend relative to the variability of the cyclical component is common.

Now if we multiply both sides of equation (6.2.2) by  $\sum_{t=1}^T (y_t - g_t)^2$  then the Lagrangian of the above minimisation problem solves:

$$\min_{[g_t]_{t=1}^T} (1 - \bar{\lambda}V) \sum_{t=1}^T (y_t - g_t)^2 + \bar{\lambda} \sum_{t=2}^{T-1} ((g_{t+1} - g_t) - (g_t - g_{t-1}))^2 \quad (6.2.4)$$

where  $\bar{\lambda}$  is the Lagrange multiplier of the transformed constraint in equation (6.2.2). The solution to this Lagrangian and the Hodrick-Prescott filter are equivalent if:

$$\lambda = \frac{\bar{\lambda}}{1 - \bar{\lambda}V} \quad (6.2.5)$$

The usual value of  $\lambda = 1600$  can then be interpreted as the value of  $\lambda$  that satisfies equation (6.2.5) when  $\bar{\lambda}$  is the Lagrange multiplier of the rewritten constraint in (6.2.2) for the US value of  $V$ . The  $\lambda$  for South Africa will be endogenously determined by solving the Lagrange multiplier for the constraint in equation (6.2.2). Computation of  $V$  requires the use of an iterative scheme, where for a given value of  $\lambda$  we compute the trend on the basis of the Hodrick-Prescott filter for each possible value of  $\lambda$  and define the function,

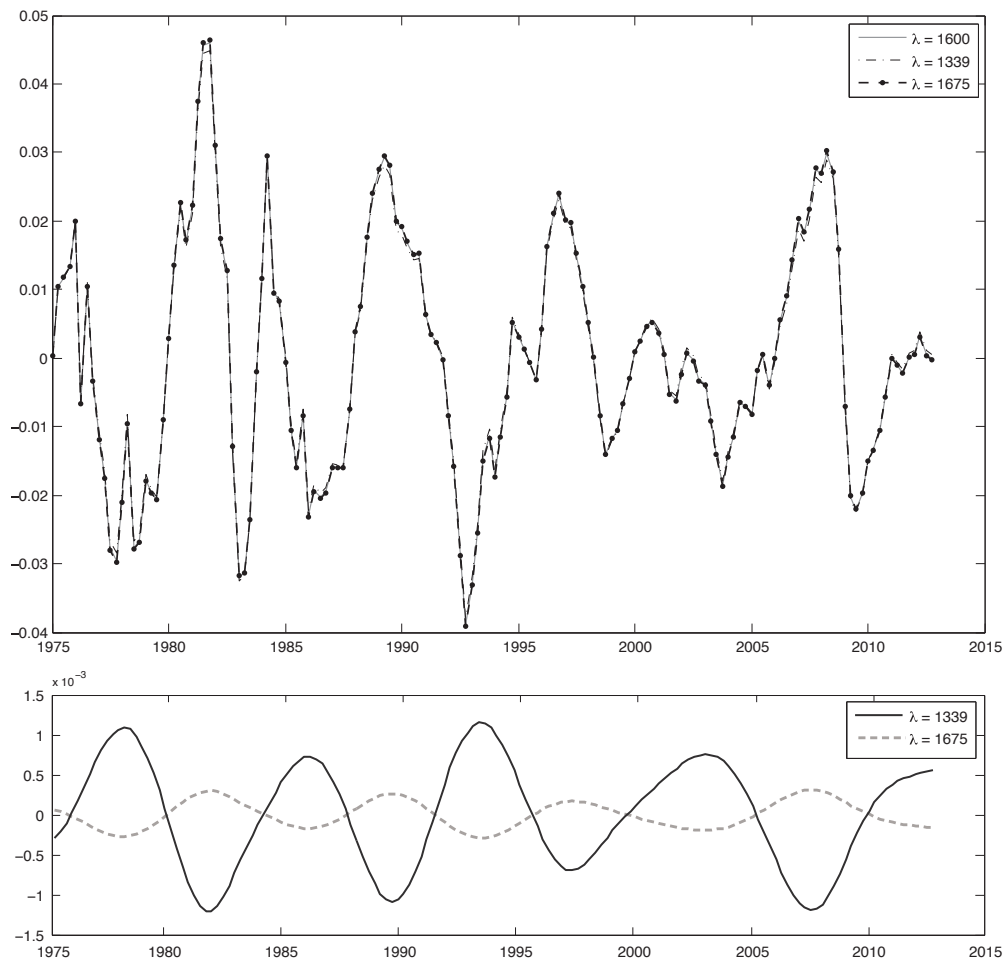
$$F(\lambda) \equiv \frac{\sum_{t=2}^{T-1} [(g_{t+1}(\lambda) - g_t(\lambda)) - (g_t(\lambda) - g_{t-1}(\lambda))]^2}{\sum_{t=1}^T (y_t - g_t(\lambda))} \quad (6.2.6)$$

where  $g_t$  refers to the trend component that relates to  $\lambda$ .

To measure the robustness of the results, Marcet and Ravn (2003) explore a second method that replaces equation (6.2.2) with the following constraint,

$$\frac{1}{T-2} \sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \leq W \quad (6.2.7)$$

This constraint restricts the variability of the acceleration in the trend component directly and  $\lambda$  will now have the interpretation of the Lagrange multiplier on equation (6.2.7). To implement this procedure we choose  $W$  on the basis of the value implied by applying the standard Hodrick-Prescott filter to the United States.



**Figure 6.3:** Output gap based on Hodrick-Prescott filter for different values of  $\lambda$  (1975Q1-2012Q4). The top panel displays the cyclical component and the bottom panel considers difference between the two optimal values and the standard value of 1600.

The difference between these two constraints is that whilst the second imposes the same variability of the growth of the trend across countries, the first constraint allows for a larger variability in the growth rate in countries with a more volatile cyclical component. i.e. you would want to impose the second constraint if you believed that South Africa and the United States share common trends, whilst the first constraint would be used where the researcher

believes that the two countries grew at very different rates. When applying this technique to the South African GDP series we note that the value of  $\lambda$  ranges from 1675 using the first constraint and 1772 using the second constraint, where the values for  $V$  and  $W$  are 0.000004 and 0.000187, respectively.

After performing this analysis, it is worth noting that the spectral technique of Pedersen (2001) suggests that one should employ a smaller value for  $\lambda$ , whilst the optimisation approach of Marcet and Ravn (2003) suggests that a larger value should be used. The inner bounds of these measures for  $\lambda$  have been used and are depicted in the top panel of figure (6.3), where it is noted that the difference would appear to be extremely marginal and the resulting turning points are much the same. The bottom panel in this figure represents the difference between the two optimal values and the standard value of 1600, where the scale of horizontal axis is extremely small.

### 6.2.1.2 The structural vector autoregressive model

The alternative method of using a structural vector autoregressive model to identify the output gap, as proposed by Du Plessis *et al.* (2008). This model is used to identify aggregate demand and supply shocks to economic activity, using measures of real output, as well as fiscal and monetary policy measures.<sup>8</sup> The identified (structural) shocks can then be used in a historical decomposition of aggregate demand to distinguish between actual and potential output. This measure is then used to derive a measure of the South African business cycle.

The result of this analysis is contained in figure (6.4), where it is suggested that the output gap exhibits a strong stochastic trend. Whilst the existence of such a trend may reflect the actual behaviour of this unobserved measure, it should be noted that this series would most probably be covariance non-stationary. Hence, if were one to use this measure for the output gap in a DSGE model, we would first need to remove the trend before it could be used in the model that we have developed. The removal of the trend from the data would also make the interpretation of the final results somewhat problematic, as they would need to be interpreted in terms of a de-trended output gap.

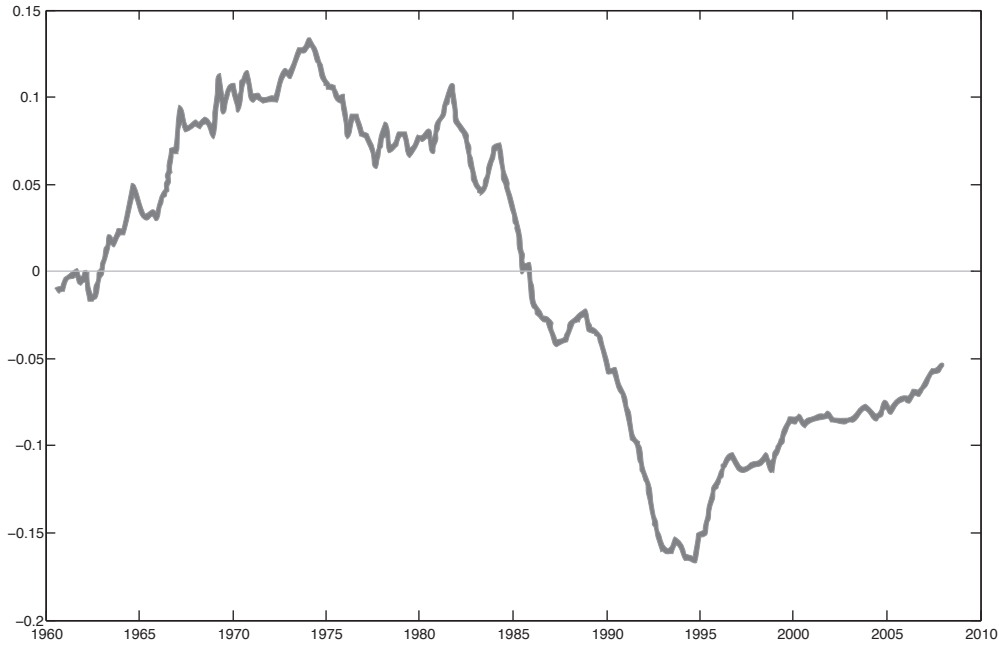
Given the results of the above analysis, it is decided to proceed with the traditional Hodrick-Prescott filter with a value of  $\lambda = 1600$  for the smoothing parameter.

## 6.2.2 Price Deflators:

Broad measures of the cyclical behaviour of price movements are described by GDP price deflators. The measurement of these differs slightly in South

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<sup>8</sup>The variables that were used in Du Plessis *et al.* (2008), include the first difference of log real GDP, ratio of government consumption to GDP, and the real interest rate.



**Figure 6.4:** SVAR measure of the output gap (1960Q2 - 2006Q4)

Africa to that of the United States (since in the US an index for price deflator is provided). In South Africa this measure is calculated from the differences between nominal and real GDP. Hence, where  $\tilde{h}_t$  represents the GDP deflator and  $\tilde{y}_t$  represents nominal GDP, the transformation for South African data is calculated as  $\tilde{h}_t = [\tilde{y}_t/y_t] \times 100$ . This represents an index of broad price movements that is equivalent to the index that is provided in the United States. To calculate the change in prices we calculate,  $\pi_t^h = \log[\tilde{h}_t/\tilde{h}_{t-1}]$  for South Africa and  $\pi_t^{h,*} = \log[\tilde{h}_t^*/\tilde{h}_{t-1}^*]$ .<sup>9</sup>

### 6.2.3 Wages & Productivity:

Wage inflation is derived from the nominal index of total remuneration per worker in the non-agricultural sector, and growth in productivity is derived from the labour productivity in the manufacturing sectors.<sup>10</sup>

<sup>9</sup>Data for measures of the Price Deflators was obtained from the South African Reserve Bank (Codes: KBP6006D & KBP6006L - Gross Domestic Product at Real Prices & Gross Domestic Product at Current Prices) and the Bureau of Economic Analysis (Code: Table 1.1.4 - Price Indexes for Gross Domestic Product). As noted previously, the United States was South Africa's single biggest trading partner over the entire sample. Therefore, we consistently make use of US data to proxy the foreign economy for measures of output, prices, interest rates and exchange rates.

<sup>10</sup>The broader measure of labour productivity for all non-agricultural sectors went through several changes in its data collection process, which resulted in noticeable break-

Both of these measures for wages and productivity are provided in the form of indices which facilitates a relatively straightforward means of transformation, where the change is represented as  $\pi_t^w = \log[w_t/w_{t-1}]$  and  $\Delta z_t = \log[z_t/z_{t-1}]$ , respectively.<sup>11</sup>

#### 6.2.4 Interest rates:

The well developed South African capital markets generate a large number of interest rates with respect to various risk and maturity profiles. To describe the conduct of South African monetary policy, following the adoption of the inflation targeting regime one would prefer to use the Repurchase Rate, which is applied to loans provided by the central bank. However, this rate only came into existence following the introduction of this operating procedure and as such we need to make use of an alternate short-term interest rate. The most widely used of these, which extends back over a sufficient period of time is the 3-month Treasury Bill rate (which is highly correlated with Repurchase Rate over the common sample). This interest rate,  $\bar{i}_t$ , is provided on a weekly basis so we would need to take the average over the respective weeks in quarter.

Thereafter, as the data is quoted as an annual rate, we convert it to the respective amount of interest that is earned over the quarter, using the annualisation formula,  $i_t = [1 + (\bar{i}_t/100)]^{0.25} - 1$ .<sup>12</sup>

Foreign interest rates are represented by the monthly United States Federal Funds rate, which is also converted into the average quarterly rate, before it is annualised.<sup>13</sup> It is represented by the expression  $i_t^*$ .

#### 6.2.5 Nominal exchange rate:

The depreciation rate for the value of South Africa currency is represented by the quarterly percentage change in the South African rand relative to the US dollar.<sup>14</sup> Hence, the data is transformed by taking  $\Delta d_t = \log[d_t/d_{t-1}]$ .

points in 2002q3, 2004q4 and 2006q2. Therefore, in this case the use of labour productivity from the manufacturing sector was deemed prudent.

<sup>11</sup>Data for measures of the Wages & Productivity was obtained from the South African Reserve Bank (Codes: KBP7013L & KBP7079L - Total remuneration per worker [non-agric] & Manufacturing: Labour productivity).

<sup>12</sup>Data for the short-term interest rate was obtained from the South African Reserve Bank (Code: KBP1405W - Discount rates on 91-day Treasury Bills).

<sup>13</sup>Data for measures of the Foreign Interest Rate was obtained from the Federal Reserve Bank (Code: H15 - Effective Federal Funds Rate).

<sup>14</sup>Data for measures of the nominal exchange rate was obtained from the South African Reserve Bank (Code: KBP5339M - Foreign exchange rate: SA cent / USA dollar).



### 6.2.6 Consumer Prices:

The construction of South African consumer price indices has recently undergone a significant structural break, to ensure that they are consistent with international practices. Details of the change in methodology are contained in a number of governmental reports, including Statistics South Africa (2009a, 2009b). In addition, the construction of this price index for this period is discussed in greater detail in chapter 8, where we also seek to identify a measure of core inflation.<sup>15</sup>

In many monetary models that are formulated for policy-making purposes central bankers are primarily concerned with an underlying measure of inflation, as they would not wish to formulate policy (that may affect the economy for many years to come) on the basis of a temporary shock to aggregate consumer price data. Such a measure of underlying inflation is often termed core-inflation and the methods used for its calculation are relatively extensive. In chapter 8 we make use of a number of these methods to derive a consistent estimate of core inflation for South Africa.

At this point it is worth noting, that to derive a measure of core inflation we use a year-on-year measure of annual inflation, which may be converted to the amount of inflation for the quarter, using the same annualisation formula that was applied to interest rates.

## 6.3 Conclusion

This chapter considers the use of various transformations that may be applied to the variables that may be included in a dynamic stochastic general equilibrium model for a small-open economy. One of the key variables that is usually included in these models is a measure of the cyclical component in economic output. We note that the use of an optimal measure for the smoothing coefficient in the Hodrick-Prescott filter, when applied to South African data, does not seem to have a large influence over the characterisation of the business cycle. In addition, this chapter also included details of the various transformations that may be applied to the other key variables in the model.

The following chapter considers the identification of an appropriate starting point for the dataset, as we would like to avoid the inclusion of large structural breaks. It includes an analysis of the first two moments of several macroeconomic variables with the aid of univariate and multivariate techniques.

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<sup>15</sup>Data for consumer prices was obtained from Statistics South Africa (Code: P0141.1 - CPI: All items - Index 2005=100 & CPI: Headline Inflation - Index 2012=100).

## Chapter 7

# Trends and Structural Changes in South African Macroeconomic Data

## 7.1 Introduction

Since the volatile 1970s, the economies of several developed and developing countries have become considerably more stable, with lower volatility for real output (and other macroeconomic series) coupled with low and stable inflation.<sup>1</sup> This development has been called the “great moderation” and was first observed in the economy of the United States, with comparable evidence soon emerging for other developed economies.<sup>2</sup> Du Plessis *et al.* (2007) and Burger (2008) have subsequently identified a similar moderation for the South African economy since the early to mid-nineties.

The existence of such phenomena may be reflected as a structural break in the behaviour of several macroeconomic variables. Ideally, one would not wish to include such an event in the dataset that is used in any empirical model that does not explicitly cater for such behaviour, as it would make it more challenging to identify consistent estimates of the parameters in the model.<sup>3</sup> Hence, whilst this event is highly desirable from a welfare perspective, some attempt should be made to understand and identify the timing of these events.

Popular hypotheses ascribe the causes of the event to “good policy”, an evolution of the economic structure, “good luck”, or a combination of these factors.<sup>4</sup> Examples of “good policy” would include successful anti-cyclical monetary and fiscal policies by government, and/or better management of inventory investment by the private sector (Du Plessis *et al.*, 2007). Structural hypotheses focus on the importance of the relative expansion of the less volatile services sectors of the economy as an explanation for the great moderation. In contrast with the structural and “good policies” hypotheses, the “good luck” hypothesis suggests that the increasingly interlinked international economy (with more trade and capital flows) coincided with an exceptionally benign period of modest shocks that followed the spate of international disturbances of the seventies.

Burger (2008) recently considered three possible causes of the great moderation in South Africa (better monetary policy, a more efficient financial sector, and improved inventory management), all of which concern domestic factors

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<sup>1</sup>A wealth of literature describes the nature and possible causes of this decline in volatility across many macroeconomic series since the volatile 1970s. The seminal papers with respect to the U.S. economy include: Kim and Nelson (1999), McConnell and Perez Quiros (2000), and Blanchard and Simon (2001).

<sup>2</sup>The early papers with evidence of the great moderation beyond the U.S. economy are Stock and Watson (2005) and Stock and Watson (2003*b*).

<sup>3</sup>The DSGE framework that is used in part three does not allow for the explicit inclusion of structural breaks.

<sup>4</sup>See, for example, Gali and Gambetti (2009).

and could be grouped under the “structure and good policy” headings. His evidence supported two of the three hypotheses, that is, better monetary policy and a more efficient financial sector contributed to greater economic stability, whilst better inventory management did not. His evidence on monetary policy extends earlier work by Du Plessis *et al.* (2007), Du Plessis *et al.* (2008) and Du Plessis (2006) who provided evidence from a number of different analytical perspectives that monetary policy explains a large part of the great moderation in South Africa, with fiscal policy also contributing to this event.<sup>5</sup>

It has further been argued that the political situation in South Africa contributed to higher levels of economic volatility during the seventies, and Burger (2008) used the Soweto riots of 1976 and the political transition of 1994 as plausible boundaries for the period of macroeconomic instability in South Africa, while Du Plessis *et al.* (2007) also used the 1994 transition as the demarcation line between the earlier period of instability and the more stable recent period.

In addition to local politics and policy, the stability of the South African economy is also affected by the international environment, especially since the economy is relatively small and open to both large capital flows and significant international trade. The international literature has attributed an important role in the great moderation to a more benign international environment,<sup>6</sup> which was transmitted to local economies via trade and capital market connections. This is an increasingly important factor in the South African context because it coincided with the liberalisation of the capital account and the promotion of greater trade liberalisation after the late eighties.<sup>7</sup>

The evidence on causes of the great moderation cannot, however, be separated from the question of dating the great moderation, as the identification of changes in policy, practice or international conditions need to match the observed changes in macroeconomic volatility to support these particular hypotheses. In contrast with the international literature, where much effort has been spent investigating this question, less attention has been directed towards this area of research in South Africa, with authors like Du Plessis *et al.* (2007) using the political transition of 1994 as an implicit start, while Burger (2008) used “ocular inspection” to split the post-1960 history into three samples: 1960 to 1976 (a period of stability), 1976 to 1994 (a period of high volatility) and

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<sup>5</sup>There has been some controversy in the South African literature over the extent to which fiscal policy has stabilised the economy. A number of authors have argued that fiscal policy has become pro-cyclical in recent years, including Burger and Jimmy (2006) and Frankel *et al.* (2007). This has been attributed to ineffective automatic stabilisers by, for example, Swanepoel and Schoeman (2003) and Swanepoel (2004). More recently, this evidence has been disputed after the cyclical component of government revenue was estimated using more rigorous techniques and model-based assessments of the cyclicity of fiscal policy, by Du Plessis and Boshoff (2007) and Du Plessis *et al.* (2007).

<sup>6</sup>See, Ahmed *et al.* (2004), Blanchard and Simon (2001), and Sensier and Dijk (2004).

<sup>7</sup>The extent of trade liberalisation in South Africa is discussed in Edwards and Lawrence (2006). A historical account of the liberalisation of the South African capital account can be found in Gidlow (2002).

post-1994, a period of great moderation.

The contribution of this chapter lies firstly in a more rigorous identification of the start of the great moderation in South Africa, and secondly, it motivates the use of time-varying stochastic volatility models to describe the nature of the declines in volatility. Such models have been used to good effect by Stock and Watson (2003a), who conduct an extensive investigation into the time-varying characteristics of volatility in the United States.

After describing the nature of changes to the first two moments of South African macroeconomic data with the aid of the stochastic volatility model, we then utilise the model to identify break-points in the variation of several macroeconomic variables (over time), using the method that is developed in Quandt (1960), Bai (1997), Hansen (1997) and Bai *et al.* (1998). This procedure also allows us to identify whether a break-point is a result of a significant change in either the first or the second moment of a time-series.

In the following sections, we describe the data before we present the formulation of the stochastic volatility model. This is followed by the results of the model and a number of break-point tests in both univariate and multivariate settings to derive a specific date for the observed changes.

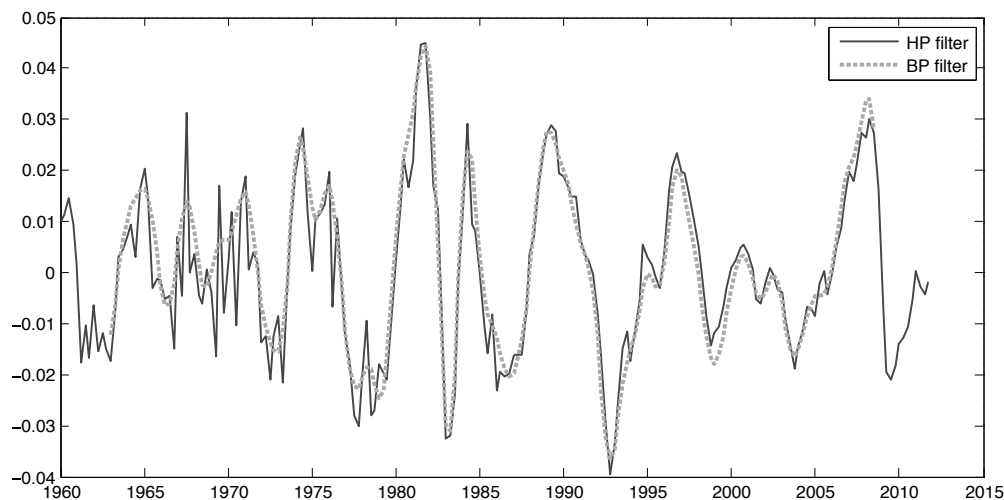
## 7.2 The volatility of South African macroeconomic data

To provide an initial description of the South African business cycle, we make use of a Band-Pass and Hodrick-Prescott filter to identify the cyclical component of South African real GDP, reported in figure 7.1. We note that during the 1960s and early 1970s the cycle seems to be relatively volatile, although the amplitude of the cycle is not too dramatic. During the mid 1970s to the early 1980s, the cycle seems to be less volatile, but in this case, the amplitude of the cycle increases significantly. During the late 1980s to the most recent financial crisis, the cycle seems to have become stretched<sup>8</sup> (i.e. longer phases) with lower volatility. This observation is confirmed in table 7.1, where we note that the year-on-year volatility of GDP first increased and then decreased over the five decades in the sample.

Such a decline in volatility has important implications for policy makers, given the welfare gains that may be derived from a less volatile business cycle. For example, lower volatility may reduce uncertainty surrounding future economic events, which contribute towards better decision making by most economic agents. In addition, lower economic volatility would also contribute towards a lower country risk-rating, which would lead to a decline in the average interest rate on national debt. Policy lessons could also be derived from understanding the nature and cause of the changes in volatility. A further

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<sup>8</sup>See also, Du Plessis (2004).

**Figure 7.1:** South African Business Cycle

(technical) reason for investigating the time varying properties of the business cycle is that it may affect the formulation of appropriate econometric business cycle models.

Period	Mean	Std Dev
1960:1 1969:4	5.6	1.8
1970:1 1979:4	3.2	2.2
1980:1 1989:4	2.2	3.2
1990:1 1999:4	1.4	2.3
2000:1 2009:4	3.6	2.1
2000:1 2011:4	3.5	1.9
1960:1 2011:4	3.1	2.7

**Table 7.1:** Year-on-year growth rates for South African gross domestic product

In the following analysis, we investigate the properties of the expenditure and production components of GDP, as well as measures of employment, wages, inflation, production, interest rates, and exchange rates. This data is measured at a quarterly frequency after eliminating trends and obvious non-stationarity. In terms of the components of GDP and prices, these are all expressed as first-difference growth rates (i.e.  $\log(y_1/y_2)$ ), and interest rates are expressed as annualised first differences (i.e.  $y_1 - y_2$ ).<sup>9</sup>

<sup>9</sup>Comprehensive details of the data sources, acronyms and transformations are provided in table A.1, which is contained in the appendix for additional details in §(A.2.1).

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To summarise the changes in the volatility of South African components that are likely to impact on the GDP growth rate, table 7.2 reports the standard deviations of 32 economic time series for each decade over the period 1960q2 to 2011q4. Each decade's standard deviation is scaled by the standard deviation of the full sample for that series, with values less than one indicating a period of low volatility relative to the full sample period.

code	Std deviation	Std deviation relative to 1960-2011					
	1960-2011	1960-1969	1970-1979	1980-1989	1990-1999	2000-2009	2000-2011
GDP	0.027	1.1	0.81	1.24	1.08	0.77	0.71
CONS	0.029	0.89	1.03	1.31	0.91	0.89	0.83
CDUR	0.122	1.04	0.98	1.45	0.56	0.83	0.86
CSDU	0.055	0.74	1.08	1.15	0.72	1.18	1.11
CNDU	0.028	0.99	0.92	1.02	1.2	0.93	0.87
CSER	0.031	0.79	1	1.52	0.8	0.62	0.6
GOV	0.046	1.32	1.08	0.9	1.03	0.28	0.27
GFCF	0.086	1.04	0.89	1.32	0.88	0.8	0.77
GFCR	0.121	1.2	0.87	1	0.73	1.14	1.14
GFCN	0.124	0.9	0.95	1.12	1.22	0.61	0.58
INV	0.024	1	1.32	1.16	0.74	0.53	0.52
GDE	0.05	1.13	1.11	1.25	0.74	0.59	0.56
EXP	0.065	0.74	0.94	1.29	1.08	1.44	1.33
IMP	0.132	1.17	1.03	1.34	0.61	0.77	0.74
GVAP	0.057	0.84	0.82	0.76	1.65	0.44	0.55
GVAS	0.049	1.26	0.79	1.21	0.93	0.93	0.87
GVAT	0.023	1.13	0.98	1.14	1.01	0.67	0.65
GVA	0.026	1.04	0.76	1.25	1.13	0.79	0.73
EMP	3.474	.	1.3	0.91	0.84	0.98	0.92
WAGE	0.041	.	0.92	0.99	0.93	1.05	1.02
CPI	0.038	.	0.23	1.12	0.7	1.23	1.25
DEF	0.05	1.31	0.88	1.29	0.65	0.56	0.54
PRND	0.289	.	0.66	1.45	0.73	0.99	0.98
PRDU	0.116	0.85	0.89	1.14	1.02	1.05	0.99
PROD	0.124	0.72	0.99	1	0.99	1.51	1.4
LPR	0.038	0.93	0.75	1.29	1.07	1.19	1.23
LCT	0.065	1.1	0.84	1.18	1	0.94	0.9
NOTI	0.009	.	.	.	0.99	1.65	1.54
TBIL	0.008	.	.	1.43	0.78	0.68	0.64
GOVS	0.005	.	.	.	0.75	1.47	1.4
ESK	0.005	0.64	0.73	1.41	1.09	1.73	1.6
EXCH	0.156	.	0.68	1.13	0.52	1.25	1.24

**Table 7.2:** Standard deviations of economic time series

According to this measure, the volatility of the majority of the variables in table 7.2 were highest during the eighties. These high levels of volatility subsequently subsided, though the disruption of the international financial crisis and associated local recession raised the volatility of a few series during the 2000s to levels above their full sample averages. The latter are consumption of semi- and non-durable goods, fixed capital formation of residential buildings, exports of goods and services, and manufacturing production. Although we do not report the results here, it is worth noting that until the recent financial crisis, none of the major components of GDP was more volatile in the 2000s than over the full period. To investigate the nature and timing of this decline

in economic volatility, we use a time-varying stochastic volatility model that is described in the following section.

### 7.3 Modelling time-varying stochastic volatility

To model changes to the behaviour of variables that may be influenced by non-stationary characteristics, we make use of a state-space model that includes time-varying parameters.<sup>10</sup> In this framework, the evolution of an observed variable of interest is modelled by latent (or unobserved) state variables in what is called the measurement equation, while the evolution of the state variables is described by respective state (or transition) equations. When this framework includes state equations that are used to describe the volatility of a time series, it is often termed a stochastic volatility (SV) model. These models facilitate an intuitive representation of volatility, which is seldom constant, predictable, or directly observable.

Stochastic volatility models have been used extensively in mathematical finance to model the volatility of securities and these models have a similar structure to the widely used generalised autoregressive conditional heteroskedasticity (GARCH) models. These two types of models have been used to describe the same stylised facts, that relate to the volatility of a time series. However, where the GARCH model is formulated to model the total conditional variance of a variable, the SV model treats volatility as a latent stochastic process (where shocks to the volatility may be isolated from the underlying signal for this process).

The basic SV model describes deviations of an observed economic variable,  $y_t$ , from its mean,  $\mu$ ; as the product of its volatility,  $\sigma_t$ , and a stochastic error term,  $\epsilon_t$ . Hence the measurement equation may be specified as,

$$y_t - \mu = \sigma_t \epsilon_t \quad \text{where } \epsilon_t \sim \text{i.i.d.} N [0, \varsigma_{\epsilon_t}^2]. \quad (7.3.1)$$

The distribution of volatility may then vary over time by including an unpredictable component,  $v_t$ , in the specification of the  $\sigma_t$  process. Hence, the specification of the state equation that may be used to describe the volatility of the observed variable,  $\sigma_t$ , may be expressed as,

$$\log \sigma_t^2 = \log \sigma_{t-1}^2 + v_t \quad \text{where } v_t \sim \text{i.i.d.} N [0, \varsigma_{v_t}^2], \quad (7.3.2)$$

where the use of logarithms ensures non-negative conditional variances.<sup>11</sup>

<sup>10</sup>See, Durbin and Koopman (2001) for a textbook reference on these methods

<sup>11</sup>For the interested reader, Shephard (1996) provides an introductory survey of SV and ARCH models. Ghysels *et al.* (1996) provides a mathematical review, and Shephard (2005) provides a more recent review of important SV papers. Note that in this specification, we are seeking to obtain parameter values for the variance of two shocks (i.e. one for  $\sigma_t$  and one for  $\epsilon_t$ ) using information from a single observed variable,  $y_t$ . This may result in several difficulties with parameter estimation in such a model.



To allow for various degrees of persistence and parameter-drift in the mean of the respective time series, we followed Stock and Watson (2003a) and assume that the data follow AR(4) processes that are subject to time-varying mean behaviour.<sup>12</sup> The inclusion of parameter drift in this instance is supported by our earlier findings, and several authors including Cogley and Sargent (2001), suggest that by allowing for such behaviour we are at least consistent with certain elements of the Lucas (1976) critique.

Therefore, we extend the above measurement equation in the model and assume that the stochastic factors are independent and identically distributed with zero mean and unit variance, such that,

$$y_t = \sum_{j=1}^4 \alpha_{j,t} y_{t-j} + \sigma_t \epsilon_t; \quad \text{where } \epsilon_t = \text{i.i.d.} N[0, 1] \quad (7.3.3)$$

with the state vector that describes the time-varying persistence of the data,

$$\alpha_{j,t} = \alpha_{j,t-1} + \theta_j \varphi_{j,t}; \quad \text{where } \varphi_t = \text{i.i.d.} N[0, 1]. \quad (7.3.4)$$

This formulation suggests that following the arrival of new information, changes to the economy take place that affect  $\alpha_t$ , which is modelled as a random walk process. The value for  $\theta_j$  is then calibrated at  $\theta_j = 7/T$ , which is consistent with the estimate of Stock and Watson (2003a) for parameter drift in autoregressions.<sup>13</sup>

The time-varying volatility of the variable may then be described by the second state equation,

$$\log \sigma_t^2 = \log \sigma_{t-1}^2 + v_t^2. \quad (7.3.5)$$

To allow for large jumps or breaks in the variance ( $\sigma_t^2$ ) we use a mixture of distributions for  $v_t$ , which include a period of low volatility,  $v_t = N(0, \tau_1^2)$  with probability  $q$ , and a period of high volatility,  $v_t = N(0, \tau_2^2)$  with probability  $1 - q$ . In this instance, we set  $\tau_1 = 0.04$  and  $\tau_2 = 0.2$ , as per the estimates of Stock and Watson (2003a).<sup>14</sup> The probability of the variable being in a state

<sup>12</sup>The use of different autoregressive structures, including AR(2), AR(3), AR(6) and AR(8) processes did not influence our findings significantly. If anything, the use of additional autoregressive lags identified more potential break-points. However, the pronounced decline during the 1980s was material and consistent, across all lag lengths.

<sup>13</sup>We considered the implications of using different measures of  $\theta_j$  and found that the respective variables are extremely insensitive to the degree to which the persistence is allowed to vary. When we decrease the value of  $\theta_j$  the results do not change materially, whilst much larger values of  $\theta_j$  increase the significance of our findings.

<sup>14</sup>Our results are not sensitive to the specific choice of these values. Varying the value of  $\tau_1$  hardly affects the results reported in the text; while a lower value of  $\tau_2$  results in slightly smoother time-varying volatility and a larger value leads to a slight increase in the volatility of each of the respective time series.

of low volatility at a particular point in time is calibrated at 0.95.<sup>15</sup>

A non-Gaussian smoothing algorithm was then used to identify the trend in the unobserved volatility of the respective time series. This was necessary as the conventional linear Gaussian model framework with a small amount of noise for the variance often fails to detect large jumps in volatility (which may be due to shocks) or gradual (smooth) changes to the trend. In addition, the exclusion of a large amount of noise in the variance of the stochastic factor,  $v_t$ , would introduce several inappropriate high-frequency elements to the trend of a linear Gaussian model (Kitagawa, 1987).

The parameters in the model are estimated using Bayesian techniques, where initial conditions are established with flat priors. Thereafter, diffuse conjugate priors were used to obtain parameter estimates. In such a setup, we are able to make use of the extremely powerful Markov Chain Monte Carlo (MCMC) methods for the non-Gaussian smoother, as suggested by Stock and Watson (2003a).

Where  $Y = y_1, \dots, y_T$ ,  $A = \alpha_{j,t}; j = 1, \dots, 4; t = 1, \dots, T$  and  $S = \sigma_1, \dots, \sigma_T$ , the MCMC algorithm is formulated to iterate between the following three conditional distributions of  $[Y|A, S]$ ,  $[A|Y, S]$ , and  $[S|A, Y]$ . The first two conditional distributions are normal and the third is computed by approximating the distribution of  $\log \epsilon_t^2$  with a mixture of normal distributions that have means and variances that match the first four moments of the  $\log \chi_1^2$  distribution, in the multi-move Bayesian sampler of Shephard (1994).

After the parameter values have been estimated with the use of non-Gaussian smoothing algorithm, the estimated instantaneous auto-covariances of  $y_t$  are then computed using  $\sigma_{t|T}^2$  and  $\alpha_{j,t|T}$ , which converge on the conditional means of  $\sigma_t^2$  and  $\alpha_{j,t}$ , given the observed time series. The smoothed instantaneous variance in the growth rates are then calculated by summing up the instantaneous auto-covariance functions in chronological order.

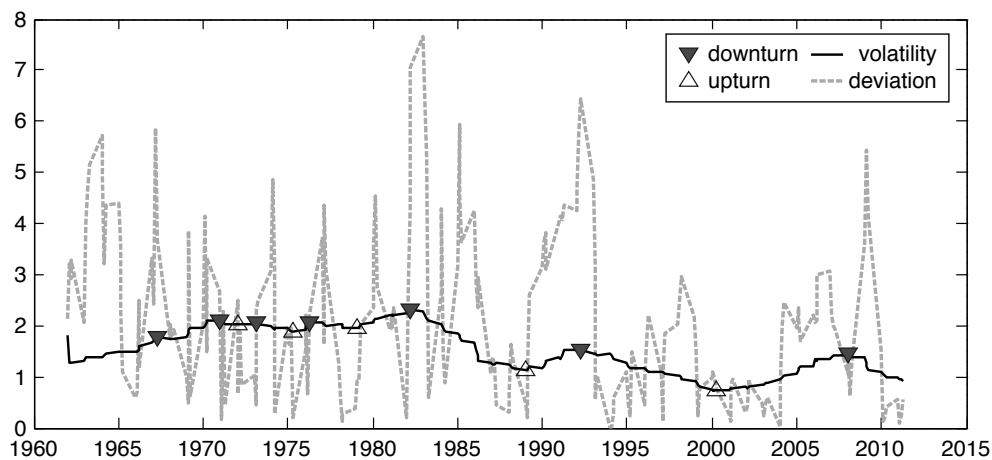
### 7.3.1 Results of time-varying stochastic volatility model

The resulting time-varying stochastic volatility of South African GDP is depicted in figure 7.2, where the line with a great degree of deviation depicts the mean absolute deviation<sup>16</sup> and the smoother line is the estimate of time-varying volatility (from the stochastic volatility model).

This graph shows that the decline in real output volatility in South Africa may have started as early as the middle of the 1980s, as the volatility of the series declines sharply between 1983 and 1987. Figure 7.2 also shows steadily declining volatility from 1992 onwards. When considering the graphs for the

<sup>15</sup>The results that we report are not largely dependent on the calibrated value of  $q$ . Decreasing the probability of an event arising during a period of low volatility increases the reported time-varying volatility, whilst increasing the value of  $q$  produces a flatter overall trend for the time varying volatility.

<sup>16</sup>The absolute difference between each  $y_t$  and the full sample mean.



**Figure 7.2:** Time varying standard deviations of South African GDP

constituent parts of South African GDP expenditure that are included in figure Du Plessis and Kotzé (2012), we note that the large decline in volatility during the 1980s is relatively widespread. For example, consumption, government expenditure, and investment (over output) all seem to experience a similar decline. Gross fixed capital formation and imports also experience such a decline, but it would appear to take place later (i.e. only starting in 1987), whilst exports maintain a relatively constant level of volatility over this period. In addition, we also detect similar findings with regard to the value-added side of the economy (with the exception of the primary sector).

With regard to employment measures, we observe little change to the volatility of the respective series, whilst the volatility of the consumer price index was higher in the eighties, declined during the nineties and has lately been on an upward trend again. The higher volatility of interest rates since the mid-nineties is clearly depicted in the remaining few graphs.

The following tentative conclusions are possible at this stage: *firstly*, real output and its major constituent parts experienced significant shifts in their mean growth rates and volatility, rejecting any notion of co-variance stationarity. *Secondly*, a sizeable downward shift in the conditional variance of real output seems to have started in the early to mid-eighties; a date which is much earlier than previously estimated in the South African literature, but in line with the estimated start of the international ‘great moderation’ (especially for the United States).

## 7.4 Break-point investigation

It is possible to identify the likely start of the great moderation in South Africa more rigorously by applying break-point tests to the conditional volatility se-

ries. We first apply a simple turning-point test that was popularised by Wecker (1979), whereby potential downturns start at local peaks in the series under investigation ( $y_{t-4}, y_{t-3}, y_{t-2}, y_{t-1} < y_t > y_{t+1}, y_{t+2}, y_{t+3}, y_{t+4}$ ) and upturns start at local troughs, that is, ( $y_{t-4}, y_{t-3}, y_{t-2}, y_{t-1} > y_t < y_{t+1}, y_{t+2}, y_{t+3}, y_{t+4}$ ).

These turning-points provides us with a set of possible break-points in the volatility of the respective variables, and they have been included in figures 7.2. To ascertain whether these turning points are statistically significant, we subject the series to a number of break-point tests that also determine whether the change in volatility is associated with changes in its time-varying mean (or trend), or with its associated conditional variance (or volatility). Distinguishing between these types of breaks is important, since changes to the trend may be associated with changes in economic policy or other events that evolve over time, whilst changes to the conditional variance would be associated with the frequency and amplitude of economic shocks that are generally of shorter duration.

Essentially, these tests seek to identify changes in the parameters of the following autoregressive model.

$$y_t = \phi_t + \beta_t(L)y_{t-1} + \varepsilon_t \quad (7.4.1)$$

where,

$$\phi_t + \beta_t(L) = \begin{cases} \phi_1 + \beta_1(L), t \leq \xi \\ \phi_2 + \beta_2(L), t > \xi \end{cases} \quad (7.4.2)$$

and,

$$\varepsilon_t^2 = \begin{cases} \varepsilon_1^2, t \leq \zeta \\ \varepsilon_2^2, t > \zeta \end{cases} \quad (7.4.3)$$

These tests make use of the heteroskedasticity-robust Quandt (1960) likelihood ratio (QLR) statistic that was implemented with the method of Bai (1997) to test for changes in the autoregressive parameters. In this specification the date of the break-point in the conditional time-varying mean is represented by  $\xi$ , which may differ from the date of the break-point in the condition variance that is given by  $\zeta$ .

The results for GDP are reported in tables 7.3 and 7.4. The first column provides the least squares estimate of the break date, the second column provides the 67% confidence interval around this date, and the final column provides an estimate of the significance of the break using the technique of Hansen (1997). The results for breaks in the trend refer to the  $\phi$  and  $\beta$  parameters in equation (7.4.1), whilst the results for the conditional variance correspond to the  $\varepsilon_t$  in the same equation.

The results in table 7.3 suggest that there are several instances where the mean growth rate has been subject to significant structural changes, whilst

Break	Confidence	<i>p</i> value
1967:4	1967:2 - 1968:2	0.00
1976:4	1976:2 - 1977:2	0.00
1981:4	1981:2 - 1982:2	0.00
1992:4	1992:2 - 1993:2	0.05
1997:2	1996:4 - 1997:4	0.00

**Table 7.3:** Break-point tests for the GDP mean

Break	Confidence	<i>p</i> value
1986:1	1985:3 - 1988:1	0.00

**Table 7.4:** Break-point tests for the GDP variance

table 7.4 indicates that there is only one instance where we observe significant changes in the conditional variance of the series. When we consider these results in conjunction with those of the SV model, we note that most of the break-points coincide with the turning points in the earlier investigation. In addition, it is particularly interesting to note that the dramatic decline in output volatility that started after the fourth quarter of 1981 may be largely attributed to changes in the time-varying mean growth rate.<sup>17</sup> This significant decline in economic volatility came to an end at the start of 1986, after which the conditional volatility remained at a more consistent (or constant) level.

During the early 1980s, the monetary authorities in South Africa moved decisively away from direct controls and towards a market-oriented framework in domestic monetary policy and capital controls, following the publication of the interim report of the De Kock Commission<sup>18</sup> in 1978. The implementation of the commission's recommendations established a single exchange rate for the South African economy, which was allowed to float against other currencies.<sup>19</sup> The improved access to international capital markets and removal of direct controls domestically were meant to provide the capital market instruments that would smooth economic activity from the early 1980s onwards.

However, the South African economy's exposure to foreign debt had in-

<sup>17</sup>There is some evidence to suggest that there could be a joint break in both the mean and the conditional variance. This is the only instance where there may be a joint break in the conditional mean and variance of GDP growth (when measured in each of the subsamples at the 5% level of significance). However, since there is no corresponding separate break in the conditional variance at this point in time, we presume that the break is largely attributed to developments in slower changing components of real output, that is, structural factors.

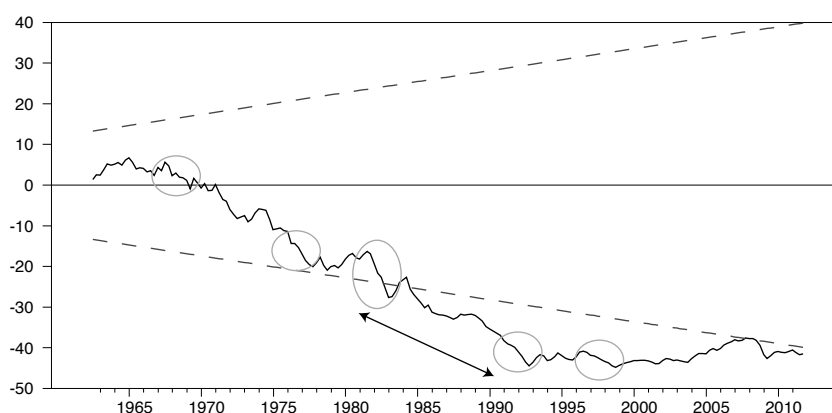
<sup>18</sup>Important impetus for this development derived from the De Kock Commission. See, De Kock Commission (1978) and De Kock Commission (1984).

<sup>19</sup>The unification of exchange rates took place in February 1983 to establish a single managed floating exchange rate mechanism.

creased dramatically by the mid 1980s (from 20.3% of GDP in 1980 to 50.1% of GDP in 1985) and, what is more, was of a short-term nature.<sup>20</sup> Servicing this debt became particularly onerous as the rand depreciated on foreign exchange markets. In addition, this change to the structure of the capital account also raised the vulnerability of the economy to the adverse economic shocks that were imposed by the trade and financial sanctions, following the perceived abandonment of political reform in President P.W. Botha's "Rubicon" speech in August 1985. The rapid decline in output volatility that had been observed up to this point was arrested by the ensuing economic disruption.

These findings are largely supported by the CUSUM test statistics that are reported in figure 7.3 and 7.4. The first of these graphs relates to the cumulative sum of the one-step ahead forecasting errors for the residuals from the AR(4) model for output growth. It suggests that in the early 1980s, the forecasting errors increased somewhat to a level where they exceed the confidence levels, before continuing in a steady downward direction throughout the rest of the 1980s and early 1990s. Similarly in the second of these graphs, which relates to forecasting errors of the residuals from an AR(1) model for the volatility of output, we see that the forecasting errors exceed the confidence level around 1986 (at a point in time where they experienced a rapid change in direction).<sup>21</sup>

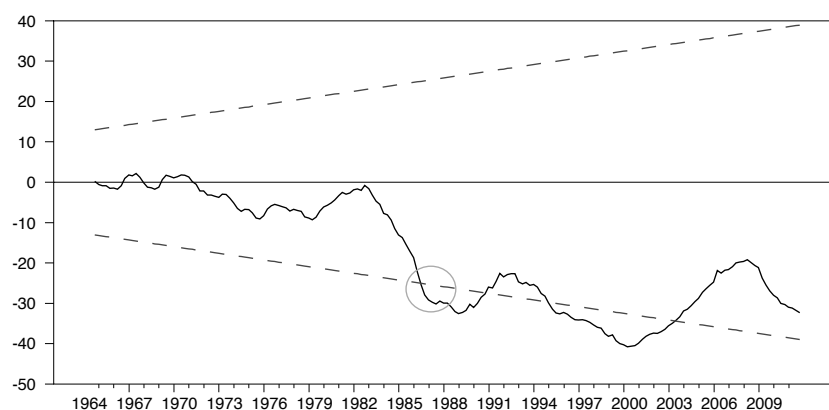
**Figure 7.3:** CUSUM test - GDP mean



Such break-points, around the early to mid-1980s, have also been found in similar analyses that have been conducted for developed world economies; however, in many of these studies, including Sensier and Dijk (2004), Ahmed *et al.* (2004), and Stock and Watson (2005), the break-points were largely

<sup>20</sup>See, Khan (1987) and Farrel and Todani (2004).

<sup>21</sup>The measure of volatility is derived from the above stochastic volatility model with the time-varying parameters. Since the model makes use of single lagged regressor in the volatility equation and four lags in the measurement equation we use an AR(4) for the mean and an AR(1) for the volatility.

**Figure 7.4:** CUSUM test - GDP volatility

attributed to changes in the conditional variance attributable to benign shocks, rather than structural developments. By contrast, owing to the prominence of the break-point in the mean of real GDP, the South African evidence suggests a greater role for structural factors in the early phases of the great moderation.

Using the breakpoints of 1981q4 and 1986q1 we then consider which of the components of GDP had the largest influence over the observed decline in output volatility in table 7.5. After weighting the components according to their contribution to output, we may then identify those variables that have a  $p$ -value that is close to zero, where the break occurs at a point in time that is similar to that of output volatility (i.e. the column for conditional variance). It is not surprising to observe that the decline in the volatility of consumption had a pronounced impact on the changes in output volatility over this period (especially as it is responsible for a large share of output). Other important contributors include changes in the volatility of government expenditure, capital formation, investment expenditure and imports. When considering those variables that contribute to output from a value added perspective, we note that the aggregate for gross value added activities is responsible for a significant contribution to the overall reduction in volatility in 1986, however, it would be difficult to attribute this break to any of the underlying components of this measure. In addition, other measures of economic activity, such as the broad price deflator and the long term interest rate, also report large declines in aggregate volatility during this period.

## 7.5 Multivariate estimation of break-points

The great moderation refers to a period of benign stability in many macroeconomic aggregates and across most sectors of the economy. To complement the results for real output reported in the previous section, this section investigates possible break-points from a multivariate setting to identify common break points across various time series.

## CHAPTER 7. TRENDS AND STRUCTURAL CHANGES IN SOUTH AFRICAN MACROECONOMIC DATA

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Series	Cond Mean			Cond Variance (break)			Cond Var (trend & break)		
	<i>p</i>	break	conf. int.	<i>p</i>	break	conf. int.	<i>p</i> trend	<i>p</i> break	break
GDP	0	1976:4	1976:2 - 1977:2	0	1986:1	1985:3 - 1988:1	0.87	0.01	1986:1
CONS	0	1976:1	1975:3 - 1976:3	0	1984:4	1984:2 - 1986:1	0.51	0	1984:4
CDUR	0.04	1997:1	1996:3 - 1997:3	0	1987:2	1987:1 - 1989:4	0.65	0.01	1987:2
CSDU	0.02	1987:1	1986:3 - 1987:3	0	1984:3	1983:3 - 1987:2	0.12	0	1984:3
CNDU	0	1984:3	1984:1 - 1985:1	0	1988:1	1987:3 - 1989:2	0.99	0	1988:1
CSER	0	1977:1	1976:3 - 1977:3	0	1994:2	1993:4 - 1996:1	0.29	0	1994:2
GOV	0	1992:3	1992:1 - 1993:1	0	1987:3	1987:1 - 1989:2	0.99	0.05	1987:3
GFCF	0	1987:3	1987:1 - 1988:1	0	1988:2	1987:4 - 1989:4	0.96	0.03	1988:2
GFCR	0	1971:3	1971:1 - 1972:1	0.01	1989:2	1987:4 - 1993:2	0	0.08	.
GFCN	0	1992:2	1991:4 - 1992:4	0	2002:2	2002:1 - 2004:1	0.03	0	1996:3
INV	0.48	.	.	0	1986:1	1985:3 - 1988:2	0	0.22	.
GDE	0.05	1973:3	1973:1 - 1974:1	0	1995:3	1995:1 - 1997:3	0	0.16	.
EXP	0	2004:2	2003:4 - 2004:4	0.06	.	.	0	0	1979:1
IMP	0.03	1973:3	1973:1 - 1974:1	0	1988:2	1987:4 - 1990:2	0.3	0	1988:2
GVAP	0.02	1970:4	1970:2 - 1971:2	0	1996:2	1995:4 - 1998:3	0.11	0	1996:2
GVAS	0	1981:3	1981:1 - 1982:1	0	1978:1	1976:4 - 1980:3	0	0	2003:4
GVAT	0	1970:1	1969:3 - 1970:3	0	1969:4	1969:3 - 1970:4	0	0	1969:4
GVA	0	1976:4	1976:2 - 1977:2	0	1986:3	1986:1 - 1988:3	0.99	0.08	.
EMP	0.07	.	.	0.02	1978:1	1977:3 - 1980:3	0.8	0.04	1978:1
WAGE	0	1992:4	1992:2 - 1993:2	0.05	.	.	0.98	0.6	.
CPI	0	1982:1	1981:3 - 1982:3	0.92	.	.	0.52	0.53	.
DEF	0	1994:1	1993:3 - 1994:3	0.01	1985:4	1985:1 - 1990:2	0	0	1968:4
PRND	0.17	.	.	0.13	.	.	0	0	1978:4
PRDU	0	1995:2	1994:4 - 1995:4	0	1980:2	1975:2 - 1982:1	0.99	0.27	.
PROD	0.14	.	.	0	1993:2	1985:4 - 1994:2	0.93	0.34	.
LPR	0	1994:1	1993:3 - 1994:3	0.88	.	.	0.33	0.24	.
LCT	0.02	1992:3	1992:1 - 1993:1	0.5	.	.	0.86	0.68	.
NOTI	0	2001:2	2000:4 - 2001:4	0.23	.	.	0.93	0.42	.
TBIL	0.85	.	.	0	1986:3	1986:1 - 1987:4	0	0	1998:2
GOVS	0	2001:1	2000:3 - 2001:3	0.25	.	.	0.99	0.69	.
ESK	0.01	2002:1	2001:3 - 2002:3	0	1975:1	1965:4 - 1975:4	0	0	1990:1
EXCH	0.05	1985:4	1985:2 - 1986:2	0.05	1998:2	1992:1 - 2000:3	0.46	0.05	1998:2

Table 7.5: Break points in mean and variance

Following Bai *et al.* (1998) we specify a low dimensional VAR model that makes use of multiple equations to obtain accurate confidence intervals for break-points in multivariate time series with either stationary or non-stationary characteristics.<sup>22</sup> This method can be regarded as an extension of the univariate regression that is described in equation (7.4.1).

Under consideration is a null hypothesis of no break-point against the alternative hypothesis of a common break in the system of equations. We tested three systems of equations with this approach: *first*, a decomposition of real GDP into its expenditure components, and *second*, a decomposition of real GDP into its value added components. The test statistic is the QLR statistic that is computed using the absolute values of the VAR residuals. The OLS estimates of the break date in the mean absolute residual and the associated confidence intervals are also reported in table 7.6.<sup>23</sup>

The results of these multivariate tests would appear to support those of the univariate studies, where the joint consideration of the expenditure components of GDP (line 1) suggests a break in volatility during the mid 1980s.

<sup>22</sup>This statistic would only be valid for large samples.

<sup>23</sup>To avoid overfitting the model we keep the VAR coefficients constant.



Variable	# variables	QLR $p$	Break	Conf. interval
consumption, gov exp, GFCF, exports, imports	5	0.00	1988:1	1987:2 - 1988:3
GVA primary, GVA second, GVA tertiary	3	0.00	1970:3	1969:3 - 1971:2
cons durable, cons semi-dur, cons non-dur, cons services	4	0.00	1987:1	1986:1 - 1988:1
gdp, gdp deflator, cpi, tbil, wages, labour prod., exch. rate	7	0.03	1988:1	1987:1 - 1989:1

**Table 7.6:** Common break-points in the variances of VAR residuals

The decomposition of output volatility into the value added components would suggest a significant decline in volatility in the late 1960s and early 1970s. From the univariate results, this finding for the value added components is probably largely influenced by the tertiary sector. However, after removing the tertiary sector data from the value added components, the most prominent suggested break in the bivariate VAR arises in 1996, with a confidence interval that would include the period of political transition in 1994. When considering these results, in conjunction with those from the univariate regressions, it would appear as if this sector is going to provide ambiguous results.

In the *third* model, we consider a decomposition of household consumption expenditure, since it has substantially influenced the decline in GDP volatility between 1981q4 and 1986q1. This model produces similar results to the VAR for the expenditure components and suggests that the most prominent change in 1987q1. The *final* model, combines the domestic variables that are discussed in chapter 6, and were also used in a recent macroeconomic forecasting model for South Africa.<sup>24</sup> These results are similar to those of the first and third model, with a suggested break taking place in 1988.

## 7.6 Conclusion

This chapter sought to identify major breakpoints in the first two moments of several South African macroeconomic time series over the period 1960q1 to 2011q4. The results suggest that the most significant change occurred during the 1980s, when output volatility declined by a significant amount between 1982 and 1986. This finding is consistent with the international literature that dates the “great moderation” during a similar period. However, in contrast with the international literature, such as what is considered in Stock and Watson (2005) and Stock and Watson (2003b), longer run factors seem to have played a more important role in the moderation locally.

<sup>24</sup>See, Alpanda *et al.* (2011) for a discussion of the variables in their DSGE and BVAR models.

The gradual decline of volatility is consistent with a description of the South African economy undergoing a slow structural shift, with the evolution of macroeconomic policy away from direct controls and towards market-based interventions and with improved private sector management of inventories. The dramatic increase in the smoothing of consumption, government expenditure, investment expenditure, capital formation and imports during the early 1980s would also suggest that economic agents were able to make better intertemporal decisions over this period of time, which would have been possible following the liberalisation of, *inter alia*, the capital account.

The role of a decline in the conditional volatility of real output suggests that in South Africa, as elsewhere, a more benign environment (fewer and less disruptive shocks) also played an important role. Since this result emerges for many countries over the same period, it has been plausibly connected with a period of benign international development, following the global shocks of the seventies. That the progress towards a more stable conditional variance was halted at the time of the most intense trade and capital sanctions provides further support for the suggestion that the international environment was an important factor in the South African great moderation.

Given the large and significant changes that affected most macroeconomic variables during the 1980s, we only use data from 1990Q1 in the dynamic stochastic general equilibrium models in part 3.

## Chapter 8

# Measures of Core Inflation

## 8.1 Introduction

The inflationary process influences future economic activity through a number of channels. It directly affects the real rate of return and the expected rate of future inflation, which influences the cost of living, increases in nominal wages, grants, agreements, and long-term contracts. In addition, inflationary pressure imposes adjustment costs on the economy, lowering the information content of price changes, while adversely affecting the distribution of income. Bernanke and Gertler (1999) have also noted that inflation contributes towards increased volatility in asset prices, which imposes a direct cost on the economy. Hence, measures of inflationary pressure play a critical role in policy-making, particularly for central banks in inflation targeting countries that are explicitly responsible for anchoring price levels.

When formulating policy, decision-makers usually focus on the persistent sources of inflationary pressures that are not influenced by temporary short-run price deviations from the underlying trend. This is because changes to policy often have lasting effects that may result in significant transition costs (Blinder, 1997). This measure of the underlying rate of inflation is often termed core inflation, and it is used by the South African Reserve Bank (SARB) to inform policy.<sup>1</sup> Furthermore, as core inflation should be less volatile than changes in the price level, by focusing on the trend in the process, the central bank would not need to make as many changes to interest rates. This would reduce the number of policy shocks, thus providing an environment that would facilitate stable economic growth.

It has also been suggested that measures of core inflation would assist in identifying the economic behaviour that is responsible for generating the underlying inflationary process, after excluding the noise in the process that is largely attributable to relative price shocks (Bryan and Cecchetti, 1994). This would imply that when using core inflation to inform policy, the central bank would not respond to the type of price shocks that are not under its control. For example, relative shortages in the supply of a particular commodity may result in a sudden (relative) price increase of that commodity, which is beyond the control of the central bank. If the measure of core inflation were to exclude the relative increase in the short-term price of this commodity, then the central bank would not respond to it.

In this chapter, we consider the relative merits of the methods that have been developed to measure core inflation. These include various trimmed

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<sup>1</sup>Ruch and Bester (forthcoming) have noted that the official statements of the central bank make reference to measures of core inflation and the forecasts of this unobserved variable. An example of such a statement is provided by Marcus (2011).

means estimates that allow for different forms of asymmetry, which have been used by many central banks (Cecchetti, 2009). An example of such a measure that has been derived for the South African economy is described in Blignaut *et al.* (2009). This measure was constructed from a subsample of the weighted disaggregated consumer prices, after the most volatile constituents have been removed. In addition, we make use of dynamic factor models to derive an index that describes the common variation in the disaggregated data.<sup>2</sup> Finally, we also use more recently developed wavelet transformations that are able to distinguish between the trend and short-term noise in the aggregated data. This computationally efficient method is able to preserve the informational content of time-series data despite the influence of non-stationarity behaviour.<sup>3</sup>

When making use of criteria for comparing these estimates of core inflation, there are largely two aspects of relevance. Firstly, Blinder (1997) emphasised the *out-of-sample* properties of these estimates, where core measures are evaluated on the basis of their ability to predict future inflation (which would imply that they should also provide an accurate measure of the *second-round* inflationary effects).<sup>4</sup> Since these measures of core inflation may also be used in various multivariate models for policy-making and forecasting purposes, this measure would need to provide insight into the evolution of the expected underlying inflationary process.<sup>5</sup> To satisfy this criteria for the evaluation of measures of core inflation, we generate out-of-sample statistics for successive 1 to 12 step ahead forecasts over the ten year period 2003M1-2012M12. These forecasts are generated with the aid of a Kalman filter in a state-space model that allows for time-varying parameters.

Then secondly, Bryan and Cecchetti (1994) and Cecchetti (2009) have noted that the *in-sample* statistics of core inflation measures are also of importance when evaluating their potential usefulness. In particular, these measures should provide an accurate estimate of the mean and a lower variance, when compared with the actual reported measure of aggregate inflation. In addition, they also note that the price indices, which may be generated from the measures of core inflation, should share a common stochastic trend with the reported aggregate consumer price index (CPI). To satisfy the aforesaid criteria, we also consider these in-sample statistics for the estimates that are derived

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<sup>2</sup>Examples of dynamic factor models that have been used for measures of core inflation include, Cristadoro *et al.* (2005) and Giannone and Matheson (2007), among others.

<sup>3</sup>Examples of measures of core inflation that have been derived by wavelet methods include Dowd *et al.* (2010) and Baqaee (2010).

<sup>4</sup>The core inflation estimate that is able to outperform other inflation forecasts would provide a better estimate of expected *second-round* effects, which are of importance to central banks. A recent statements by the Governor of the SARB was as follows: “Whilst the current rate of inflation has increased above the target range, we have not increased interest rates, since we believe that the present rise is transitory and will have little second-round effects.” (Marcus, 2011).

<sup>5</sup>This measure of core inflation will be used in the models that are contained in part three of this dissertation.

from the respective models.

In addition to the computation and evaluation of various measures of core inflation, this chapter also considers the properties of the South African price indices, and the effect of changes to the methodology that is used to compile these indices. While changes in the behaviour of economic agents necessitate changes to the way in which we measure economic activity, a change in the technique that is used to measure broad-based prices may result in the imposition of a structural break in the data. When deriving estimates for South African core inflation we need to be cognisant of such an event, as it will affect the way in which certain methods may be applied, particularly when using disaggregated data or when applying these methods to year-on-year, as opposed to month-on-month, inflationary data.

The following section of this chapter contains a discussion on the various methods that have been used to measure price movements in South Africa. This section also considers the implications of changes to the methodology that is used to calculate price indices, when deriving various measures of core inflation. Thereafter, in section 8.3 we provide an explanation of the trimmed means, while section 8.4 contains a discussion on the estimation of the dynamic factor model that is used to measure core inflation. Section 8.5 seeks to explain the use of wavelet methods for this purpose, while section 8.6 contains the comparative results of the out-of-sample and in-sample analyses.

## 8.2 Constructing Price Indices for South Africa

Over the period January 1975 to December 2008, consumer price data was reported monthly for 33 sub-categories.<sup>6</sup> The basket of goods remained consistent over this period, but the weights were amended periodically to reflect changes in consumer preferences. Aron and Muellbauer (2004) suggest that the construction of South African price indices over this period was subject to several methodological inconsistencies and a general lack of transparency, where the construction of seasonally adjusted CPI involved the assimilation of various seasonally unadjusted weighted subcomponents. After attempting to recreate the index from the sub-categories, they suggested that this measure was constructed as a chained Laspeyres index, which compares the old basket of goods,  $q_0$ , at the respective old prices,  $p_0$ , and new prices  $p_t$ . This would imply that the aggregate price index has been constructed from

$$P_t = \frac{\sum_{n=1}^N (P_{n,t} \cdot q_0)}{\sum_{n=1}^N (P_{n,0} \cdot q_0)}, \quad (8.2.1)$$

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<sup>6</sup>Consumer price data is available for earlier periods, but for these earlier periods, the data was reported for fewer sub-categories. For the purposes of this investigation, a starting date of 1975 gives us more than enough data.

where level factors are used to ensure that the data from different base years are comparable.<sup>7</sup>

This measure of consumer prices was complemented by a second measure that was termed CPIX, which was provided over the period 1997M1 to 2008M12 by government's official reporting agency, Statistics South Africa (StatsSA). It effectively sought to exclude the effects of mortgage costs from the consumer price index, in order to avoid the self-referential impact of changes due to the central bank interest rate.<sup>8</sup> This measure of prices was used to determine a rate of inflation, which provided the initial target in the SARB's inflationary target framework (Van der Merwe, 2004).<sup>9</sup>

Following a review of the practices that were used to construct these indices by StatsSA, it was decided that a new methodology for calculating consumer inflation would be adopted from January 2008. This revision in methodology ensures that the current calculation is consistent with international practice, and a new basket has been used to reflect consumers' current preferences, which would have been influenced by changes in taste and technology over the last three decades.<sup>10</sup> As a result, CPIX was no longer calculated and the more encompassing measure, which is termed *headline inflation*, has been used as the target for monetary policy.

This process was conducted in a highly transparent manner, and we now have the first release of a document that describes the methodology that is followed to construct this important index.<sup>11</sup> As a result, we can now confirm

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<sup>7</sup>Note, that while Aron and Muellbauer (2004) suggests that the construction of the aggregate measure of total inflation prior to 2008 was derived from seasonally adjusted composite series, we found that when using a seasonal filter on the composites, the aggregate measure of total inflation was extremely close to that of the combined seasonally unadjusted weighted inflation rates, to the extent that there are no obvious gains from applying the seasonal filter. It is also worth noting that since the early 1990s, the new approach of using the weighted price indices approximates the reported measure of inflation more closely than the weighted inflation rates of the composites. Aron and Muellbauer (2004) also suggest that the use of a logged seasonally adjusted series may be appropriate; however, we found that the construction of such a series does not replicate that of reported inflation. This may in part be due to the different sample that we have used in this study.

<sup>8</sup>Since, the interest rate on mortgages is highly correlated with the central bank interest rate, any increase in the rate of the policy instrument would positively influence inflation.

<sup>9</sup>In certain respects, this measure is similar to those estimates of core inflation that have been derived by other central banks, which often exclude the effects of energy and food. These methods of calculating core inflation have fallen out of favour, as the estimates are often more volatile than the measure of reported aggregate consumer price inflation (Cogley, 2002).

<sup>10</sup>The new methodology is in accordance with the practice of the International Labour Organisation (ILO) through the application of the United Nations Statistical Division (UNSD) Classification of Individual Consumption by Purpose (COICOP).

<sup>11</sup>See the website <http://www.statssa.gov.za/cpi/index.asp> for a list of articles that have been published by Statistics South Africa regarding the change in methodology. Much of what is contained in this discussion relates to the documents: "Shopping for two: The CPI new basket parallel survey - results and comparisons with published CPI data"

that the current practice for calculating inflation in South Africa makes use of a Jevons Index for the prices of disaggregated goods and services,

$$P_{n,t} = \prod_{k=1}^K \left( \frac{p_{k,t}}{p_{k,t-1}} \right)^{\frac{1}{K}}, \quad (8.2.2)$$

where  $p_{k,t}$  is the price of a particular good that may be found on the shelf of a particular store at a particular point in time. The aggregate price of these goods across all stores and regions is then given as  $P_{n,t}$ . The price indices for the respective goods are then multiplied by the basket weights to compile a Lowe's index for the aggregate CPI,<sup>12,13</sup>

$$CPI_t = \sum_{n=1}^N (P_{n,t} \cdot q_n^m). \quad (8.2.3)$$

This aggregation is performed on highly disaggregated Jevons indices that are not reported by StatsSA; however, by combining the indices of the reported disaggregates, the truncating error is extremely small.<sup>14</sup> This measure is used to derive the month-on-month rate of inflation, which is calculated as a simple growth rate,

$$\pi_t = \left( \frac{CPI_t}{CPI_{t-1}} \right) - 1. \quad (8.2.4)$$

For the purpose of this analysis, it is worth noting that it is possible to arrive at a similar result for reported *headline inflation* by calculating the rate of inflation for each of the underlying price indices and multiplying them by their respective basket weights,<sup>15</sup>

$$\tilde{\pi}_t = \sum_{n=1}^N (\pi_{n,t} \cdot q_n^m). \quad (8.2.5)$$

In addition to establishing a clear methodological framework, this measure of headline inflation makes use of information from the new Income and Expenditure Survey (IES) to determine the weights of the basket items, which (Statistics South Africa, 2009a) and “The South African CPI Sources and Methods Manual” (Statistics South Africa, 2009a).

<sup>12</sup>A Lowe's index may be interpreted as a modified Laysperes Index.

<sup>13</sup>Prior to the changes that took place in 2008, the aggregate price index was termed ‘CPI - all items’.

<sup>14</sup>There are 33 reported disaggregates for the older measure of inflation, while there are 44 reported disaggregates for the new measure over the sample that has been used in this chapter. From January 2013, the number of disaggregated items increased to 45, which further complicates the calculation of measures of core inflation that are provided by trimmed means and dynamic factor models (but not those that are provided by wavelet estimates).

<sup>15</sup>This is of particular importance for when we start calculating trimmed means and dynamic factor model estimates.



now include a larger number of disaggregate price series that are reported by StatsSA. The statistics agency is also no longer reliant on retailers to provide information on prices, as it currently employs field teams to collect this information. Importantly, interest rates are also no longer used as an indicator of housing costs, as this measure has been replaced by a measure of owners' equivalent rent.<sup>16</sup>

During the implementation phase of this project, data was collected according to both methodologies from January 2008 to December 2008.<sup>17</sup> This allows for twelve overlapping data points for the price indices - which enables one to smooth over the month-on-month inflation measures for eleven observations - to construct a combined measure of inflation over the structural break, where the weighting is calculated as

$$\hat{\pi}_t = \sum_{\omega=1}^{11} \left( \frac{\omega}{12} \cdot \tilde{\pi}_{[\text{New}, t]} \right) + \sum_{\omega=1}^{11} \left( \left[ 1 - \frac{\omega}{12} \right] \cdot \tilde{\pi}_{[\text{Old}, t]} \right) . \quad (8.2.6)$$

To calculate year-on-year inflation from this measure, we can convert the month-on-month series into a index of prices and recalculate the rate inflation series to allow for the maximum amount of smoothing.<sup>18</sup> It is also worth noting that as the broad-based measures of inflation are calculated at a more disaggregated level, than what is provided to the public, there is a slight difference between the weighted value of the disaggregate items and the reported broad-based measures of inflation. This difference reaches a year-on-year maximum of 2.8% when comparing the old measure of inflation, while a maximum discrepancy of 1.9% is found for the new measure of inflation. Figure 8.1 depicts the respective year-on-year measures of inflation over the out-of-sample period, taking into account the period where there were two overlapping measures of inflation.

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<sup>16</sup>In addition, the old alcoholic beverages has been split up into spirits, wine and beer; the old housing has been split into actual rentals, owners equivalent rent, maintenance and repairs, water and other services, and electricity and other fuels. The old line items for household services has been incorporated into domestic workers' wages; the old healthcare has been split into medical products and medical services; petrol is now included as a separate, weighted component; and the old communication has been split into postal services and telecommunication. The old recreation has been split into recreational equipment and recreational cultural services; the old education has been split into primary and secondary, and tertiary education. A number of other items have been included for restaurants, hotels, insurance, and financial services.

<sup>17</sup>Although new data was collected from 2008, comparable data for the new price indices has been provided back to 2007. However, it is important to note that these values are only approximations and they do not include estimates for some of the new sub-categories, notably, the way in which changes to house prices and rentals are now calculated.

<sup>18</sup>Alternatively, one could combine the year-on-year estimates from the two price indices, but this would not allow for any overlap.

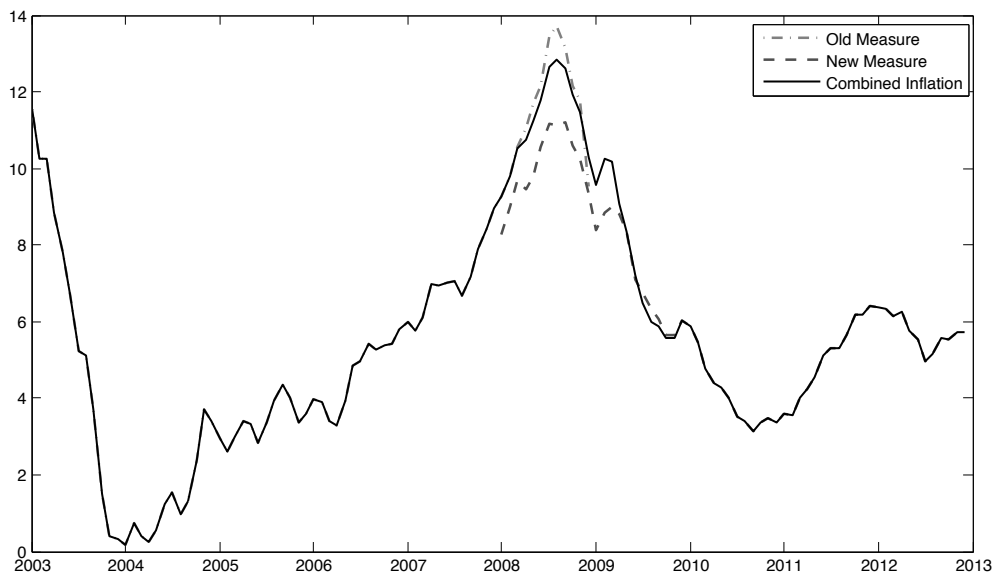


Figure 8.1: Year-on-year inflation (2003M1-2012M12)

### 8.3 Trimmed Means Estimates of Core Inflation

A trimmed means estimate of core inflation would exclude extreme price movements in the disaggregated data items, as these may be attributed to short-term phenomena that may not influence the persistent rate of inflation. For example, if there is a temporary shortage in the supply of sugar products, we would expect that the relative price of sugar would increase sharply for a short period of time. However, this shortage may be quickly rectified, and as a result, we would not wish for this temporary disturbance to affect long-term monetary policy decisions.

To derive this statistic, we follow Blignaut *et al.* (2009), who make use of an asymmetric trimmed mean estimate, which is calculated after arranging the disaggregated data in ascending order. Thereafter, a number of disaggregated indices may be excluded from the combined measure by setting to zero the corresponding weights for the items that find themselves at the upper and lower ends of the distribution. Hence, this statistic may be calculated as

$$\tilde{\pi}_{[\gamma_1, \gamma_2; t]}^T = \frac{\sum_{n=1}^N \pi_{n,t} \cdot \omega_{n,t}}{\sum_{n=1}^N \omega_{n,t}}. \quad (8.3.1)$$

where  $\gamma_1$  and  $\gamma_2$  refer to the amount that is trimmed from each side of the distribution.<sup>19</sup>

<sup>19</sup>An asymmetric trimmed mean estimate would allow for  $\gamma_1$  to differ from  $\gamma_2$ .

While this statistic may be calculated for both month-on-month and year-on-year inflation, it is worth noting that when performing it on month-on-month inflation, where the variation in each disaggregate series is often zero, the combined trimmed means estimates are largely determined by arbitrary rules, and the resulting measures of core inflation are often extremely volatile.<sup>20</sup> In contrast, the use of year-on-year data has more variation, and as such, the resulting measure would appear to be more reasonable, although this would not allow one to smooth over the structural break, since the first measure of year-on-year inflation for the new series is January 2009 and the last measure of year-on-year inflation that uses the old method is December 2008.

As an alternative to this trimmed means estimate, we derive an additional trimmed means estimate that excludes those items that experienced the largest deviation, irrespective of whether this change was positive or negative. To derive this measure, we take the absolute value of the respective disaggregated inflation rates to identify the subcategories that are to be trimmed, prior to sorting the observations. Thereafter, we construct the trimmed means estimate using the data that is available prior to taking the absolute value. Doing so allows us to provide more reliable estimates for month-on-month rates of core inflation that are substantially less volatile. However, while this method is intuitively appealing, it is noted that the mean of the process is much lower than the reported measure of aggregate inflation, as large positive price shocks are more common than large negative shocks.

When reporting on the results, we refer to the *asymmetric trimmed mean* estimate for the traditional trimmed mean estimate that was calculated according to the method employed in Blignaut *et al.* (2009), while the *absolute value trimmed mean* estimate refers to the result from the alternative trimmed means method.

## 8.4 Dynamic Factor Model Estimates of Core Inflation

Dynamic factor models have been utilised by several researchers to construct measures for core inflation in a number of countries.<sup>21</sup> These models rely on the assumption that the underlying core inflation rate is driven by a small number of common factors, and that the noise in the inflation process is related to

<sup>20</sup>For example, where one is looking to trim a single series from a number of zero rates of inflation, the choice of which zero to trim would influence the result depending on whether its respective weight is relatively large or small. Unfortunately, there is no intuition that could be used to guide this decision, and as a result the choice is largely arbitrary.

<sup>21</sup>Recent studies for estimating core inflation with dynamic factor models include: Amstad and Potter (2009) for the United States, Cristadoro *et al.* (2005) for the European Union, Amstad and Fischer (2004) for Switzerland, and Giannone and Matheson (2007) for New Zealand.

localised shocks that affect a limited number of the disaggregated price series. Therefore, the model may be specified as

$$\tilde{\pi}_{n,t} = \mu_n + \lambda_n F_t + \epsilon_{n,t} , \quad (8.4.1)$$

where,  $F_t = (f_{1,t}, \dots, f_{q,t})'$  represents the  $r$  common dynamic factors and  $\lambda_n$  is the matrix of factor loadings that has dimensions  $n \times q$ . The variable-specific shocks are then contained in the vector  $\epsilon_{n,t}$ . To give the model a dynamic characterisation, it is assumed that the factors have a vector autoregressive structure, where  $\chi_q(L)F_t = v_t$ .

To estimate this model we make use of the procedure of Doz *et al.* (2011), as it allows for the model to be estimated in the presence of missing data. This is of importance in the current setting, since we need to make use of the data that seeks to approximate the new method for calculating consumer prices for the in-sample period 2002M1-2007M12 to derive a consistent estimate for core inflation in 2009M1.<sup>22</sup>

This procedure also facilitates the procedure for generating forecasts within the unified framework, using the Kalman filter with the measurement equation in the state-space representation of the model that treats the vector of factors as an unobserved process. In this case, the autoregressive state equation allows for time-varying parameters in all of the models that are used to generate forecasts.

The number of factors that are included in the optimal dynamic factor model is determined by out-of-sample statistics, allowing various combinations for a maximum of  $q = 10$  and  $s = 1$ . Note that the decision to utilise a single autoregressive element is due to the need to restrict the dimensionality of the comparative investigation, where all the forecasts for competing models make use of an equivalent state-space framework.<sup>23</sup>

## 8.5 Wavelets Estimates of Core Inflation

Most of the commonly used decompositions in economics, such as those that were designed by Hodrick and Prescott (1997), Baxter and King (1999) and Christiano and Fitzgerald (2003), make use of expressions from the time domain. All of these techniques seek to approximate ideal filters, where one is able to identify the trend, cycle and noise components that are located at different periodicities.

Alternatively, these time domain decompositions may be represented in the frequency domain, where the components may be expressed as elements that

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<sup>22</sup>Such data was approximated at a disaggregated level for most, but not all respective series; for example, there is no data for owner's equivalent rental income for 2002M1-2007M12.

<sup>23</sup>If we made use of two autoregressive elements, we would not only generate 20 forecasts for the dynamic factor models, but we would also need to generate twice as many forecasts for all the other models, for which we make use of a similar state-space model.

arise at different Fourier frequencies.<sup>24</sup> This methodology effectively breaks down a time series into a number of sine and cosine functions, which define the rate at which the time series oscillates. It is important to note, that applying these transformations results in the loss of all time-based information, where it is assumed that the periodicity of all the components are consistent throughout the entire sample.

To allow for changes in the periodicity of the respective components, Gabor (1946) developed the Short-Time Fourier Transform (STFT) technique, which involves applying of a number of Fourier transforms to different subsamples of the data.<sup>25</sup> Although this technique would provide potentially useful information on the timing of an event that may have arisen at a particular frequency, it is limited in that the precision of the analysis is affected by the size of the subsample. For instance, one would need a large subsample to identify changes that arise at a low frequency, and small subsamples to identify changes in the higher frequency components.

To overcome the limitations of the above frequency domain techniques, wavelet transformations were developed to capture features of time-series data across a wide range of frequencies that may arise at different points in time. This technique makes use of a number of wavelet functions that are stretched and shifted to describe features that are localised in frequency and time. For example, the wavelet function would be expanded over a relatively long period of time when identifying low-frequency events, and it would be relatively narrow when describing high frequency events.<sup>26</sup> After shifting all of these wavelet functions that have different amplitudes over the entire sample of data, one is able to associate the components with specific time horizons that occur at different locations in time.

Early work with wavelet functions dates back to Haar (1910), who used a number of square-wave functions to decompose time-series data. Unfortunately, the properties of square-wave functions were found to be limited, and as such, a number of alternatives were developed, including those that are discussed in Grossmann and Morlet (1984) and Daubechies (1992).<sup>27</sup> For the computation of these transformations, which make use of various wavelet functions at different scales, most studies currently employ the multiresolution decomposition of Mallat (1989) and Strang and Nguyen (1996).

To describe the use of this technique, one could allow for the case where a

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<sup>24</sup>Shumway and Stoffer (2010) suggest that the use of the frequency domain is more appropriate for decompositions of time-series variables into trend, cycle and noise components.

<sup>25</sup>Gencay *et al.* (2010) refer to the subsample as a data *window*, where the technique involves sliding the window across the time series and taking a Fourier transform of each subsample.

<sup>26</sup>The wavelets literature refers to the use of scales rather than frequency bands, where the highest scale refers to the lowest frequency and vice versa.

<sup>27</sup>See, Hubbard (1998) and Heil and Walnut (2006) for a detailed account of the history of wavelet analysis.

variable is composed of a trend and a number of higher-frequency components. In this instance, the trend may be represented by a father wavelet,  $\phi(t)$ , while the mother wavelets,  $\psi(t)$ , are used to describe information at lower scales (i.e. higher frequencies). Using an orthogonal wavelet transformation, one could then describe variable  $x_t$  as

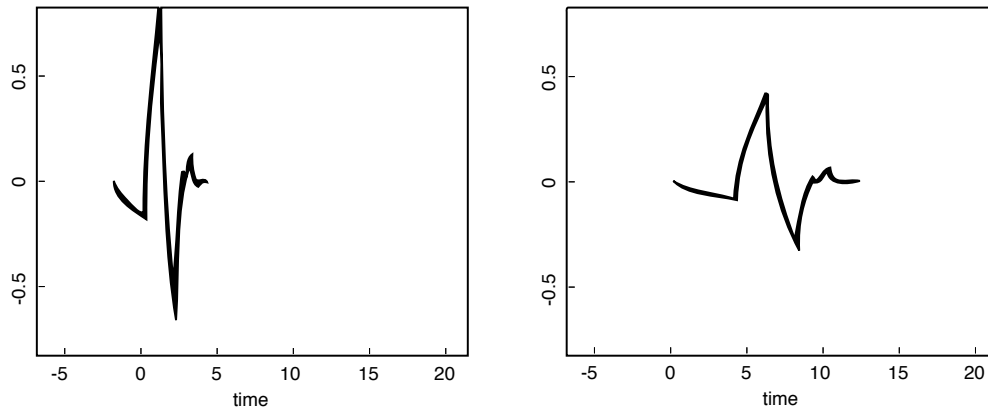
$$x_t = \sum_k s_{0,k} \phi_{0,k}(t) + \sum_{j=0}^J \sum_k d_{J,k} \psi_{J,k}(t), \quad (8.5.1)$$

where  $J$  refers to the number of scales, and  $k$  refers to the location of the wavelet in time. The  $s_{0,k}$  coefficients are termed smooth coefficients, since they represent the trend, and the  $d_{J,k}$  coefficients are termed the detailed coefficients, since they represent finer details in the data.

The mother wavelet functions,  $\psi_{J,k}(t), \dots, \psi_{1,k}(t)$ , are then generated by shifts in the location of the wavelet in time and scale, such that

$$\psi_{j,k}(t) = 2^{-j/2} \psi \left( \frac{t - 2^j k}{2^j} \right), \quad j = 1, \dots, J, \quad (8.5.2)$$

where the shift parameter is represented by  $2^j k$  and the scale parameter is  $2^j$ .<sup>28</sup> As depicted in the daublet wavelet functions in figure 8.2, smaller values of  $j$  (which produce a smaller scale parameter  $2^j$ ), would provide the relatively tall and narrow wavelet function on the left. For larger values of  $j$ , the wavelet function is more spread out and of lower amplitude. In addition, after shifting this function by one period, we produce the function that is depicted on the right of figure 8.2.



**Figure 8.2:** Daublet (4) wavelet functions -  $\psi_{1,0}(t)$  and  $\psi_{2,1}(t)$

Early applications of wavelet methods in economics include the work of Ramsey and Zhang (1997), which made use of a wavelet decomposition of

<sup>28</sup>This choice of dyadic scaling factors is arbitrary but efficient (Daubechies, 1992).

exchange rate data to describe the distribution of this data at different frequencies. In addition, Ramsey and Lampart (1998*b*) made use of a decomposition of money and income data to describe the relationship between these variables at different frequencies, while Ramsey and Lampart (1998*a*) considered the relationship between income and expenditure (i.e. permanent income hypothesis) at different time scales.<sup>29</sup> More recently, Dowd *et al.* (2010) and Baqaee (2010) describe the use of wavelets in the derivation of measures for core inflation in the United States and New Zealand.<sup>30</sup>

In this study we make use of a maximum overlap discrete wavelet transform (MODWT), which does not restrict the sample size to a multiple of  $2^j$ . In addition, this technique is also able to preserve the phase properties of the data, where it can match the smoothed terms to the underlying price index. As we are primarily interested in removing the noise from the data, we make use of various smoothed wavelet functions that include different daubelets, coiflets and symlets, where  $J$  is set at 2 and 3, respectively.<sup>31, 32</sup>

## 8.6 Results

### 8.6.1 Out-of-sample results

To generate forecasts for each of the respective models we make use of a Kalman filter, after the model has been cast in the following state-space framework, with respective measurement and state equations,<sup>33</sup>

$$\hat{\pi}_t = \mu + \alpha_t \hat{\pi}_{t-1} + \varepsilon_t \quad (8.6.1)$$

$$\alpha_{t+1} = \alpha_t + \zeta_t \quad (8.6.2)$$

where  $\alpha_t$  is the time-varying parameter. This would ensure that the framework that is used to generate the forecasts for all the models is equivalent to that which is employed by the dynamic factor model. For completeness, we have also included forecasts that were generated from the reported measure of

<sup>29</sup>Using a wavelet decomposition, Ramsey and Lampart (1998*b*) is able to explain the conflicting results from Granger causality tests that were usually performed on the levels of these variables.

<sup>30</sup>See Ramsey (2002), Schleicher (2002) and Crowley (2007) for a more general overview of the use of these methods in economics.

<sup>31</sup>Specifically, we make use of daubelets 2-6, coiflets 1-5, and symlets 2-6, where higher orders refer to functions that focus more intently on higher frequency information.

<sup>32</sup>In this study we perform a simple wavelet analysis that seeks to identify the trend, or father wavelet. Note that these methods could also be used to remove noise from each of the respective scales, should they extend over a particular threshold, before each of the signals is combined to represent the de-noised signal.

<sup>33</sup>Note, that if we had made use of more than one autoregressive element in the dynamic factor model, we would have needed to make use of more than one state-space framework to generate equivalent forecasts for the other models. This would have resulted in an extremely large number of different forecasts that would need to be evaluated.

aggregate consumer price inflation. This model is used to generate forecasts that extend from 1-12 steps ahead, over the ten-year period, 2003M1-2012M12. These forecasts are compared to the own forecast values of future inflation, by calculating the root-mean squared errors and the statistics of Diebold and Mariano (1995).<sup>34</sup> To calculate the own forecasts, we make use of the reported measure of headline inflation in the above state-space model to generate the respective forecast.

Figure 8.3 displays the respective root-mean square error statistics for the average of the 1-12 step ahead forecasts. In this case we report only on the optimal specifications for each of the above classes of models, which are those that have the lowest average root-mean squared errors over time. All of these measures report relatively high forecasting errors from 2003-2004 and from the period 2007-2009. These periods relate to those where year-on-year inflation was at a reasonably high level (i.e. well in excess of the target rate, as may be seen in figure 8.1).

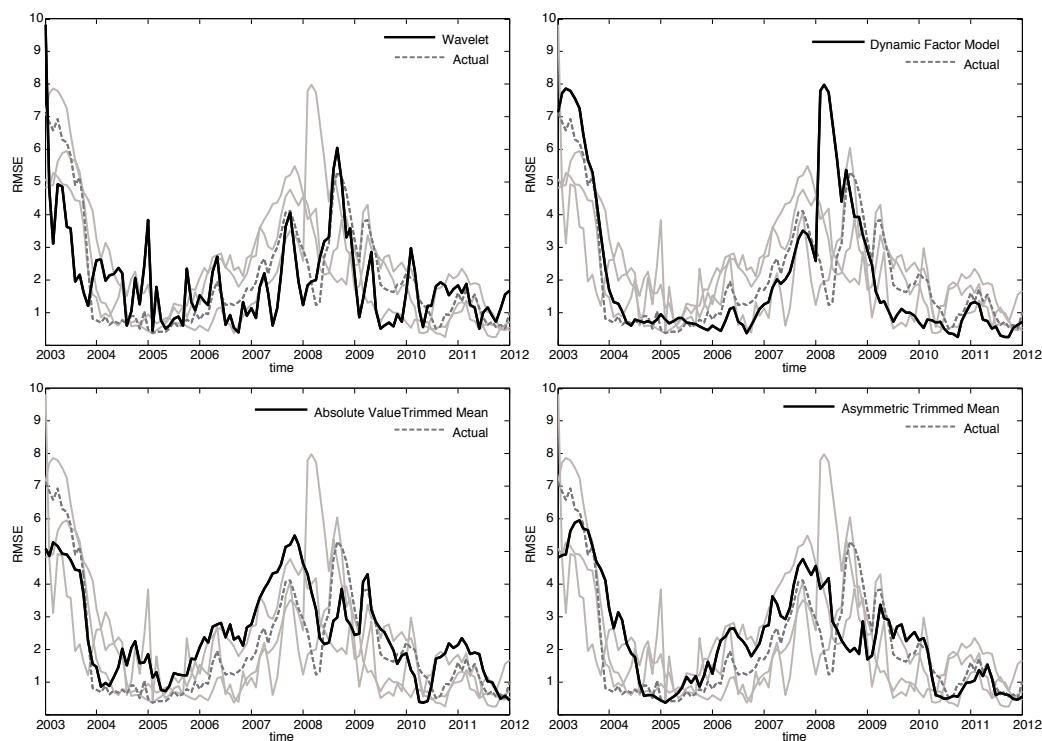
During these periods, the dynamic factor model would appear to do relatively poorly (particularly after a change in the trend of the process), while it does particularly well when inflation is more stable and predictable. The asymmetric trimmed means estimate would appear to do relatively well after the process has remained stable for an extended period of time, but at other times its forecasting error is relatively large. Similarly so for the absolute value trimmed mean estimate, although at almost all points in time its forecasting error is larger than the asymmetric trimmed means estimate. The wavelets estimate would appear to provide relatively smaller errors when inflation is high (or trending either upwards or downwards), while its errors are often somewhat larger when inflation is low (where there is less of a positive or negative trend). The wavelets estimate would also appear to perform poorly when there is a change in the direction of the trend.<sup>35</sup>

To determine whether the forecasting errors from the core inflation measures are significantly different from those that are provided by the own forecasts that make use of the reported measure of headline (aggregate) inflation, table 8.1 summarises the sum of the significant Diebold and Mariano statistics. These results suggest that on 46 occasions the dynamic factor model produced forecasts that were significantly superior to those that were generated by the equivalent models for the reported measure of headline inflation, while on 20 occasions it was significantly inferior. The wavelets estimates also generated a large number of forecasts that significantly outperformed those of the own-inflation forecasts, while both trimmed means estimates generated the largest

<sup>34</sup>The Diebold and Mariano statistics may be used to determine whether the differences in the root-mean squared errors of competing models are statistically significant.

<sup>35</sup>These results for the wavelets estimates should not be surprising, since the technique is able to explicitly model any trend in the process, including those that may exhibit stochastic tendencies.





**Figure 8.3:** Root-mean squared error (2003M1-2012M12): year-on-year inflation

number of estimates that are significantly inferior.<sup>36</sup>

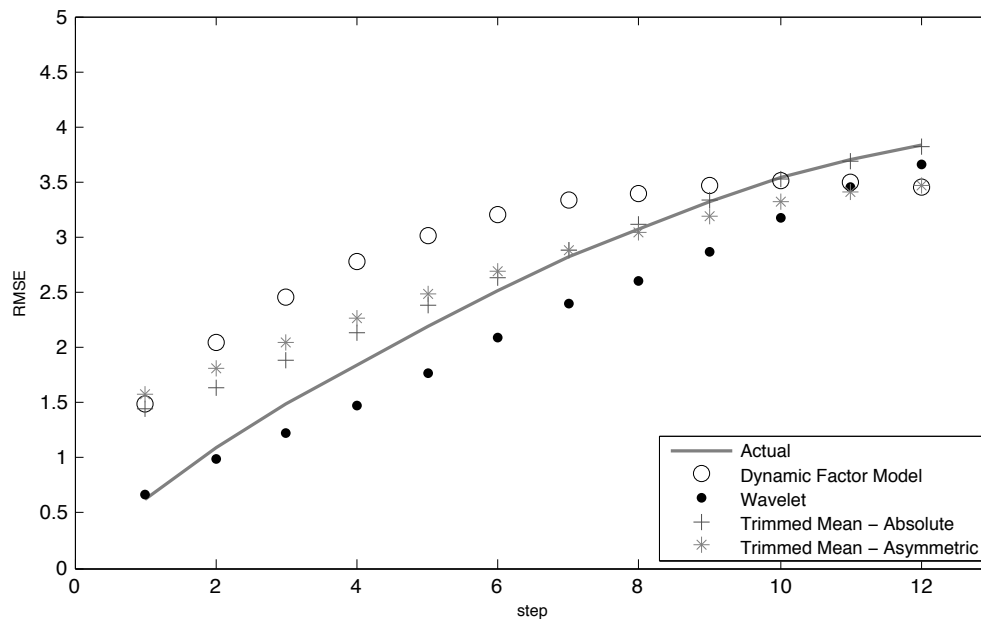
	Factor Model	Wavelet	Trim - Absol	Trim - Asymm
Superior	46	46	25	30
Inferior	20	38	69	52

**Table 8.1:** Summary of Diebold and Mariano statistics (2003M1-2012M12): year-on-year inflation

We turn our attention to each of the individual step ahead forecasts, where we report on each 1-12 step ahead forecast, after taking the average of these measures for the entire out-of sample period. The results are shown in figure 8.4 where we note that the wavelets measure of core inflation is consistently below that which is provided by the own inflation forecast at almost all horizons. At short to medium horizons, the dynamic factor model would appear to perform rather poorly; however, as the forecast horizon increases, so too does

<sup>36</sup>Tables A.2 to A.4 contains the relative root-mean squared errors and Diebold and Mariano statistics for each period of time, where the relative root-mean squared error is calculated as  $[(\text{RMSE}_{\text{DFM}}/\text{RMSE}_{\text{actual}}) - 1] \times 100$ .

its relative forecasting potential. The trimmed mean estimates would appear to provide similar results, although the trimmed mean absolute value hardly manages to outperform the forecasts that use aggregated inflation data, while at longer horizons, the errors from the asymmetric trimmed mean estimate are equivalent to those of the dynamic factor model.



**Figure 8.4:** Root-mean squared error (1-12 step ahead): year-on-year inflation

To investigate the statistical significance of these results, we have summarised the Diebold and Mariano statistics for the 1-12 step ahead forecasts in table 8.2. In this case there are four occasions where the wavelets measure is superior, while there are no occasions where it is inferior. This contrast strongly with the other measures, which are never significantly superior, while they are inferior on a number of occasions.

	Factor Model	Wavelet	Trim - Absol	Trim - Asymm
Superior	0	4	0	0
Inferior	5	0	3	4

**Table 8.2:** Summary of Diebold and Mariano statistics (1-12 step ahead): year-on-year inflation

## 8.6.2 In-sample results

Bryan and Cecchetti (1994) suggest that estimates of core inflation should provide an accurate estimate of the first moment of actual inflation, while the second moment for core inflation should be lower than actual inflation. In addition, they also suggest that the measure of core inflation should be cointegrated with the actual measure of inflation. When we consider the in-sample statistics for the models that were generated over the full year-on-year sample of 1976M1-2012M12, we note in table 8.3 that the estimates that provide the most appropriate approximation of the mean and median of actual inflation are derived from the wavelets and dynamic factor models, while the greatest discrepancy is provided by the two trimmed means estimates. The standard deviation for all the estimates are below actual inflation, which implies that they all satisfy this criterion.

	Actual	Factor Model	Wavelet	Trim - Absol	Trim - Asymm
mean	9.95	9.95	9.95	8.5	9.78
median	10	9.89	10.25	8.79	9.79
std	4.5	4.38	4.46	3.84	4.4

**Table 8.3:** In-Sample Descriptive Statistics (1976M1 - 2011M4): Year-on-Year

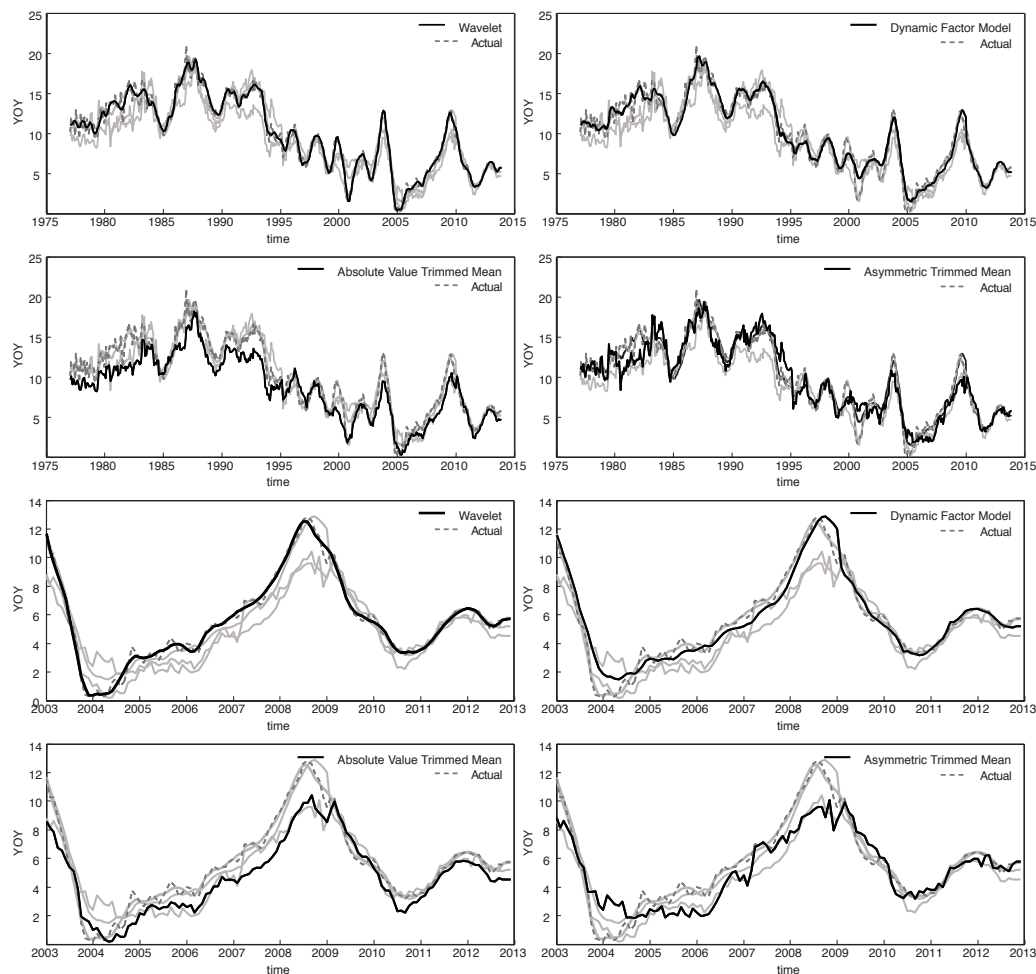
In addition, we note that with the exception of the absolute-value trimmed mean estimate for core-inflation, all of the measures of core inflation would appear to be cointegrated when using the Engle-Granger method.<sup>37</sup> The final optimal estimates are then illustrated in figure 8.5 over both the entire sample period and the out-of-sample period.

## 8.7 Conclusion

This chapter considers the use of various methods that may be used to provide an estimate for core inflation in South Africa. It focuses on the application of asymmetric trimmed mean, dynamic factor models, and wavelet approaches, where we estimate a total of 224 different estimates for this key variable. The results are then evaluated using both out-of-sample and in-sample methods.

We find that when comparing the out-of-sample estimates over the ten-year period, 2003M1-2012M12, the only measure of core inflation that is able to improve upon the actual inflation forecasts over all (or for most) horizons,

<sup>37</sup>When testing for cointegration, we first convert the year-on-year measures of inflation (including those for core inflation) into approximate price indices, which are integrated of the first order.



**Figure 8.5:** Estimates of core inflation (year-on-year)

is provided by wavelet estimates. This measure would also appear to provide suitable estimates of future inflation during most periods of time when compared with other models. The only exception would appear to be when there is a change in the trend of the underlying process. For this reason, we make use of this measure of core inflation in the subsequent dynamic stochastic general equilibrium model that is described in part 3.

Before closing this chapter, it is also worth noting that over much longer forecasting horizons (i.e. one year), the dynamic factor model and the asymmetric trimmed means approach improve upon the out-of-sample results of the wavelets methods. In addition, the results from these estimates would also appear to be most encouraging when the trend in the underlying process is relatively flat. Therefore, these results suggest that the long-term forecasts of most measures of core inflation would appear to outperform equivalent forecasts that make use of actual headline inflation, which should encourage further research into measures of this key unobserved variable.

## Part III

# Applied Dynamic Stochastic General Equilibrium Models for South African Data

## Chapter 9

# Estimating a Small Open-Economy Model for South Africa

## 9.1 Introduction

Theoretical macroeconometric models provide economists with an organised framework that can be used to analyse economic phenomena. When utilising a Dynamic Stochastic General Equilibrium (DSGE) framework, such models may include forward looking behaviour, several nominal and real rigidities, a combination of observed and unobserved variables, as well as a relatively large number of shocks. These models provide a popular framework for policy analysis and are used by most central banks for forecasting purposes (Tovar, 2008). Examples of small open-economy DSGE models for the South African economy may be found in Ortiz and Sturzenegger (2007), Steinbach *et al.* (2009a), Alpanda *et al.* (2010a), Alpanda *et al.* (2010b), Alpanda *et al.* (2011) and Steinbach (2013).

To close off the open-economy features of this model, the debt-elastic interest rate (risk) premium that is described in chapter five, has been employed in the initial specification of the model. As noted in Lubik (2007) and Justiniano and Preston (2010), this specification allows for the introduction of country-risk premium shocks that accounts for deviations from uncovered interest rate parity. This condition would prevent unstable borrowing through a relationship that attributes the difference in interest rates to changes in the exchange rate and a risk premium (which is a function of the net foreign asset position and a stochastic shock).

The model in this chapter differs from those of Ortiz and Sturzenegger (2007), Steinbach *et al.* (2009a), Alpanda *et al.* (2010a), and Alpanda *et al.* (2010b) in number of other important ways. Firstly, we make use of a large number of observed variables to identify the variance of the shocks and the mean of the structural parameters. In many of the earlier DSGE models, researchers advocated the use of a large set of shocks to generate persistence, as in Christiano *et al.* (2005), Smets and Wouters (2003) and Smets and Wouters (2007).<sup>1</sup> However, more recently Iskrev (2010), Canova and Sala (2009) and Chari *et al.* (2008) have suggested that the inclusion of too many of these elements may result in identification problems, particularly when the number of observed variables is small. Therefore, we specify the model with an equal number of shocks and observed variables, where the stochastic elements in the model are effectively ‘just identified’ by actual data. In total we make use of ten observed variables, as we are of the opinion that the use of a relatively

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<sup>1</sup>These models were developed for the United States economy. They include a number of open-economy features, where the domestic economy is relatively influential (when compared to the models for a small-open economy that have been used for South African data).

large set of variables provides for a interesting characterisation of the dynamic features of the economy.<sup>2</sup>

Another important feature of this model is that it makes very little use of calibration, as the vast majority of the structural parameters are estimated using Bayesian techniques. In addition, we also allow for an inflation-forecasting central bank; as it is our understanding that this is consistent with the current practice of the SARB.<sup>3</sup>

After estimating the model with Bayesian techniques, the results are then assessed by considering the posterior values of the model parameters, before we discuss the results of the simulated impulse response functions. Further details of the estimates for the central bank's reaction function are also discussed, with reference to the degree to which monetary policy has been conducted in response to past interest rates, expected inflation, current output gap, and the exchange rate; where monetary policy follows a generalised Taylor rule.

The essential features of this model are then compared to a model that makes use of an alternative method for closing off the open-economy features of the model. Following the specification that is include in Galí and Monacelli (2005) and Steinbach *et al.* (2009a), we now assume that international asset markets are complete (i.e. a complete set of contingent claims can be traded across international borders). In addition, they also assume that the risk-premium on domestic assets relative to foreign assets is influenced by the difference between foreign and domestic demand shocks. Therefore, as described in chapter five, such a condition would prevent unstable borrowing as the difference in interest rate would be curtailed by movements in the exchange rate.

In the following section, we describe the basic features of the model. Section 9.3 contains details of the estimation and prior distributions for the parameters. Section 9.4 discusses the results from the model, including posterior parameter estimates, simulated moments, impulse response functions and variance decomposition. Thereafter, section 9.5 describes the use of the alternative method for closing-off the open economy features in the model, which are then compared with the aid of historical variance decompositions. Section 9.6 concludes.

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<sup>2</sup>Whilst having many more shocks than observed variables may give rise to identification problems, having more observed variables than shocks will result in stochastic singularity problems. Ruge-Murcia (2007) discuss the implications of stochastic singularity in DSGE models that make use of frequentist estimation techniques.

<sup>3</sup>See, Kahn (2008) for details on the activities and challenges that are encountered when making Monetary Policy decisions in emerging market economies. Svensson (1997), Svensson (2005) and Woodford (2007) establish the general theory on the practice of inflation-forecast targeting.



## 9.2 The Model Economy

### 9.2.1 Households

The model utilises a unit measure of identical and infinitely lived representative households. The household's preferences over consumption,  $C_t$ , and labour,  $N_t$ , may be described by the following utility function:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \Theta_{\tau} \left\{ \frac{(\bar{C}_{\tau} - \zeta C_{\tau-1})^{1-\sigma}}{1-\sigma} - \frac{N_{\tau}^{1+\gamma}}{1+\gamma} \right\} \right] \quad (9.2.1)$$

where  $t$  indexes time,  $\beta$  is the time-discount factor,  $\sigma$  is the inverse intertemporal-elasticity of substitution, and  $\gamma$  is the inverse Frisch labour supply elasticity. Habits in consumption are represented by  $\zeta$ , which depend on past aggregate consumption,  $C_{t-1}$  (Abel, 1990).<sup>4</sup>  $\Theta$  is an exogenous demand shock, whose natural logarithm follows an AR(1) process, with persistence parameter  $\rho_c$  and error,  $\epsilon_{c,t}$ .

In a closed-economy model, the budget constraint for the representative household may then be expressed as,

$$P_t C_t + E_t [Q_{t|t+1} D_{t+1}] \leq D_t + W_t N_t$$

where dividends and interest earned on investment in the following period are denoted  $D_{t+1}$ , the stochastic discount factor is  $Q_{t|t+1}$ , and nominal wages are represented by  $W_t$ . The term for dividends and interest would incorporate the returns from domestic bonds,  $B_t$ , which pay a gross nominal interest of  $i_t$ . However, in an open-economy model, the household is also able to hold foreign bonds,  $B_t^*$ , which pay a gross nominal interest of  $i_t^* \phi_{t-1}$ , where  $i_t^*$  is the foreign gross nominal interest rate, and  $\phi_t$  is a risk-premium factor. Where  $e_t$  is the nominal exchange rate, we may express the households bond holdings with,

$$B_t + e_t B_t^* \leq i_{t-1} B_{t-1} + i_{t-1}^* \phi_{t-1} e_t B_{t-1}^*$$

Hence, the budget constraint for the open-economy household may be given by,

$$P_t C_t + B_t + e_t B_t^* \leq i_{t-1} B_{t-1} + i_{t-1}^* \phi_{t-1} e_t B_{t-1}^* + W_t N_t \quad (9.2.2)$$

which is similar to the condition that is contained in Justiniano and Preston (2010).<sup>5</sup>

<sup>4</sup>In equilibrium,  $C_t = \bar{C}_t$ , and the degree of habit persistence is treated as an externality by the households.

<sup>5</sup>As is discussed below, in equation (9.2.7), the term  $\phi_{t-1}$  will include a stochastic term for imperfect risk-sharing.

After introducing the foreign economy, we need to expand the consumption index,  $C_t$ , which combines a composite of a home good,  $C_{h,t}$ , and a foreign good,  $C_{f,t}$ . The broad consumption index may therefore be described as,

$$C_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{h,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (9.2.3)$$

where  $\alpha$  may be termed the level of openness, which is expressed as a level parameter that is determined by the importance of foreign goods in overall consumption. The parameter  $\eta > 0$  is then used to represent the elasticity of substitution between home and foreign goods.

Using this notation, it should be noted that the home good,  $C_{h,t}$ , are then purchased from the domestic final goods producers, at a price of  $P_t$ . Similarly, the foreign good that are imported into the domestic economy,  $C_{f,t}$ , at a price of  $P_{f,t}$ , which is denominated in local currency. As noted in chapter 5, the consumption price index,  $P_{c,t}$ , is related to the domestic and foreign goods prices, through the expression,

$$P_{c,t} = \left[ (1 - \alpha) P_t^{1-\eta} + \alpha P_{f,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (9.2.4)$$

and the consumer price inflation is defined as  $\pi_t^C = P_{c,t}/P_{c,t-1}$ . The price of the composite consumption good, as  $P_{c,t}$ , allows us to write the consumption aggregate,

$$C_t = \frac{P_t}{P_{c,t}} C_{h,t} + \frac{P_{f,t}}{P_{c,t}} C_{f,t} \quad (9.2.5)$$

where the aggregate consumption index implies that in equilibrium the share of home goods and foreign goods in overall consumption are given respectively by,

$$\frac{C_{h,t}}{C_t} = (1 - \alpha) \left( \frac{P_t}{P_{c,t}} \right)^{-\eta} \quad \text{and} \quad \frac{C_{f,t}}{C_t} = \alpha \left( \frac{P_{f,t}}{P_{c,t}} \right)^{-\eta}. \quad (9.2.6)$$

The risk-premium factor,  $\phi$ , is then expressed as,

$$\phi_{t-1} = \exp(\Phi_{t-1} - \chi a_{t-1}) \quad (9.2.7)$$

where,  $\Phi_t$  is an exogenous risk-premium shock that has stochastic properties, and  $\chi > 0$  regulates the sensitivity of the risk-premium to changes in the ratio of foreign bond holdings to the trend in output,  $a_t$ , which is defined as,

$$a_{t-1} = \frac{e_{t-1} B_{t-1}^*}{\bar{Y}} \quad (9.2.8)$$

where  $\bar{Y}$  is the steady-state value of real output and the exogenous risk-premium shock is assumed to follow an AR(1) process.

It is also assumed that the household supply of labour is provided to firms that produce intermediate goods in a way that is described by monopolistic competitive behaviour. In addition, the labour supply may be differentiated over the continuous interval,  $k \in [0, 1]$ , where the constant elasticity of substitution is used to provide the aggregate wage index,

$$W_t = \left[ \int_0^1 W_{k,t}^{1-\xi_w} dk \right]^{\frac{1}{1-\xi_w}} \quad (9.2.9)$$

When setting wages, it is assumed that households employ staggered contractual agreements, as in Erceg *et al.* (2000), where in every period there is the probability  $1 - \theta_w$  for each household to reset its existing wage. Hence, the aggregate overall wage evolves according to the condition,

$$W_t = \left[ \theta_w (W_{t-1} \pi_{t-1})^{1-\xi_w} + (1 - \theta_w) \bar{W}_t^{1-\xi_w} \right]^{\frac{1}{1-\xi_w}} \quad (9.2.10)$$

where  $\bar{W}_t$  is the new wage that is applied by those households that reset their wage.

### 9.2.2 Final Goods Producers

The producers of final-goods are perfectly competitive. They purchase from a selection of  $j$  goods from intermediate goods producers, which are differentiated over the continuous interval,  $j \in [0, 1]$ . Aggregating over all the differentiated goods would yield the final good,  $y_t$ , with the aid of the function:

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{\theta_t-1}{\theta_t}} dj \right]^{\frac{\theta_t}{\theta_t-1}} \quad (9.2.11)$$

where  $\theta_t$  is the elasticity of substitution between the intermediate goods, and  $\theta$  is the steady-state value of  $\theta_t$ . The gross mark-up over marginal cost that monopolistically competitive *intermediate* firms charge when they make their pricing decisions at the steady-state is then given as,  $\theta/(\theta - 1)$ .<sup>6</sup>

This specification allows for the incorporation of a mark-up shock,  $\mu_t$ , which is modelled as a AR(1) process that is related to the above expression through,  $\mu_t = \theta_t/(\theta_t - 1)$ , such that at the steady-state,  $\bar{\mu} = \theta/(\theta - 1)$ .<sup>7</sup>

The goods of the final producers are either consumed domestically or they are exported abroad,  $C_{h,t}^*$ . Hence, from a production perspective,  $Y_t = C_{h,t} + C_{h,t}^*$ . This would imply that since the final-goods producers are perfectly

<sup>6</sup>A description of the monopolistically competitive intermediate firms is provided in the subsequent subsection.

<sup>7</sup>See, Rabanal and Rubio-Ramirez (2005) and Smets and Wouters (2003) for more on this.

competitive, their profit maximisation problem would be,

$$\max P_t C_{h,t} + e P_{h,t}^* C_{h,t}^* - \int_0^1 P_{j,t} Y_{j,t} dj \quad (9.2.12)$$

where  $P_{j,t}$  is the price of the intermediate good  $j$ . The export price of the home-origin good in units of the foreign currency is  $P_{h,t}^*$ , and  $e_t$  is the nominal exchange rate (in units of domestic currency per unit of foreign currency). We assume that the final-goods producers do not price discriminate when they export, hence,  $e_t P_{h,t}^* = P_t$ . Therefore, the final-goods producer maximisation problem yields the demand function for intermediate goods,

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t \quad (9.2.13)$$

The foreign demand for home-goods is determined by the relationship,

$$C_{h,t}^* = (C_{h,t-1}^*)^\delta \left[ \alpha^* Y_t^* \left( \frac{P_t}{e_t P_t^*} \right)^{-\eta} \right]^{1-\delta} \quad (9.2.14)$$

where  $\delta$  is a persistence parameter determining the extent to which current level of exports are determined by past exports.<sup>8</sup> The share of the home-produced consumption goods in the overall expenditure of foreigners is summarised by the parameter,  $\alpha^*$ , and the foreign aggregate output level,  $Y_t^*$ , follows an AR(1) process.

### 9.2.3 Intermediate Goods Producers

The production function for the intermediate goods producers is specified as,

$$Y_{j,t} = Z_t N_{j,t} \quad (9.2.15)$$

where  $Z_t$  is the aggregate productivity shock, and  $N_{j,t}$  is the amount of (homogeneous) labour input used in the production of the intermediate  $j$  goods. The aggregate productivity shock,  $Z_t$ , is then specified to follow an AR(1) process.

We incorporate the sticky-price mechanism of Calvo (1983), to describe the manner in which the firms for intermediate goods set prices, where each firm has the probability  $(1 - \theta_h)$  of resetting its price. In addition, as we allow for an array of  $j$  different goods, the aggregate price index may be given as,

$$P_{h,t} = \left[ \theta_h (P_{h,t-1} \pi_{h,t-1}^\delta)^{1-\xi_h} + (1 - \theta_h) P_t^{1-\xi_h} \right]^{\frac{1}{1-\xi_h}} \quad (9.2.16)$$

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<sup>8</sup>This persistence can be motivated by a habit specification in the utility of foreigners Lim and McNelis (2008).

These firms discount future earnings at the same rate as households, such that their objective function may be expressed as,

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} Q_{t,t+k} Y_{j,t+k} \left[ \overset{\circ}{P}_{h,t} \pi_{h,t+k-1}^{\delta} - MC_{t+k} P_{h,t+k} \right] \quad (9.2.17)$$

where the total nominal cost function is given as,  $TC_t = W_t N_{j,t}$ .

### 9.2.4 Foreign Producers

Foreign producers can also be modelled in a similar fashion, where we assume that foreign goods are imported directly from foreigners who engage in monopolistic competition themselves, and that they price-to-market when they sell their goods to the domestic market.<sup>9</sup> The evolution of prices for foreign goods that are consumed by the domestic economy is then expressed as,

$$\pi_{f,t} - \varphi_f \pi_{f,t-1} = \beta E_t [\pi_{f,t+1} - \varphi_f \pi_{f,t}] + \frac{\theta - 1}{\kappa^*} (q_t - S_t + \Psi_t) \quad (9.2.18)$$

where  $\pi_f$  is the inflation in the foreign goods price;  $\pi_{f,t} = P_{f,t}/P_{f,t-1}$ . The parameter,  $\varphi_f$ , is for indexation purposes, and  $\kappa^*$  is the foreign cost of price-adjustment. Analogous to the domestic mark-up shock,  $\Psi_t$  is an exogenous cost-push shock whose logarithm is assumed to follow that of an AR(1) process.

The real exchange rate,  $q_t$ , is defined as  $q_t = e_t P_t^*/P_t$ , and the terms-of-trade,  $S_t$ , is defined as  $P_{f,t}/P_t$ .<sup>10</sup> Their difference is analogous to the marginal cost of foreign producers (actually intermediaries) who buy the product at  $e_t P_t^*$  and sell it at  $P_{f,t}$ . This difference can also be thought of as the deviation from the law-of-one-price,  $\psi_{f,t} = e_t + P_t^* - P_{f,t} = q_t - S_t$ .<sup>11</sup>

### 9.2.5 Central Bank

The central bank makes use of the nominal interest rate as its policy instrument in generalised Taylor rule that allows for the inclusion of the exchange rate in its reaction function. In addition, we assume that the central bank targets the expected future value of inflation, and as such we make use of an expectational operator for this critical variable. Hence,

$$i_t = \rho i_{t-1} + (1 - \rho) [\varrho_{\pi} E_t (\pi_{t+1}^c) + \varrho_y \tilde{y}_t + \varrho_d d_t] + \varepsilon_{i,t} \quad (9.2.19)$$

where  $\rho$  determines the extent of interest rate smoothing. The parameters  $\varrho_{\pi}$ ,  $\varrho_y$ ,  $\varrho_d$  determine the importance of CPI inflation, de-trended output and the

<sup>9</sup>A related approach would be to assume that foreign goods are intermediated by *domestic importers* which mark up the foreign price in a staggered fashion Justiniano and Preston (2010).

<sup>10</sup>We assume that the law of one price holds at the steady-state; hence,  $\bar{q} = 1$ , where a bar over a variable indicates its steady-state value.

<sup>11</sup>See, Monacelli (2003).

nominal depreciation of the exchange rate in the Taylor rule. The last term,  $\varepsilon_i$ , is an AR(1) interest rate shock.<sup>12</sup>

### 9.2.6 Log-linear conditions

In what follows, we briefly describe the equations that characterise the equilibrium conditions of the model after all variables are log-linearised around their steady-state.

The domestic household's Euler condition yields a partially forward-looking IS curve in consumption:

$$c_t = \frac{1}{1+\zeta} E_t[c_{t+1}] + \frac{\zeta}{1+\zeta} c_{t-1} - \frac{1-\zeta}{\sigma(1+\zeta)} (i_t - E_t[\pi_{t+1}^c] - \Theta_t) \quad (9.2.20)$$

where  $\sigma$  is the inverse intertemporal-elasticity of substitution and habits in consumption are represented by  $\zeta$ . The exogenous demand shock, is represented by  $\Theta$ , whose natural logarithm follows an AR(1) process, with persistence parameter  $\rho_c$ , and error,  $\epsilon_{c,t} \sim \text{i.i.d.} N[0, \sigma_c^2]$ . The rate of consumer price inflation is expressed as  $\pi_t^c$ .

The relation between consumption and domestic output can be derived from the goods market clearing condition as:

$$y_t = (1-\alpha)c_t + [(1-\alpha)\eta\alpha + \eta\alpha] s_t + \alpha y_t^* + \eta\alpha\psi_{f,t} \quad (9.2.21)$$

where  $\alpha$  is the share of imports in consumption,  $\eta$  is the elasticity of substitution between domestic and foreign goods,  $y_t$  and  $y_t^*$  are domestic and foreign output, respectively, whilst  $s_t = p_{f,t} - p_{h,t}$  is the terms of trade, and  $\psi_{f,t}$  is the deviation of imported goods prices from the law-of-one-price.

Time differencing the terms-of-trade yields  $s_t = s_{t-1} + p_{f,t} - p_{h,t}$ , where  $p_{h,t}$  and  $p_{f,t}$  are inflation rates associated with the domestic and foreign goods prices, respectively. The domestic producer's problem yields a partially forward-looking New Keynesian Phillips curve for domestic price inflation:

$$\pi_{h,t} = \frac{\delta}{1+\delta\beta} \pi_{h,t-1} + \frac{\beta}{1+\delta\beta} E_t[\pi_{h,t+1}] + \frac{(1-\theta_h)(1-\theta_h\beta)}{\theta_h(1+\delta\beta)} mc_t \quad (9.2.22)$$

where  $\beta$  is the time-discount parameter,  $\delta$  determines the degree with which prices are indexed to past domestic price inflation, and  $\theta_h$  is the probability that the firms cannot adjust their prices in any given period. The above Phillips curve ties current domestic inflation rate to past and expected future inflation as well as the marginal costs of the firm. Marginal cost is  $mc_t = \varpi_t - a_t + \gamma s_t + \eta_t^p$ , where  $\varpi_t$  is the real wage rate,  $a_t$  is the level of productivity

<sup>12</sup>We also estimated our model using current inflation and output in the Taylor rule, but this alternative specification generated very similar results. This is in line with the findings in Taylor (1999).

in the production function that follows an exogenous AR(1) process, and  $\eta_t^p$  is a domestic cost-push shock that also follows an AR(1) process.

Similarly, foreign goods price inflation follows a forward-looking Phillips curve:

$$\pi_{f,t} = \beta E[\pi_{f,t+1}] + \frac{(1 - \theta_f)(1 - \theta_f \beta)}{\theta_f} \psi_{f,t} \quad (9.2.23)$$

where  $\theta_f$  is the probability that the importers cannot adjust their prices in any given period. Overall consumer price inflation in the domestic country is given by  $\pi_t = (1 - \alpha)\pi_{h,t} + \alpha\pi_{f,t}$ .

Staggered wage setting by households yields the following wage inflation Phillips curve:

$$\pi_{w,t} - \varphi_w \pi_{t-1} = \beta E_t[\pi_{w,t+1}] - \varphi_w \beta \pi_t + \frac{(1 - \theta_w)(1 - \theta_w \beta)}{\theta_w(1 + \xi_w \gamma)} \mu_t^w \quad (9.2.24)$$

where  $\pi_{w,t}$  is the nominal wage inflation,  $\varphi_w$  is a parameter determining the degree of inflation indexation of nominal wage inflation,  $\gamma$  is the inverse of the elasticity of labour supply, and  $\epsilon_w$  is the elasticity of substitution between differentiated labour services of households in the labour aggregator function. The wedge between the real wage and the marginal rate of substitution between consumption and labour in the household's utility function is  $\mu_w$ , which may be expressed as,

$$\mu_t^w = \frac{\sigma}{1 - \zeta} (c_t - \zeta c_{t-1}) + \gamma(y_t - a_t) - \varpi_t + \eta_t^w \quad (9.2.25)$$

where  $\eta_t^w$  is a wage cost-push shock that follows an AR(1) process. The relationship between nominal wage inflation and real wages can be expressed as  $\pi_{w,t} = \varpi_t - \varpi_{t-1} + \pi_t$ .

The uncovered interest parity (UIP) condition is given by,

$$E[q_{t+1}] - q_t = (r - E[\pi_{t+1}]) - (r_t^* - E_t[\pi_{t+1}^*]) + \phi_t \quad (9.2.26)$$

where  $q_t = e_t + p_t^* - p_t$  is the real exchange rate, which is related to the terms-of-trade and the gap from the law-of-one-price as  $q_t = (1 - \alpha)s_t + y_{f,t}$ . Time differencing the real exchange rate yields the relationship between real and nominal depreciation rates as  $q_t - q_{t-1} = \Delta e_t + \pi_t^* - \pi_t$ . The variable  $\phi_t = \mu_t^\phi + \chi \cdot nfa_t$  captures time-varying country risk-premia, and is the sum of an exogenous component,  $\mu_t^\phi$ , which follows an AR(1) process, and the net foreign asset position of the country,  $nfa_t$ , where  $\chi$  is an elasticity parameter. The net asset position of the country evolves over time according to

$$nfa_t - \frac{1}{\beta} nfa_{t-1} = y_t - c_t - \alpha(s_t - \phi_{f,t}). \quad (9.2.27)$$

Monetary policy is then conducted via a Taylor rule for the nominal interest rate, as in (9.2.19), which is already linear and requires no further manipulation.

The rest of the world is modelled as a closed economy version of the domestic economy, which can be represented by an IS curve:

$$y_t^* = \frac{1}{1+\zeta} E_t[y_{t+1}^*] + \frac{\zeta}{1+\zeta} y_{t-1}^* - \frac{1-\zeta}{\sigma^*(1+\zeta)} (r_t^* - E_t[\pi_{t+1}^*] + \mu_t^{d,*}) \quad (9.2.28)$$

a New Keynesian Phillips curve,

$$\pi_t^* = \frac{\delta^*}{1+\delta^*\beta} \pi_{h,t-1} + \frac{\beta}{1+\delta^*\beta} E_t[\pi_{h,t+1}^*] + \frac{(1-\theta^*)(1-\theta^*\beta)}{\theta^*(1+\delta^*\beta)} mc_t^* \quad (9.2.29)$$

where the foreign marginal cost is given by,

$$mc_t^* = \left( \frac{\sigma^*}{1-\zeta} + \gamma^* \right) y_t^* - \left( \frac{\sigma^*\zeta}{1-\zeta} \right) y_{t-1}^* - (1+\gamma^*) a_t^* + \mu_t^{w,*} \quad (9.2.30)$$

and a foreign Taylor rule specified as,

$$i_t^* = \rho^* i_{t-1}^* + (1-\rho^*) [\varrho_\pi^* \pi_t^* + \varrho_y^* y_t^*] + \epsilon_t^{i,*} \quad (9.2.31)$$

## 9.3 Estimation, Data and Prior Distributions

### 9.3.1 Bayesian Estimation

In this model we estimate most of the parameters and make very little use of calibration. The structural parameters are stacked in the vector  $\Xi$ , which may be expressed as

$$\Xi = [\beta \ \zeta \ \sigma \ \gamma \ \eta \ \alpha \ \chi \ \varphi_h \ \varphi_f \ \varphi_w \ \theta_h \ \theta_f \ \theta_w \ \dots \dots \rho \varrho_\pi \ \varrho_y \ \varrho_d \ \rho_z \ \rho_{\bar{c}} \ \rho_h \ \rho_f \ \rho_w \ \rho_i \ \rho_{\bar{\Phi}} \ \rho_\pi^* \ \rho_y^* \ \rho_i^*]' \quad (9.3.1)$$

The dynamic linear system of equations characterising equilibrium can be summarised as

$$E_t[f(\xi_{t+1}, \xi_t, \xi_{t-1}, v_t; \Xi)] = 0, \quad v_t \sim NID[0, V_{(\Xi)}] \quad (9.3.2)$$

where  $\xi_t$  is the vector of variables,  $v_t$  is the vector containing the orthogonal Gaussian shocks whose variance-covariance matrix is given by the diagonal matrix  $V$ , which is also estimated using Bayesian techniques.<sup>13,14</sup> For given parameter values, the Blanchard-Kahn method can be used to find the *policy*

<sup>13</sup>Since the foreign shock processes are specified as AR(1) processes, the vector of current variables  $\xi_t$  includes the first lags of the foreign variables.

<sup>14</sup>Note that we set  $\bar{\Phi} = -\log(\beta \bar{i}^*)$  to ensure that  $\overline{(nx/Y)} = \bar{a} = 0$ , but we do not need to set specific values for  $\bar{\Phi}$  and  $\bar{i}^*$  since they do not enter any of the log-linearised equilibrium conditions. Similarly, the export parameter,  $\gamma^*$ , does not enter any of the log-linearised equations; hence, is ignored in the estimation.



*functions* that describe how the variables,  $\xi_t$ , evolve over time as a function of their past values,  $\xi_{t-1}$ , and the current realisation of shocks,  $v_t$ , under rational expectations. These policy functions,  $g$ , are linear in the variables, and can be written as<sup>15</sup>

$$\xi_t = g(\xi_{t-1}, v_t; \Xi) = g_\xi(\Xi) \xi_{t-1} + g_v(\Xi) v_t. \quad (9.3.3)$$

The above solution can be thought of as the *transition equation* of a state-space representation, describing the evolution of all variables in the model, including the unobservables. The *measurement equation* describes how the full set of variables are related to the observed variables,  $\xi_t^*$ . Since we assume that the observed variables encounter a very small measurement error,  $\check{\xi}_t$ , our measurement equation is given by<sup>16</sup>

$$\xi_t^* = M \xi_t + \check{\xi}_t \quad (9.3.4)$$

where  $M$  is a matrix that picks the elements of  $\xi_t$  that are observable.

Given a prior density for the parameters,  $\Upsilon(\Xi)$ , and the observable series  $\xi^* = \{\xi_t^*\}_{t=1}^T$ , Bayes' rule implies that the posterior distribution of the parameters is proportional to the product of the prior and the likelihood function

$$\Upsilon(\Xi|\xi^*) \propto L(\xi^*|\Xi) \Upsilon(\Xi) \quad (9.3.5)$$

where the likelihood function,  $L(\xi^*|\Xi)$ , is evaluated using the Kalman filter (Hamilton (1994) and Ireland (2001)). To construct the entire posterior distribution and identify its corresponding moments, Markov Chain Monte Carlo (MCMC) simulation methods are employed (An and Schorfheide (2007) and Fernandez-Villaverde and Rubio-Ramirez (2004)).<sup>17</sup>

### 9.3.2 Data

The dataset that we have used to estimate the model has been discussed extensively in part two, for the period 1990Q1 to 2012Q4. Essentially, we estimate the model with ten observed variables for measures of: domestic output gap,  $\tilde{y}$ , GDP-deflator inflation,  $\pi$ , the wavelets measure of core-inflation,  $\pi^c$ , nominal interest rate,  $i$ , nominal wage inflation,  $\pi^w$ , nominal productivity,  $z$ , nominal currency depreciation,  $d$ , foreign output gap,  $y^*$ , foreign GDP-deflator inflation,  $\pi^*$ , and foreign nominal interest rate,  $i^*$ .

<sup>15</sup>See Blanchard and Kahn (1980), Uhlig (1999), and Adjemian *et al.* (2011) for more on this.

<sup>16</sup>Since the observed data are not perfectly measured we apply a small measurement error of 0.25% to capture the high frequency component that would not necessarily relate to business cycle phenomena.

<sup>17</sup>Christopher Sims' *csminwel* optimisation algorithm is used to find the mode (Sims, 2001), and the *Metropolis-Hastings* algorithm provides the corresponding moments of the posterior distribution. The estimated means of the posterior distributions are then used in constructing the policy functions using the *Blanchard-Kahn* method. All of these calculations are performed with the aid of the *Dynare* software that is currently developed by Adjemian *et al.* (2011).

### 9.3.3 The Prior Distributions

The prior distributions for the parameters, which are similar to the ones used in Alpana *et al.* (2011), are provided in table 9.1.<sup>18</sup> The only parameters that we calibrate pertain to the time-discount parameter,  $\beta$ , and the import-share parameter,  $\alpha$ . Doing so ensures that the model's steady-state will be able to match the features of the observed data (Ireland, 2001). Therefore, we set  $\beta = 0.99$ , reflecting a 4% annual real interest rate at the steady-state. The ratio of South African Imports to GDP averaged 28% over the sample period; hence we set  $\alpha = 0.28$ . All of the remaining parameters in the model are estimated.

The estimation had a little trouble identifying the elasticity of the risk-premium with respect to the ratio of foreign debt to GDP parameter,  $\chi$ ; hence we follow Justiniano and Preston (2010) and set its mean equal to 0.01 and specify a relatively narrow standard deviation of 0.001. The prior for the habit parameter,  $\zeta$ , is slightly more informative than that of Alpana *et al.* (2010b), and has a beta distribution with a mean of 0.7 and standard deviation of 0.1. The risk-premium for the South African economy is captured by the modified UIP condition,  $\phi$ , which takes the form of an elasticity and is assumed to be positive, with a mean of 0.01 and a standard deviation of 0.2.

For  $\sigma$ , the inverse of the elasticity of intertemporal substitution, we specify a gamma prior with a mean of 1.5 and a standard deviation of 0.37. The parameter  $\eta$  has a gamma prior with a mean of 1.5 and a standard deviation of 0.25, reflecting *a priori* expectation of a high elasticity of substitution between home and foreign goods. The parameter  $\gamma$  also has a gamma prior with a mean of 2 (reflecting a Frisch-elasticity of labour supply of 0.5) and a standard deviation of 0.75.

The use of uninformative priors for the price-indexation parameters  $\varphi_h$  and  $\varphi_f$  generated low posterior estimates; which results in the model failing to generate inflation persistence, and hump-shaped impulse responses for inflation. Therefore, we made use of beta distributions with a mean of 0.7 and a standard deviation of 0.05 for these parameters.<sup>19</sup> The same prior is used for the degree of indexation in wages,  $\varphi_w$ .<sup>20</sup>

The degree of price-stickiness, which appear in the respective domestic and foreign Phillips curve expressions are estimated with a prior mean for the  $\theta_h$  and  $\theta_f$  parameters of 0.5. We use the same prior for the degree of price-stickiness in the staggered wage contracts. The relatively high and informative

<sup>18</sup>The abbreviations used in the table for the prior distributions are: C: calibrated; B: beta; N: normal; G: gamma; IG: inverse gamma. The mean and standard deviations of the prior densities are given in parentheses.

<sup>19</sup>These priors are slightly more informative than Justiniano and Preston (2010), but in line with Smets and Wouters (2003).

<sup>20</sup>In contrast with Steinbach *et al.* (2009a), who calibrate all but one of these parameters, we are able to estimate of the parameters in the staggered wage contracts, as we include data on wages and productivity.

priors that are used for the price-adjustment cost parameters help match the magnitudes of the impulses generated from our model, especially for inflation and output, to the corresponding impulses generated by the core forecasting model of the South African Reserve Bank.<sup>21</sup>

For the Taylor rule parameters, we follow Justiniano and Preston (2010), and assume that the priors for  $\varrho_\pi$ ,  $\varrho_y$ , and  $\varrho_d$  all have a gamma distribution with means of 1.5, 0.25 and 0.25, respectively. A relatively large second moment is assigned to each of these parameters, to allow for a significant amount of revision after the priors have been taken to the data. The prior for the interest rate smoothing parameter,  $\rho$ , is assigned a beta distribution with a mean of 0.5 and a standard deviation of 0.25. Similar priors have been used in other models that have been applied to South African data, such as Steinbach *et al.* (2009a) and Alpanda *et al.* (2010a).<sup>22</sup>

The prior distributions used for the persistence in all the foreign and domestic shocks are assigned relatively uninformative priors where the persistence is moderate. The distributions for the persistence parameters take a beta distribution with a mean of 0.5, and the standard deviation of 0.2. Similarly, the standard deviations of the shocks are assumed to be uninformative i.i.d processes that take inverse-gamma distributions, with a mean of 0.5% and an infinite standard deviation.

## 9.4 Results

### 9.4.1 The Posterior Moments

The estimates for the mean and the 10%-90% confidence marks of the posterior distributions of the estimated parameters are reported in table 9.1. The density functions for the posteriors are plotted along with their priors in figures (9.1)-(9.2).<sup>23</sup>

The mean estimates for the Taylor rule parameters are consistent with previous estimates in the literature regarding monetary policy in South Africa, including Woglom (2005), Ortiz and Sturzenegger (2007), and Steinbach *et al.* (2009a). The Taylor rule is fairly persistent with mean  $\rho$  equal to 0.7, and the mean estimates for  $\varrho_\pi$ ,  $\varrho_y$ , and  $\varrho_d$  are 1.8, 0.3 and 0.03 respectively, implying

<sup>21</sup>See, Smal *et al.* (2007) for a description of the core forecasting model of the domestic central bank.

<sup>22</sup>Alpanda *et al.* (2010a) note that their results were very similar when they made use of slightly different priors for the Taylor rule parameters. In particular, they tried a gamma distribution with a mean of 0.125 and a standard deviation of 0.125 for the output coefficient  $\varrho_y$ , and a beta prior for  $\rho_i$  with a mean of 0.7 and a standard deviation of 0.1, as in Rabanal and Rubio-Ramirez (2005). This would suggest that the data contains sufficient information to identify these parameters.

<sup>23</sup>For the Metropolis-Hastings algorithm in *Dynare*, we use two chains of 100,000 draws each with a 45% initial burn-in phase. The acceptance rate for each chain is about 28%.

		Prior distribution	Posterior mean	[10%	90%]
<b>Structural Parameters</b>					
$\beta$	Time discount factor	C	[0.99]		
$\zeta$	Habit in consumption	B	[0.70, 0.100]	0.854	0.812 0.897
$\sigma$	Inverse intertemp. subst.	G	[1.50, 0.370]	1.196	0.891 1.501
$\gamma$	Inverse of labour supply	G	[2.00, 0.750]	2.606	1.778 3.434
$\eta$	Home/foreign subst.	G	[1.50, 0.250]	0.641	0.579 0.704
$\alpha$	Import share in consumption	C	[0.28]		
$\chi$	Debt elasticity risk premium	N	[0.01, 0.001]	0.010	0.009 0.011
$\phi$	Modified UIP condition	B	[0.10, 0.200]	0.269	0.247 0.291
$\theta_h$	Calvo: domestic prices	B	[0.50, 0.100]	0.850	0.824 0.876
$\theta_f$	Calvo: foreign prices	B	[0.50, 0.100]	0.450	0.418 0.483
$\theta_w$	Calvo: wage prices	B	[0.50, 0.100]	0.789	0.756 0.823
$\varphi_h$	Indexation: domestic prices	B	[0.70, 0.050]	0.584	0.524 0.645
$\varphi_f$	Indexation: foreign prices	B	[0.70, 0.050]	0.679	0.627 0.732
$\varphi_w$	Indexation: wage prices	B	[0.70, 0.050]	0.694	0.642 0.745
<b>Taylor Rule Parameters</b>					
$\rho$	Interest rate smoothing	B	[0.75, 0.100]	0.680	0.633 0.727
$\varrho_\pi$	Reaction to inflation	G	[1.50, 0.250]	1.807	1.490 2.124
$\varrho_y$	Reaction to output gap	G	[0.25, 0.120]	0.297	0.196 0.399
$\varrho_d$	Reaction to exchange rate	G	[0.12, 0.050]	0.029	0.017 0.042
<b>Persistence Parameters</b>					
$\rho_z$	AR(1): productivity	B	[0.50, 0.200]	0.826	0.795 0.856
$\rho_w$	AR(1): wages	B	[0.50, 0.200]	0.879	0.850 0.909
$\rho_h$	AR(1): domestic cost push	B	[0.50, 0.200]	0.270	0.210 0.33
$\rho_f$	AR(1): foreign cost push	B	[0.50, 0.200]	0.952	0.938 0.966
$\rho_w$	AR(1): wage cost push	B	[0.50, 0.200]	0.331	0.265 0.398
$\rho_d$	AR(1): depreciation (risk)	B	[0.50, 0.200]	0.826	0.785 0.867
$\rho_i$	AR(1): monetary	B	[0.50, 0.200]	0.662	0.597 0.727
$\rho_{y^*}$	AR(1): foreign output	B	[0.50, 0.200]	0.924	0.907 0.942
$\rho_{\pi^*}$	AR(1): foreign inflation	B	[0.50, 0.200]	0.61	0.542 0.679
$\rho_{i^*}$	AR(1): foreign interest rate	B	[0.50, 0.200]	0.932	0.915 0.949
<b>Shocks</b>					
$\epsilon^z$	i.i.d.: productivity	IG	[0.0050, $\infty$ ]	0.020	0.018 0.021
$\epsilon^c$	i.i.d.: wages	IG	[0.0050, $\infty$ ]	0.003	0.002 0.003
$\epsilon^h$	i.i.d.: domestic cost push	IG	[0.0050, $\infty$ ]	0.008	0.008 0.009
$\epsilon^f$	i.i.d.: foreign cost push	IG	[0.0050, $\infty$ ]	0.045	0.038 0.051
$\epsilon^w$	i.i.d.: wage cost push	IG	[0.0050, $\infty$ ]	0.014	0.013 0.016
$\epsilon^d$	i.i.d.: depreciation (risk)	IG	[0.0050, $\infty$ ]	0.005	0.004 0.006
$\epsilon^i$	i.i.d.: monetary	IG	[0.0050, $\infty$ ]	0.003	0.002 0.003
$\epsilon^{y^*}$	i.i.d.: foreign output	IG	[0.0050, $\infty$ ]	0.012	0.011 0.013
$\epsilon^{\pi^*}$	i.i.d.: foreign inflation	IG	[0.0050, $\infty$ ]	0.002	0.002 0.002
$\epsilon^{i^*}$	i.i.d.: foreign interest rate	IG	[0.0050, $\infty$ ]	0.001	0.001 0.001

Table 9.1: Estimated Posterior Parameters Values

that the central bank reacts strongly to changes to inflation, whilst its reaction to changes in exchange rate are extremely small. Note that the estimates for  $a_\pi$  and  $\varrho_y$  are influenced by the choice of the prior, as can be seen from by the respective prior and posterior densities.

The shocks are also fairly persistent, partly due to the prior distributions assumed for these parameters. The innovations to the risk premium and the external cost-push shocks have fairly large standard deviations, while the innovation to the Taylor rule has a standard deviation of 0.3% (i.e. just over 1% when annualised), similar to the estimates for the U.S. and the European Union (Smets and Wouters, 2003). The persistence and the standard deviation parameters of the productivity shock are not well identified by the data, as the prior and the posterior distributions for these parameters are almost identical.<sup>24</sup>

The habit parameter,  $\zeta$ , has a mean of 0.85, which is fairly high, despite the uninformative prior that was imposed in the estimation. The indexation parameters in the Phillips curves,  $\varphi_h$  and  $\varphi_f$ , have estimated means of 0.58 and 0.68; whilst the indexation in wages,  $\varphi_w$ , is slightly higher at 0.69. When we initially estimated these parameters with uninformative priors, we found that the mean of their posteriors had very low values; this did not generate hump-shaped impulse responses for output and inflation, which is more in line with previous VAR evidence. Smets and Wouters (2003) find that habits and price indexation play an important role in generating intrinsic persistence in the model; this led us to employ more informative priors for these parameters.

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<sup>24</sup>Both the mark-up shock and the productivity shock affect marginal costs in a similar fashion, so separate identification of these shocks requires more data than was used in the estimation. We choose not to do this here since we have abstracted from capital in the production function.

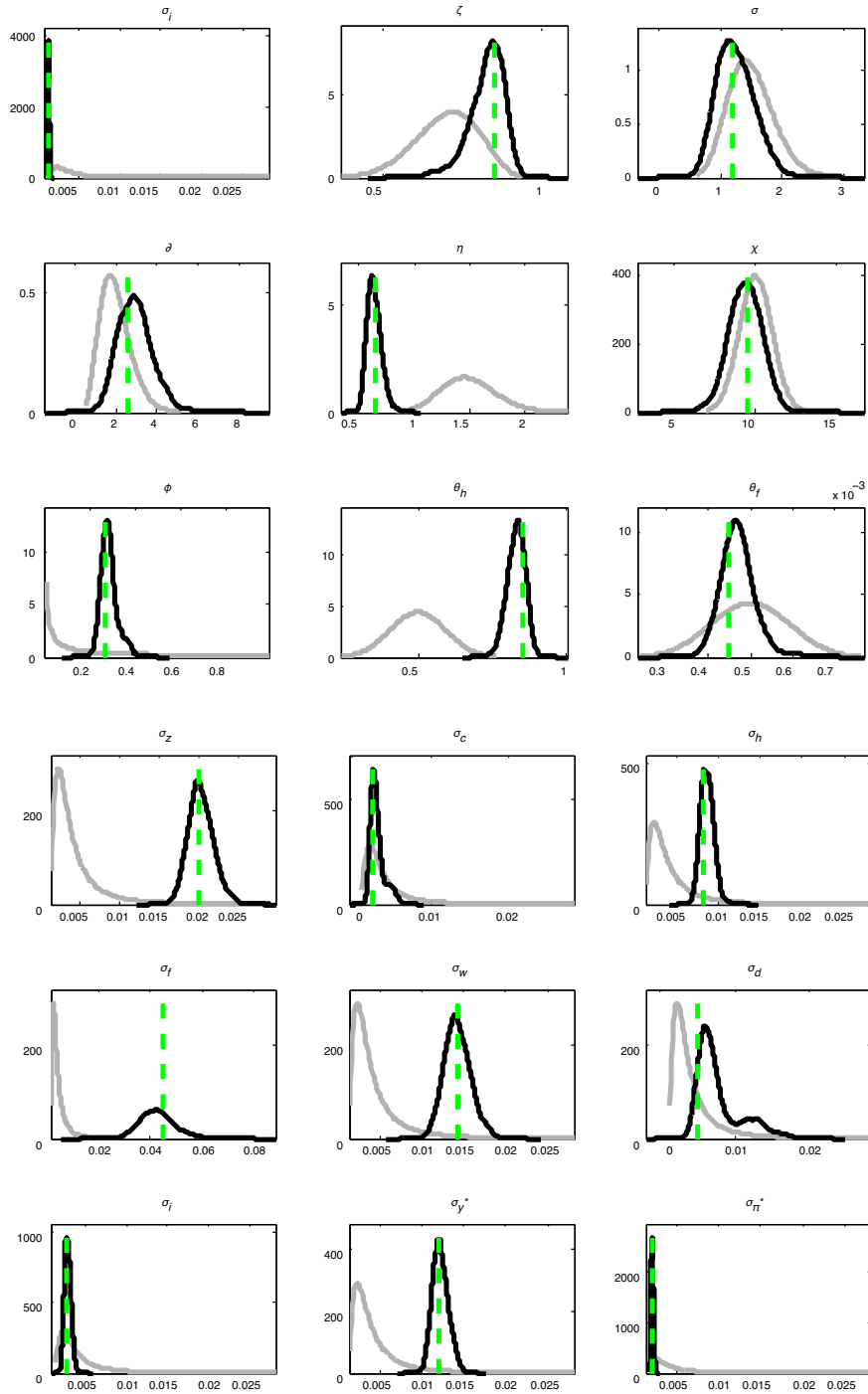


Figure 9.1: Posterior Parameter Estimates

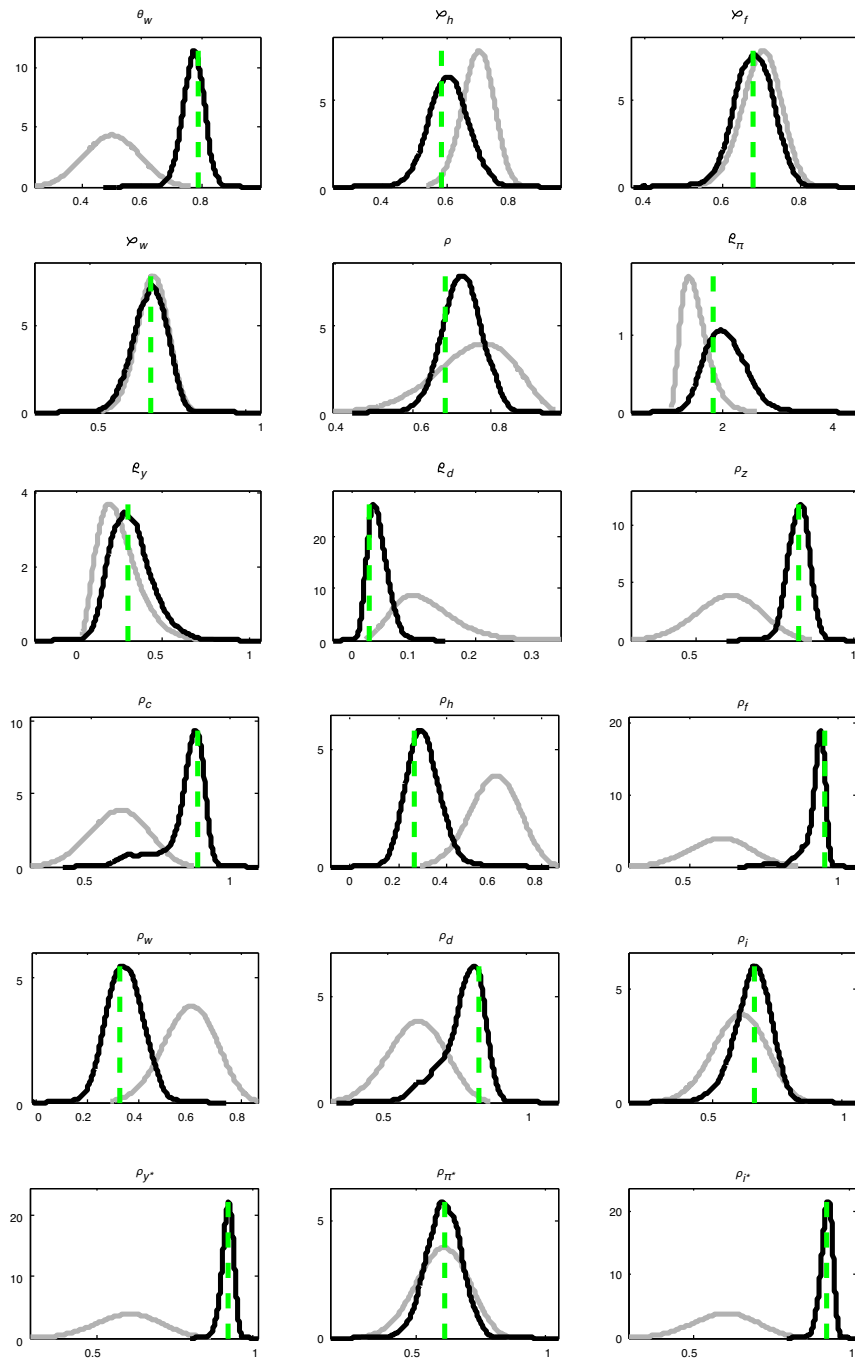


Figure 9.2: Posterior Parameter Estimates

The mean of the posterior distribution for  $\gamma$  is 2.47, corresponding to a labour supply elasticity of 0.4, which is within the range of values typically obtained in the literature. The elasticity of substitution between home and foreign goods,  $\eta$ , is estimated at 0.7, which is similar to the results of Justiniano and Preston (2010), despite the high prior mean. The mean estimates for the price-stickiness parameters,  $\theta_h$  and  $\theta_f$ , are 0.92 and 0.44, which suggests that there are a number of nominal rigidities in the domestic market. Similarly, the price-stickiness of wages in the domestic market is also relatively high, as  $\theta_w$  is 0.9. These parameters are all fairly well identified.<sup>25</sup> The estimates for domestic price-stickiness are somewhat higher than those found in other international studies such as Smets and Wouters (2003), but consistent with those that have been applied to South African data.

### 9.4.2 Impulse Responses

The Bayesian impulse response functions for model are contained in figures (9.4)-(9.7). These impulse responses indicate the response of key variables to a one standard-deviation innovation in each shock, for a maximum of forty quarters. When considering the effects of the respective shocks on the nominal and real exchange rates, the impulse response functions reflect the depreciating values of the respective measures of the external value of the currency.<sup>26</sup>

Following a positive innovation in the Taylor rule (i.e. a positive innovation on  $\varepsilon_i$ ), output, consumption, and domestic inflation all decline, while the currency strengthens at impact (where we have negative currency depreciation). The effect on the real exchange rate is more persistent, as the rate of inflation declines. The real interest rate is positively influenced by the increase in nominal interest rates and the decline in inflation. The shape of the response by output and consumption is hump-shaped, which is consistent with literature. In this model, the effect of an interest rate shock on output is both much greater and more persistent than the effect of this shock on inflation.

A positive innovation to productivity,  $\varepsilon_z$ , increases output in a hump-shaped manner. The shock also eases inflationary pressure, which leads to a reduction in the interest rate (since the Taylor rule coefficient on inflation is stronger than the coefficients on output and depreciation).

The impulse responses also move in the expected directions following an innovation to the consumption demand shock,  $\varepsilon_c$ . A positive demand shock increases consumption and output, which fuels inflation and causes the interest rate to rise. In addition, the currency depreciates along with a deterioration in the trade-balance-to-GDP ratio.

<sup>25</sup>This is in contrast with the results of Alpanda *et al.* (2010b), which uses the Rotemberg (1982) mechanism for price-stickiness. In addition, the model of Alpanda *et al.* (2010b) does not include any wage or productivity data.

<sup>26</sup>Where applicable, the impulse response functions reflect annual rates. They include a confidence interval of one standard deviation.



A positive innovation to the mark-up shock,  $\varepsilon_h$ , increases inflation and lowers output and consumption (through its affect on labour). As such, a positive mark-up shock acts as a cost-push shock, which shifts the Phillips curve and presents a less favourable tradeoff between inflation and output to the central bank. The currency depreciates after the impact period while interest rates rise after two quarters, since the Taylor rule places more emphasis on rising inflation.

A shock to the risk premium through a positive innovation to  $\varepsilon_d$  raises the domestic interest rate, causing a depreciation of the currency and higher inflation. Output increases due to the improved terms of trade, even though consumption initially declines. However, the persistence of the increase in output ensures the consumption eventually increases.

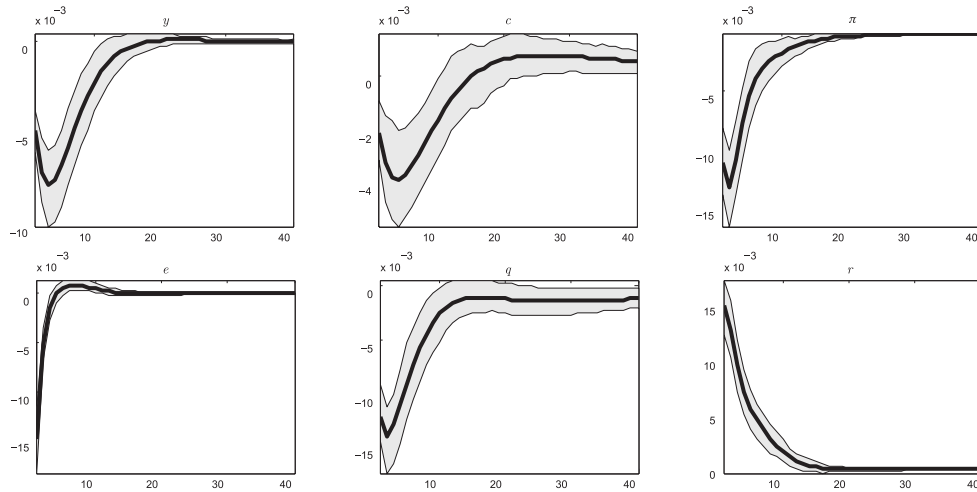


Figure 9.3: Bayesian Impulse Response Function for  $\epsilon^i$

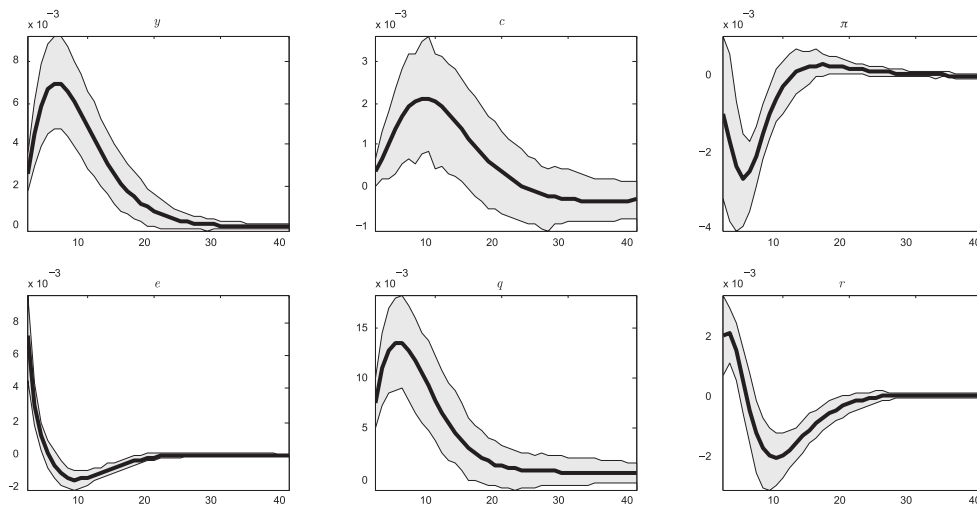


Figure 9.4: Bayesian Impulse Response Function for  $\epsilon^z$

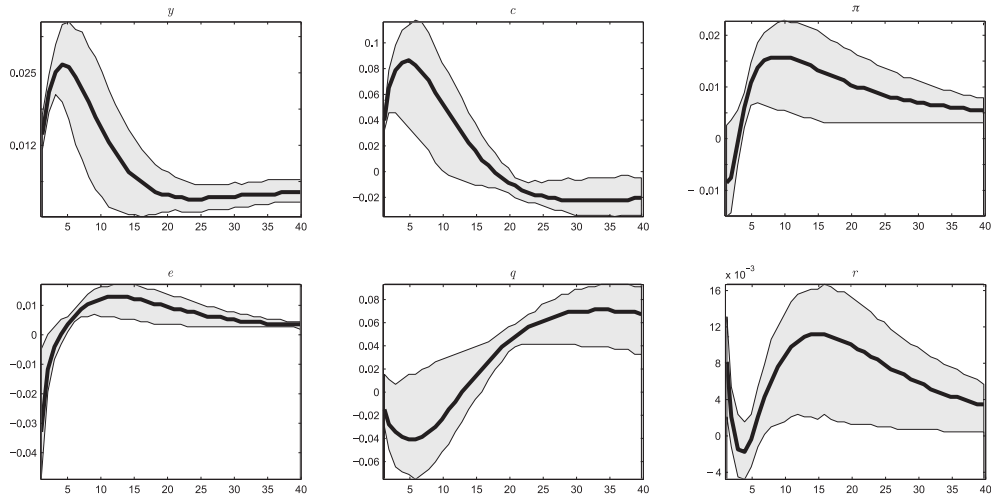


Figure 9.5: Bayesian Impulse Response Function for  $\epsilon^c$

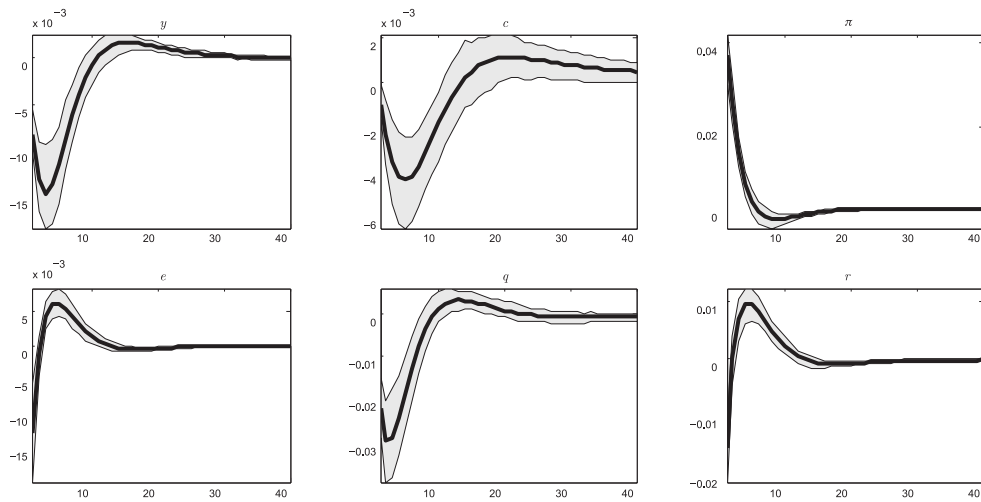
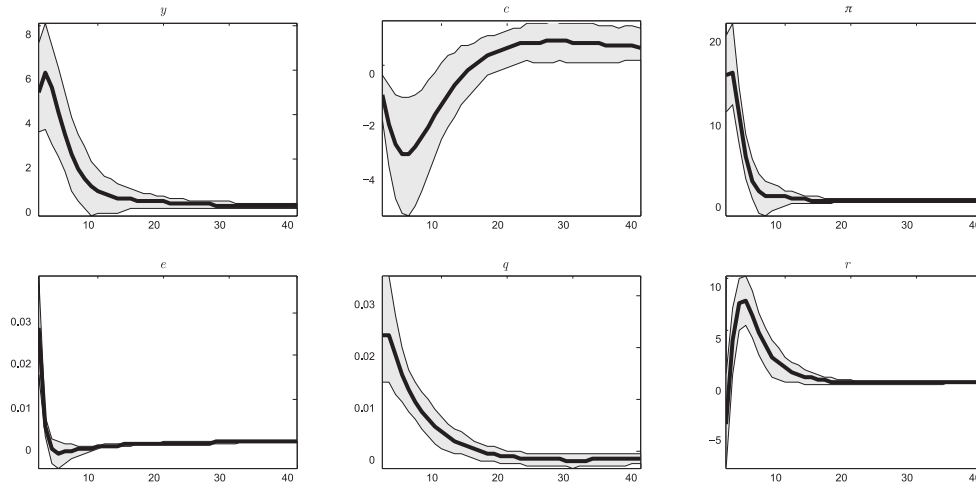


Figure 9.6: Bayesian Impulse Response Function for  $\epsilon^h$


 Figure 9.7: Bayesian Impulse Response Function for  $\epsilon^d$ 

## 9.5 Alternative Method for Closing-Off the Model

To close the model with an international risk-sharing condition, as in Galí and Monacelli (2005) and Steinbach *et al.* (2009a), replace the domestic IS curve (equation 9.2.20) and the balance-of-payments expression (equation 9.2.27) with the condition:<sup>27</sup>

$$\frac{\sigma}{1-\zeta}(c_t - \zeta c_{t-1}) = \frac{\sigma^*}{1-\zeta}(y_t^* - \zeta y_{t-1}^*) + q_t \quad (9.5.1)$$

and replace the risk variable in the interest parity condition (equation 9.2.26) with the difference of the demand shocks in the domestic and foreign economies,

$$E[q_{t+1}] - q_t = (r - E[\pi_{t+1}]) - (r_t^* - E_t[\pi_{t+1}^*]) + (\mu_t^d - \mu_t^{d*}) \quad (9.5.2)$$

The international risk-sharing condition (equation 9.5.1) implies that the marginal utility of consumption in South Africa is proportional to the product of marginal utility of consumption in the foreign sector and the real exchange rate (Galí and Monacelli, 2005). Since domestic and foreign output and inflation rates are relatively smooth in the data, this condition helps generate low volatility in the exchange rate. It also creates a singularity problem in the estimation when the depreciation rate is used as an observable along with

<sup>27</sup>Part one contains a more detailed discussion of alternative methods for closing-off the small open-economy model.

domestic and foreign output and inflation rates. This is because the model cannot match the observed series on interest rates, inflation rates and devaluations rates point-by-point while satisfying equation (9.5.1).

The UIP condition in the risk-sharing model (i.e. equation 9.5.2) ties the country risk-premium to the difference between demand shocks in the domestic and foreign economies. Under this condition, it is assumed that the interest rate on household assets is equal to the policy rate plus a demand/risk-premium shock in the two respective economies. The application of UIP and complete risk-sharing conditions then imply that the country risk-premium is captured by the difference in these shocks. This feature of relating risk-premia to demand shocks is due to Smets and Wouters (2007), which makes use of this condition in a closed production economy with capital inputs. The demand/risk-premia shocks in this model would then link the required return on household assets to the cost of capital faced by firms to capture the financial accelerator mechanism, as in Bernanke *et al.* (1999).

In contrast with Smets and Wouters (2007), the model of Steinbach *et al.* (2009a) does not make provision for capital or investment, and as such, their UIP condition equates the expected depreciation rate with the difference in the foreign and domestic interest rates faced by the respective households in each country (rather than the interest rate differentials in the policy rates set by the respective central banks.) This assumption is somewhat restrictive, because it links foreign and domestic demand shocks with expected depreciations, and thus reduces the role of demand shocks in the model.

### 9.5.1 Historical Decompositions

Using the dataset that is described in part two, we make use of historical decompositions to compare the results of the models that make use of different ways of closing-off the open-economy features of the model. Figures 9.8 through 9.11 reflect these decompositions for the output gap, consumer price inflation, nominal interest rate, and the exchange rate; for the sample period 1990-2012. The top panel in each of these figures is used to show the results for the risk-premium condition and the bottom panel is used for the risk-sharing condition.

To calculate the historical contribution of a given shock, we isolate its effects by setting all other shocks to zero, and simulate the key model variables using the (Kalman-smoothed) estimates of the given shock. This procedure is then repeated for all shocks, where the sum of the simulations for a given variable matches the models overall estimate of that variable (which would match the data if the variable is observed). For example, the top panel of figure 9.9 suggests that the decline in consumer price inflation, during the recent global financial crisis (2009-2011), may largely be attributed to changes in the risk-premium, demand, production and external shocks. In contrast, the use of the risk-sharing condition, which is displayed in the bottom panel, would

describe this decline in consumer price inflation, as the result of production, demand and monetary shocks.<sup>28</sup>

An immediate, apparent difference in the two panels of each of the figures is the importance of the risk-premium shock in the top panels. For example, output is dominated by demand shocks in the risk-sharing model, which is often offset by monetary policy shocks, whilst changes to the risk-premium also influence the value of this variable in the top panel of figure 9.8. Similar results are apparent for consumer price inflation and interest rates in the respective top and bottom panels of figures 9.9 and 9.10. For the nominal exchange rate, the top panel would suggest that changes to the risk-premium are largely responsible for the value of this variable, however, in the risk-sharing model the value of the variable is largely determined by demand shocks. Note also that the scale of the vertical axis suggests that the volatility in the unobserved exchange rate (in the bottom panel) is significantly lower than the volatility in the top panel, which is consistent with the observed values for this variable.

It is also interesting to note that when we compare the results for interest rates during the more recent emerging market currency crisis in 2001 and the more recent global financial crisis, the influence of the risk-premium would appear to have declined (in the top panel). This would suggest that the central bank does not react strongly to factors that influence the nominal exchange rate, which supports the findings of Woglom (2003), Ortiz and Sturzenegger (2007) and Alpanda *et al.* (2010a).

After considering all of the graphs together, we note that the influence of the external shock in the risk-sharing models (that are not able to include the exchange rate as an observed variable) is very small. Hence, this model would suggest that the economy is not influenced by external shocks, which is a characteristic of a closed-economy. In contrast with this result, the sum of the risk-premium and external shocks in the specification of the risk premium model (that includes the exchange rate as an observed variable) is relatively large. This would suggest that the economy is influenced by global economic events, as is the case for most small open-economies.

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<sup>28</sup>The early years of these plots are less reliable because there is no information about the cumulated shocks in the first observation. Instead, it is assumed that the share of each shock in explaining the first observation is the same as its share in the asymptotic variance decomposition.

## 9.6 Conclusion

This chapter considered the construction of a small open-economy New Keynesian DSGE model that may be used to describe the South African economy. The model makes use of ten observed variables which are used to identify an equivalent number of shocks. Almost all of the parameters are estimated with the aid of Bayesian techniques, where only two of the thirty-eight parameters are calibrated. Using priors that are consistent with those that are found in the existing literature, we find reasonable the posterior distributions, when compared to previous studies for the South African economy. In addition, the more complicated moments, as represented by the impulse response functions, would appear to provide a suitable description of the effects of suitably identified shocks to key variables in the model.

After describing the basic features of the model, we then make use of an alternative method for closing-off the open-economy features of a small open-economy DSGE model. As noted in Alpanda *et al.* (2010a), when we treat the exchange rate in each model as an unobserved variable, then the results are similar, which supports the findings of Schmitt-Grohe and Uribe (2003), which were obtained with the use of a calibrated Real Business Cycle model for the United States.

Furthermore, due to the difference in the degree of volatility between the nominal exchange rate and the respective demand shocks in the domestic and foreign economies, one is not able to include the exchange rate as an observed variable in the model that imposes risk-sharing conditions. However, after making use of the risk-sharing condition (which no longer associates changes in the exchange rate with differences in demand shocks), we are able to include the relatively volatile nominal exchange rate as an observed variable in the model. The nominal exchange rate is then used to partially identify the shocks in the risk premium, which would appear to influence most of the observed variables in the model.

Hence, after considering the results of the historical decompositions, we note that the information that is contained in the exchange rate may be used to infer that external shocks and changes to the risk-premium influence the variables that affect the South African business cycle. This would be consistent what one would expect for a small open-economy, such as South Africa. In addition, the findings that are reported in this chapter also support those of Alpanda *et al.* (2010a), despite the fact that the dataset and model structure that were utilised in this chapter differ to those that were used in this paper.

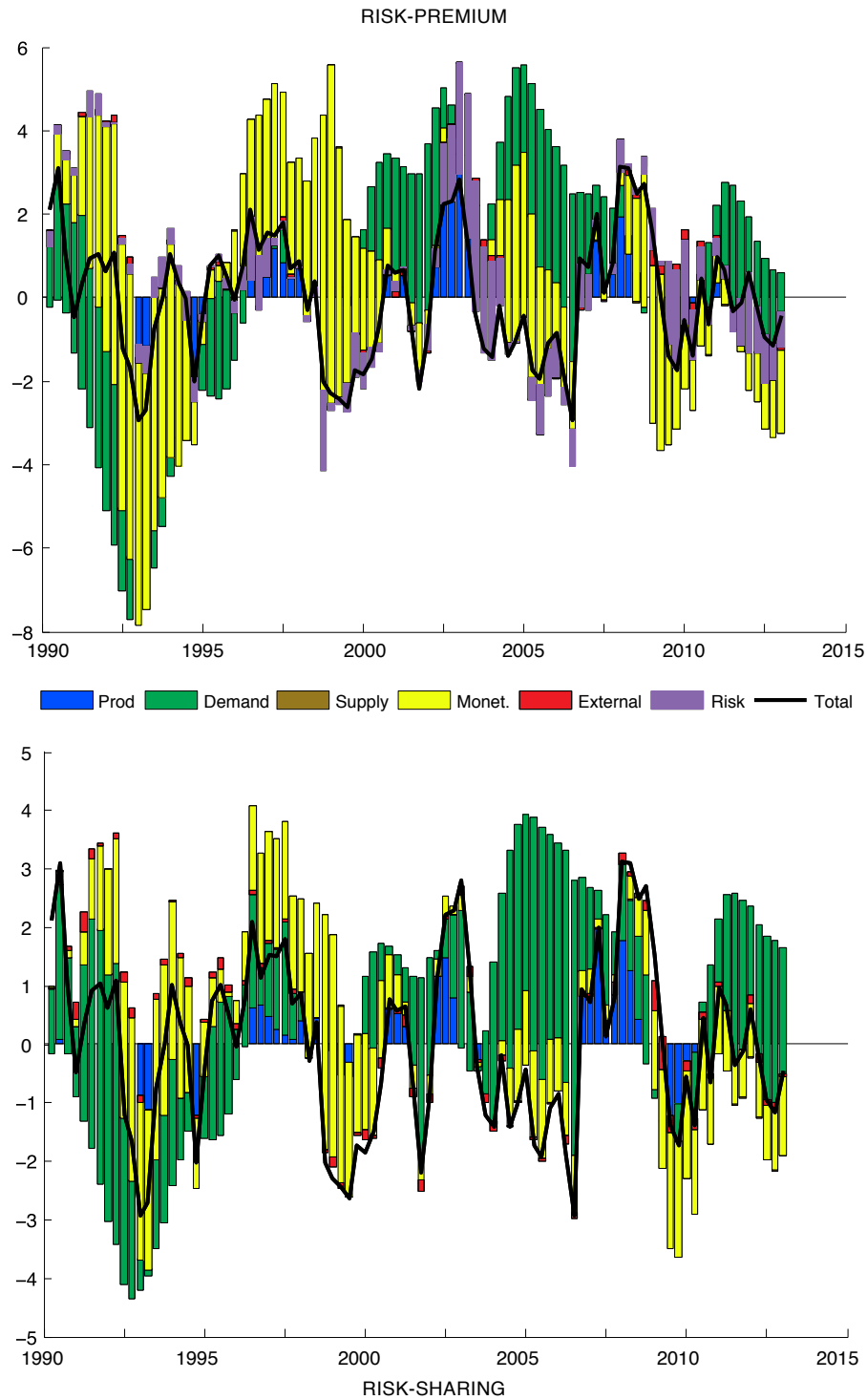


Figure 9.8: Historical Decomposition - Output Gap



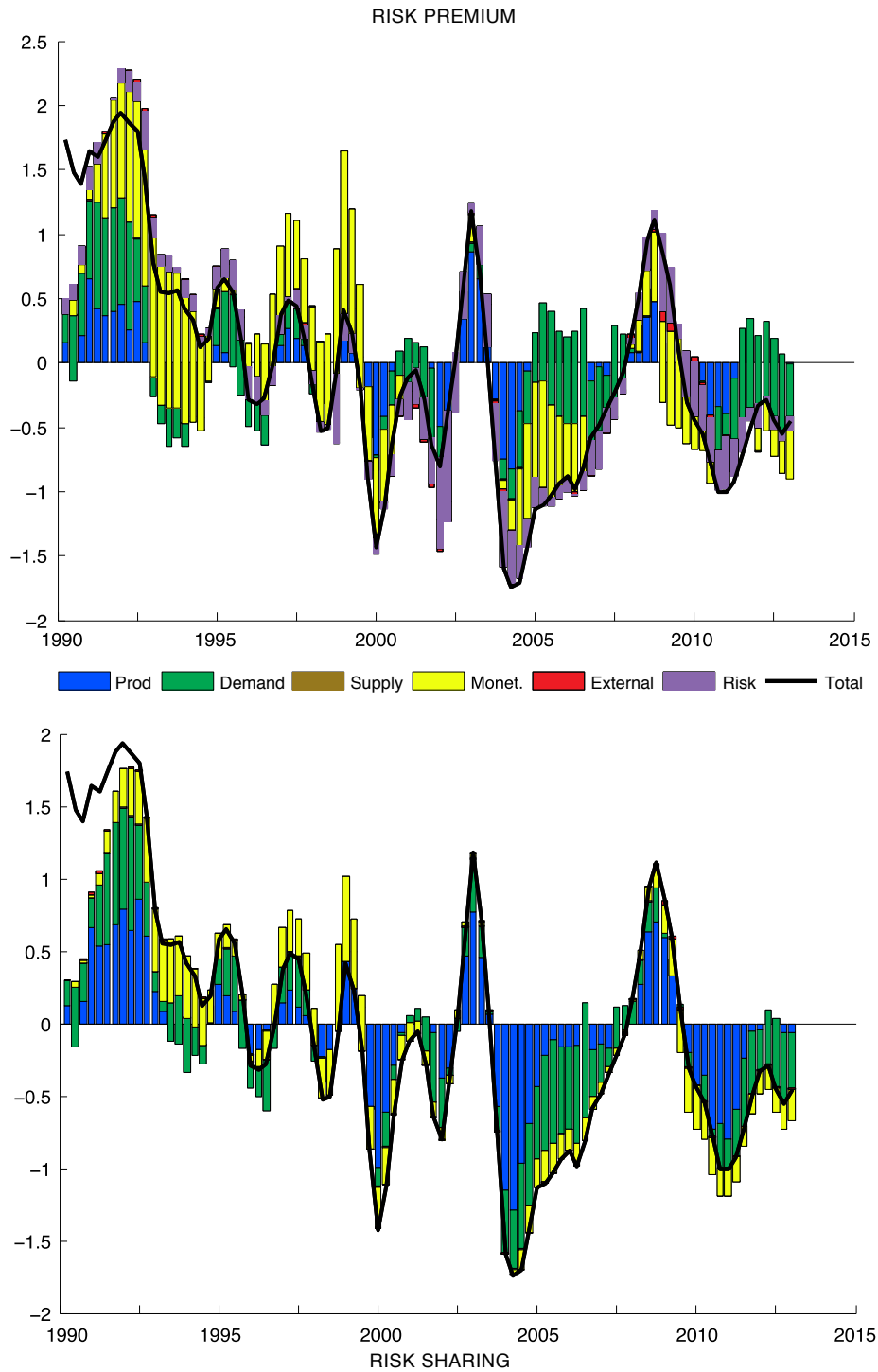


Figure 9.9: Historical Decomposition - Consumer Inflation

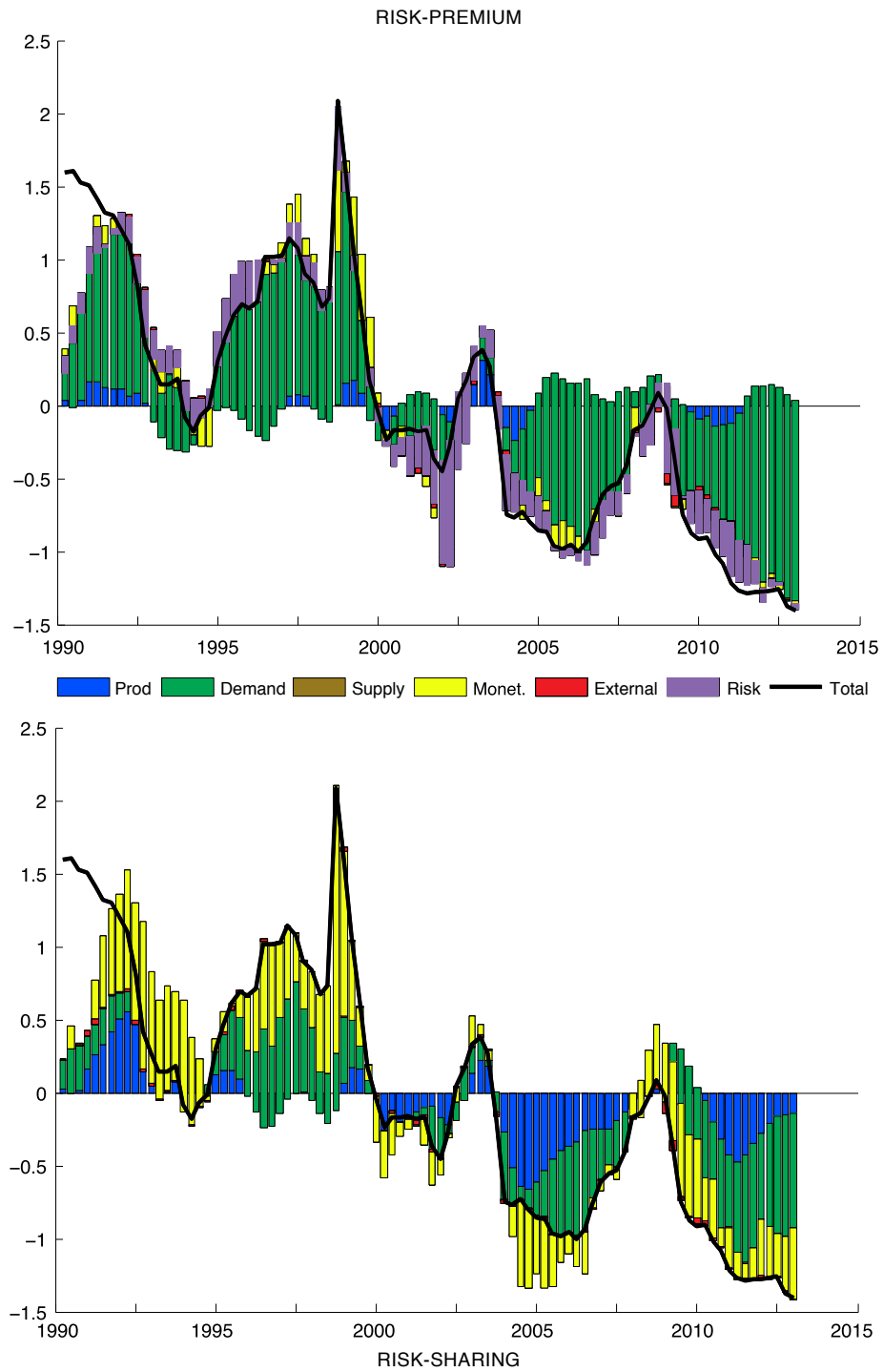
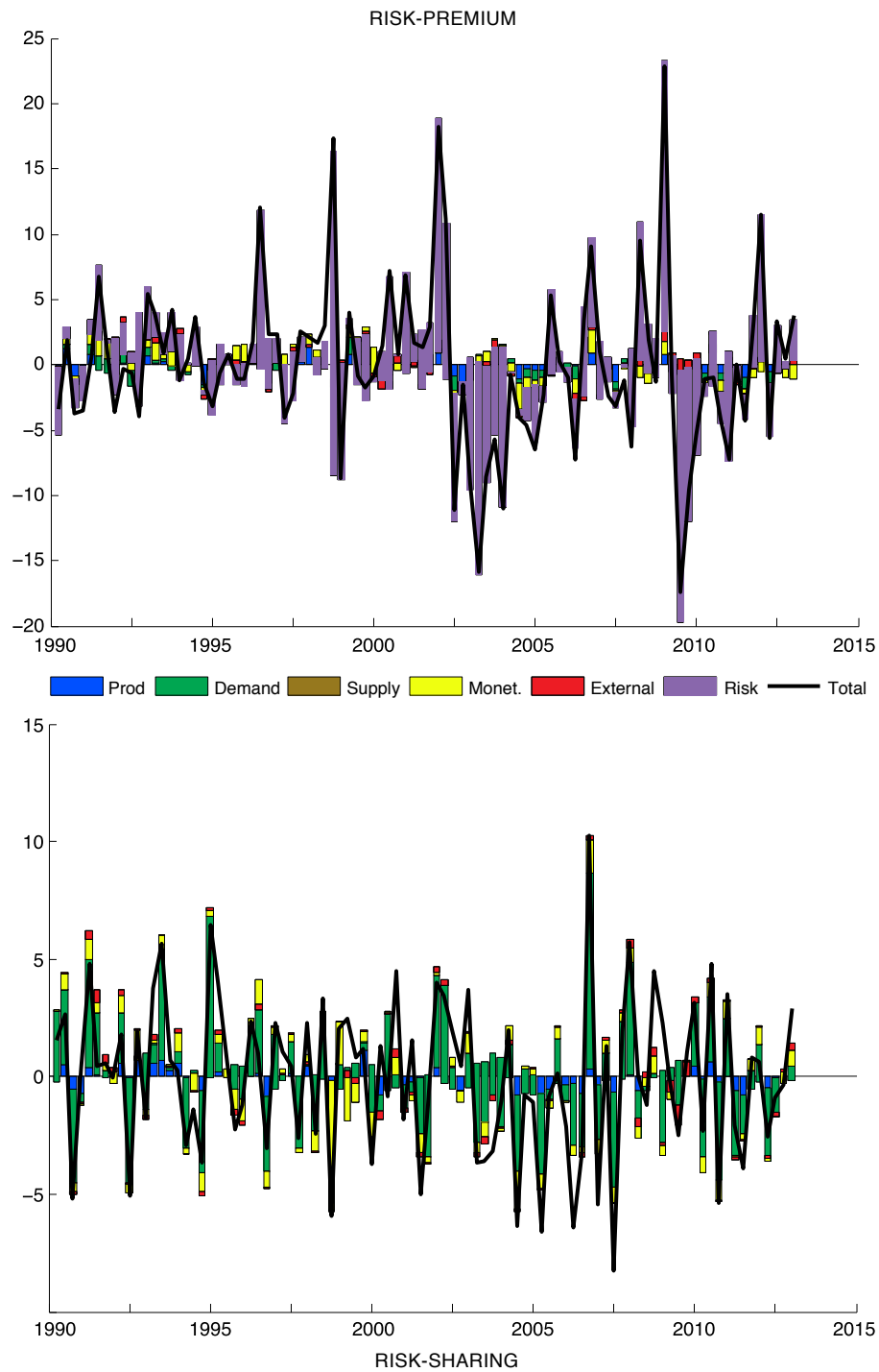


Figure 9.10: Historical Decomposition - Interest Rates



**Figure 9.11:** Historical Decomposition - Nominal Exchange Rate

## Chapter 10

# Optimal Monetary Policy in South Africa

## 10.1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models allow for researchers to ask ‘what if?’ questions, as it is assumed that the parameters in the model are based on fundamental aspects of decision making that will not change after a policy intervention.<sup>1</sup> In this chapter we investigate the effects of a change to the monetary policy rule in terms of its impact on economic volatility. We assume that monetary policy is conducted with a forward-looking generalised Taylor rule that conditions the central bank’s response to past interest rates, expected future inflation, as well as the current output gap and exchange rate.<sup>2</sup>

Following the framework of Alpanda *et al.* (2010b), we initially consider the effects of varying each of the coefficients in the reaction function individually, to identify any constraints that may prevent us from obtaining reasonable estimates for the *optimal* coefficients when all of the coefficients are allowed to vary simultaneously. After identifying these constraints we are then able to derive optimal estimates for each of the coefficients for a number of different loss functions. The results suggest that the central bank should possibly consider a stronger reaction to changes inflation and a smaller reaction to changes in the output gap, which may allow for greater interest rate smoothing.

In the final part of this analysis we construct an efficiency frontier for the generalised Taylor Rule as in Clarida *et al.* (1999), to consider the trade-off between output and inflation variability. These results suggest that when on the efficiency frontier, a relatively small change in inflation volatility may result in a relatively larger change in output volatility. However, it is also noted that the points on the efficiency frontier are only obtained with unreasonably large coefficients in the reaction function.

In what follows, we consider the general framework for an optimal policy investigation in section 10.2, before we derive partially optimal values for the central bank’s reaction function in 10.3. The result from this investigation are considered before we conduct a fully optimal investigation in 10.4. Section 10.5 then considers the tradeoff between output and inflation variability with the aid of efficiency frontiers before section 10.6 concludes.

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<sup>1</sup>The critique of Lucas (1976) noted that one would not be able to consider the effects of a change to policy in a model where the parameters are not structural (or time invariant), as in the case of most reduced-form models.

<sup>2</sup>Optimal policy investigations within open-economy DSGE models are considered in Justiniano and Preston (2010) for Australia, Canada, New Zealand; Alpanda *et al.* (2010b) for South Africa; and Smets and Wouters (2002) for the Euro area.

## 10.2 Optimal Policy in a Small Open-Economy

The structure of the model that we have used for the optimal policy investigation follows chapter 9, which assumes that monetary policy is conducted with the aid of a generalised Taylor rule that takes the form:

$$i_t = \rho i_{t-1} + (1 - \rho) [\varrho_\pi E_t (\pi_{t+1}^c) + \varrho_y \tilde{y}_t + \varrho_d d_t] + \varepsilon_{i,t}$$

When estimating the parameters for this reaction function in chapter 9, we found that  $\rho = 0.7$ ,  $\varrho_\pi = 1.5$ ,  $\varrho_y = 0.4$ , and  $\varrho_d = 0.04$ . To conduct an optimal policy investigations we may then construct a loss function that is dependent on the volatility of critical macroeconomic variables. Such a loss function may take on a number of different functional forms and this section we consider the results from four different functions.

Such policy investigations have been used to investigate whether a central should react to changes in the exchange rate when it sets its interest rate policy. Exchange rate movements directly affect the foreign component of consumer price inflation, and indirectly affect the domestic component of consumer price inflation through their effect on the marginal cost of domestic producers. Hence, higher currency depreciation warrants a contractionary response by the central bank through an increase in the interest rate. In the presence of volatile exchange rates, however, this would cause frequent changes in interest rates and increase the variability of output. The optimal response of a central bank is therefore ambiguous and depends on the quantitative importance of these effects (Monacelli (2003) and Justiniano and Preston (2010)). For the specification of the Taylor rule that has been used in this model, where the central bank conditions on future expected inflation rates and the current rate of currency depreciation (which may impact on inflation in the near future - i.e. over a period that is longer than one quarter), there may be an informational gain on the part of policy-makers to warrant conditioning on exchange rate movements.

In Alpanda *et al.* (2010b) it is suggested that it would not be optimal for South Africa to condition on exchange rate movements, which supports the findings of Justiniano and Preston (2010) who make use of a similar model for other small open-economies. The analysis in this chapter differs slightly to that of Alpanda *et al.* (2010b) in that it focuses on the tradeoff between conditioning on changes in output and inflation.

### 10.2.1 The Loss Function of the Central Bank

The loss function for policy-maker's follows the specification in Justiniano and Preston (2010), which depends on the variation in consumer price inflation, the output gap, and the nominal interest rate. The inclusion of these variables ensures that the loss function is consistent with the specification of decision

making rule of the central bank in equation (10.2.1).<sup>3</sup>

$$L_t(\varrho_\pi, \varrho_y, \rho_i) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} [(\pi_t^c)^2 + \lambda_y \tilde{y}_t^2 + \lambda_i (i_t)^2] \quad (10.2.1)$$

where  $\lambda_y, \lambda_i \geq 0$  are the weights on variation in output and interest rates relative to the variation in inflation.

The loss function (10.2.1) can be evaluated for each set of policy parameters in the Taylor rule,  $\varrho_\pi$ ,  $\varrho_y$ , and  $\rho_i$ . Since all the other parameters are structural, we assume that they cannot be affected by monetary policy makers. We also ignore parameter uncertainty, and keep the values of the structural parameters at the estimated means of their posterior distribution.

Considering the limiting case with  $\beta = 1$ , the objective function of the policy makers is analogous to minimising a weighted sum of the unconditional variances:

$$L(\varrho_\pi, \varrho_y, \rho_i) = \text{var}(\pi_t^c) + \lambda_y \text{var}(\tilde{y}_t) + \lambda_i \text{var}(i_t). \quad (10.2.2)$$

For given values for the weight parameters,  $\lambda_y$  and  $\lambda_i$ , we calculate the set of policy parameters that minimises the above loss function. We restrict attention to policy parameters which are consistent with long-run stability; hence  $\varrho_\pi > 1$  and  $0 \leq \rho < 1$ . Since the choice of weights is somewhat arbitrary, we repeat this procedure for different values of  $\lambda_y$  and  $\lambda_i$ .

### 10.3 Partially Optimal Taylor Rule Coefficients

As a preliminary exercise, we first fix three of the four policy parameters to the estimated values of their posterior mean, and then compute the loss function for all possible values of the remaining policy parameter. The results are given in figure (10.1), where we care equally about the variation in inflation, output and interest rates, by setting  $\lambda_y = 1$  and  $\lambda_i = 1$ . The sum of standard deviation of these variables is depicted on the vertical axis whilst the coefficient values are on the horizontal axis.

The partially optimal policy (keeping two of the three coefficients equal to their estimated values) prefers higher long-run response coefficients for inflation, where we can achieve a significant reduction in volatility (note the relative scale of the vertical axis). When, after the coefficient exceeds a value of 3.4, it is not possible to obtain a further reduction in economic volatility. For output, it would appear as if the model may prefer smaller values although the increase in volatility for higher values is only marginal greater. Similarly, for interest rates, where slightly higher rates of interest rate smoothing may

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<sup>3</sup>Note that from a utility maximising perspective, minimising the variation in output may not be optimal if most of the variation in detrended output is due to changes in productivity (i.e. changes in the natural rate of output).

reduce volatility by a small amount. These partially optimal coefficients are 3.4, 0.2 and 0.8 on inflation, output, and interest rate smoothing, respectively.

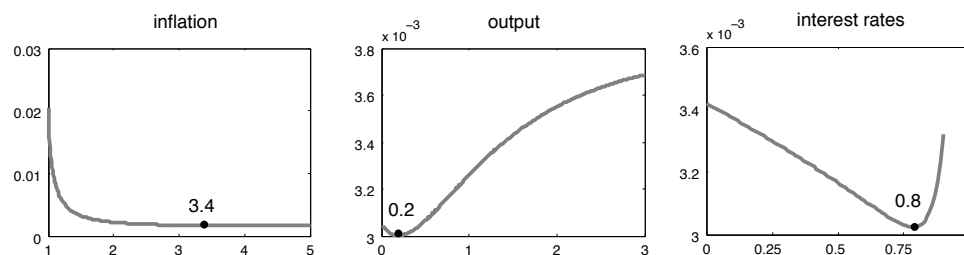


Figure 10.1: Loss functions for individual policy parameters

## 10.4 Optimising over the loss function

During the subsequent exercise, we allow the  $\varrho_\pi$ ,  $\varrho_y$ , and  $\rho_i$  policy parameters to vary simultaneously and report on the results in table 10.1. For reference, the estimated values from the model are given in column (1), for which we provide the value of the loss function, as well as the standard deviations.<sup>4</sup>

In column (2) we make use of the loss function, where  $\lambda_y = 0.5$  and  $\lambda_i = 1$ . In this case the optimal long-run response coefficients for inflation, output and interest rates are found to be 2.13, 0.35, and 0.89, respectively. Note that the coefficients on inflation and interest rates are higher than the estimated Taylor rule coefficients, and the coefficient on the output gap is somewhat reduced. This results in lower variability in the rate of inflation, interest rate and exchange rate; whilst the variability in the output gap has increased.

Columns (3)-(4) illustrate how the optimal policy varies for different policy weights on the output variance, where  $\lambda_y = 1$  and  $\lambda_i = 0$ . Column (5) illustrates the sensitivity of the results, where the relative weight on interest rate smoothing,  $\lambda_i = 0.5$  and  $\lambda_y = 0.5$ . In each of these case, one is able to obtain a lower loss function by increasing the reaction to changes in inflation, whilst maintaining a higher level of interest rate smoothing, which comes at the expense of a decrease in the reaction to changes in the output gap.

<sup>4</sup>When we allow the policy parameter that is associated with the exchange rate to vary simultaneously, we note that there is very little difference in the loss function (i.e. economic volatility) for different values of this coefficient. Hence, the forward-looking inflationary policy reaction function may account for most of the pass-on inflationary effects that may result from any currency depreciation. Furthermore, when making any changes to the loss function the optimal coefficients will vary to a degree that is unreasonable, which would suggest that the optimal coefficients for the central bank's reaction to exchange rate may not be suitably identified. Therefore, we follow Alpanda *et al.* (2010b) and set the coefficient to its estimated value, which is very small and consistent with the policy recommendation of Justiniano and Preston (2010).



		(1)	(2)	(3)	(4)	(5)
		Est.	Optimal Rule with weights $(\lambda_y, \lambda_i)$			
Coefficients			(0.5, 1)	(1, 1)	(0, 1)	(0.5, 0.5)
Inflation	$\varrho_\pi$	1.55	2.13	2.65	2.02	2.5
Output	$\varrho_y$	0.45	0.35	0.35	0.32	0.32
Interest rate	$\rho$	0.71	0.89	0.86	0.93	0.87
Exchange rate	$\varrho_d$	0.04	-	-	-	-
Loss fnc. ( $\times 10^{-4}$ )		3.05	1.81	1.93	1.31	1.19
<b>std. dev. (%)</b>						
Inflation	$\pi_t$	3.04	2.16	1.8	2.54	1.87
Output	$y_t$	2.23	3.27	2.96	4.21	3.04
Interest rate	$i_t$	4.03	2.84	2.71	2.58	2.73
Exchange rate	$d_t$	8.64	7.93	7.49	8.93	7.57

Table 10.1: Optimal Taylor Rule Coefficients

When we consider these changes in variability in percentage terms, we note the the difference that are affected by changes in the central bank's reaction function are relatively large. For example, the result of increasing the response to changes in inflationary bring about a reduction in the variability of inflation by between 16% and 41%, whilst output variability rises to between 32% and 89%. The increase in the interest rate smoothing, would also reduce interest rate volatility, which decreases by between 29% and 36%. All of these changes have a negligible effect on the volatility of the exchange rate, which decreases by a maximum of 13%.

We would expect to find that the model's optimal Taylor rule coefficient values are larger than the estimated coefficients, since the SARB would be more cautious in its monetary policy in the presence of data, model and parameter uncertainty (which we do not account for in the model). We indeed find larger optimal response coefficients for inflation and output, but surprisingly the optimal coefficient for currency depreciation is much smaller than the estimated value, and close to zero.

In support of the findings in Justiniano and Preston (2010) we find a relatively large tradeoff between output and inflation volatility, where policy can has a significant influence on variability of these variables. In addition, with the addition of more recent data we find that the optimal coefficients for interest rate smoothing should be larger than what has been estimated, which would obviously reduce the standard deviation in the interest rate. This would imply that over the recent global financial crisis, the central bank may not have been as systematic as what it was prior to the crisis (when we compare these findings to Alpanda *et al.* (2010b)).

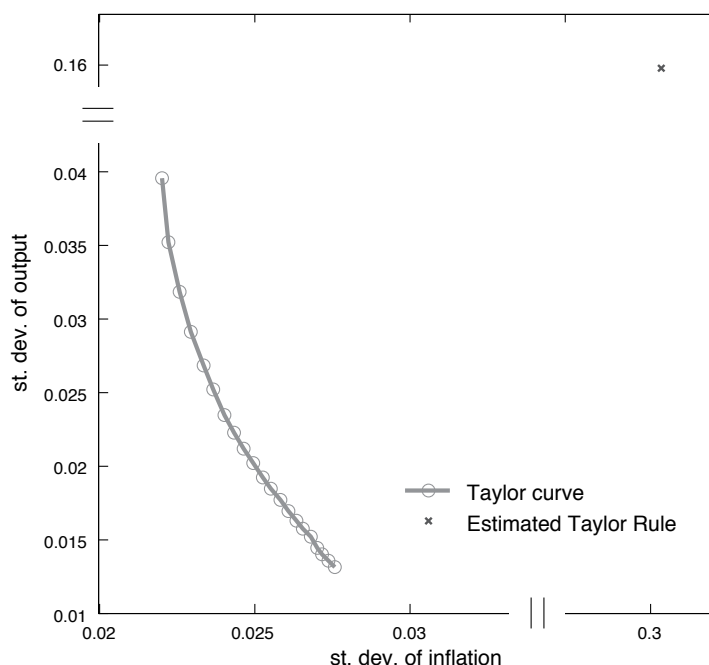


Figure 10.2: Efficiency frontier

## 10.5 Policy Analysis with Efficiency Frontiers

To further consider the tradeoff between output and inflation variability we make use of efficiency frontiers for the central banks reaction function. Similar tools were employed in Taylor (1979), Clarida *et al.* (1999) and Cecchetti *et al.* (2001); to determine whether an economy is at a point that is close to the frontier for stable inflation and output growth.

When applying this methodology to South African data, Du Plessis and Smit (2003) and Du Plessis (2003) found that the volatility of output and inflation between 1994 and 2002, was much lower than the volatility in the preceding period (between 1986 and 1993). After decomposing this improvement into a part that may be attributed to potentially better policy-making and a part that may be attributed to a more stable economic environment, they found that 55% of the decline in output and inflation volatility may be attributed to improved policy-making.

The objective of the investigation in this chapter, is to ascertain whether the central bank has operated close to the efficiency frontier, over the period 1990Q1 to 2012Q4, to ascertain whether or not there is scope for potential improvement. This investigation has been conducted within the context of a rational expectations model, as per Taylor's (1979) original suggestion, and is similar to that which was employed in Alpanda *et al.* (2010a).

Once again, it is assumed that the reaction function of the central bank follows the generalised Taylor rule that is provided above. In addition, we

assume that the objective of the central bank is to minimise a loss function composed of a weighted average of the variance of output, inflation and nominal interest rate, which is consistent with what is provided above:

$$L_t(\varrho_\pi, \varrho_y) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} [(\pi_t^c)^2 + \lambda_y (\tilde{y}_t)^2 + \lambda_i (i_t)^2] \quad (10.5.1)$$

In this exposition, we once again assume that the central bank places equal weights on interest rate and inflation volatilities. For a given  $\lambda_y$ , we then calculate the long-run response coefficients in the Taylor rule,  $\varrho_\pi$  and  $\varrho_y$ , which minimises the central bank's loss function, keeping all the other parameter values the same.<sup>5</sup> We repeat this procedure for different values of  $\lambda_y$  between 0 and 2, and calculate the optimal Taylor rule coefficients and the corresponding standard deviation of inflation and output implied by these coefficients. This procedure derives the efficiency frontier for the central bank as in (Clarida *et al.*, 1999), which is plotted in figure 10.2, along with the volatility implications of the Taylor rule parameters that were estimated in chapter 9.

The efficiency frontier points to the short run trade-off faced by the central bank with respect to the variability of output and inflation. This is reminiscent of the results in Clarida *et al.* (1999). In their model, optimal policy does not include a smoothing motive, and the output variable in the loss function of the central bank is the output gap (i.e. per cent deviations from a time-varying model-implied natural rate of output) rather than the de-trended level of output we use here (i.e. per cent deviation from the stochastic trend that is not necessarily model-consistent). This set-up dictates that demand shocks be fully offset and productivity shocks be fully accommodated by the central bank; this leaves the cost-push shocks that present a trade-off between inflation and output volatility at the efficiency frontier. Since we include an interest rate smoothing motive, our results do not imply a full and immediate offset for demand shocks, but rather a gradual one. Also, since we do not use the model implied output gap in our loss function, productivity shocks are treated analogous to cost-push shocks by the central bank in our calculations. Despite these differences, our results are fairly similar in spirit to those of Clarida *et al.* (1999).

The results point to a substantial reduction in volatility if the central bank were able to move towards the efficiency frontier, from the estimated Taylor rule. In the previous sections it was noted that this could possibly be achieved with the aid of a stronger reaction by the central bank to changes in the price level. Furthermore, when one is on the efficiency frontier, we also note that a

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<sup>5</sup>We set the smoothing parameter,  $\rho$ , to its estimated value, and do not search for the optimal coefficient on this Taylor rule parameter. This would ensure that our results are comparable to Justiniano and Preston (2010) and Alpanda *et al.* (2010a), who find that the smoothing parameter is driven to unreasonably high numbers in the context of the above loss function.

small reduction in inflation volatility would result in a slightly larger increase in output volatility. Note however that these results are somewhat misleading, since the Taylor rule coefficients associated with the efficiency frontier are both extremely large and unrealistic. Similar results were obtained for the efficiency frontier for the model of Steinbach *et al.* (2009a) and Alpanda *et al.* (2010a).

## 10.6 Conclusion

In this paper, we make use of the small open-economy DSGE model that was described in chapter 9, to analyse the conduct of optimal monetary policy in South Africa. The optimal coefficients for the policy rule are obtained by minimising a loss function that includes the variance of inflation, output, and the interest rate. In the initial analysis we find that the optimal policy places a heavier weight on the central bank's reaction to inflation and a smaller possible value to the reaction to changes in output, when compared to the estimated Taylor rule for South Africa.

The results support the finding of Alpanda *et al.* (2010b) in that it suggests that the central bank could afford to react more vigilantly to changes in inflation.<sup>6</sup> However, it should be noted that this result should be interpreted with caution, as we do not allow for parameter uncertainty, which would imply that the optimal coefficients that we have reported are usually much larger than they would have been, if parameter uncertainty were included.

In the final part of the analysis we make use of efficiency frontiers for the policy reaction function to investigate the tradeoff between inflation and output variability. These results suggest that reductions in inflation and output volatility could be achieved by moving from the estimated Taylor rule to the model-implied efficiency frontier. However, it was noted that this would require the use of much larger response coefficients in the Taylor rule.

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<sup>6</sup>This is despite the fact that the use of the model structure and dataset differ to those that were utilised in this study.

## Part IV

# Advances in Dynamic Stochastic General Equilibrium models

## Chapter 11

# Nonlinear DSGE Models and their Forecasting Potential

## 11.1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are frequently used by central banks and other policy-making institutions for forecasting purposes.<sup>1</sup> Smets and Wouters (2007) suggest that when these models incorporate sufficient nominal and real rigidities, as well as a relatively large number of shocks, they are able to outperform other multivariate models when applied developed-world macroeconomic data. Most of the DSGE forecasting models that are used by policy-making institutions make use of linearised state-space system to characterise the equilibrium dynamics of business cycle fluctuations; however, as noted in Del Negro and Schorfheide (2011), such a linear approximation may be unreliable when applied to an economy that is affected by large shocks, as is the case for most emerging market economies.<sup>2</sup>

Against this backdrop, the objective of this chapter is to analyse whether the nonlinearities would improve upon the out-of-sample fit of a New Keynesian DSGE model that may be applied to macroeconomic data from an emerging market economy. In addition, we also compare the forecasting performance of the nonlinear DSGE model with a large variety of BVAR models. In this study, the data for the emerging market economy pertains to South Africa, which was recently included in the BRICS nations of fast-growing newly industrialised or emerging economies. When compared with Brazil, Russia, India and China, it is worth noting that South Africa is significantly smaller, accounting for approximately 2% of combined BRICS economic output, which may imply that it would be susceptible to large shocks, where nonlinearities could be of greater importance.<sup>3</sup>

The forecasting performance of linear DSGE models that have been applied to South African data has been somewhat mixed. Initial studies by Liu and

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<sup>1</sup>See, Tovar (2009) for an overview of the use of these models in central banks. Edge *et al.* (2010) provides details of the DSGE model that has been used at the Federal Reserve Bank, while Ratto *et al.* (2008) describe the model that has been used at the European Central Bank. An early exposition of the multi-country DSGE model that has been used at the International Monetary Fund (IMF) is provided in Carabenciov *et al.* (2008).

<sup>2</sup>In addition, Fernández-Villaverde and Rubio-Ramirez (2005) and Fernández-Villaverde (2010) note that linearisation would result in an approximation error that influences the likelihood function and the eventual parameter estimates. When combined with the assumption of Gaussian errors, it would also eliminate the possibility of investigating asymmetries, threshold effects and time-varying volatility (as in Fernández-Villaverde *et al.* (2011) and Binsbergen *et al.* (2012)).

<sup>3</sup>South Africa is classified as an emerging market economy by the International Monetary Fund 2013, as well as by the FTSE, S&P, Dow Jones, and MSCI. The data for nominal GDP in USD terms for each of the BRICS nations was obtained from the International Monetary Fund (2013).

Gupta (2007), Liu *et al.* (2009), Liu *et al.* (2010), and Gupta and Kabundi (2011) suggest that the forecasting potential of Bayesian vector autoregressive (BVAR) models may be superior to that of small closed-economy DSGE models, for key macroeconomic variables. However, studies by Steinbach *et al.* (2009a), Gupta and Kabundi (2010), Alpanda *et al.* (2011) and Gupta and Steinbach (2013) indicate that when one allows for open-economy features and a relatively large number of rigidities, DSGE models would appear to compete favourably with BVAR models.

The structure of the DSGE model in this chapter follows the specification of Pichler (2008), who found that there was little difference in the forecasting performance of linear and nonlinear models, when applied to data for the United States economy. Therefore, the use of this specification would allow for us to compare the results from an emerging market with those from a developed-world economy. In addition, the results of previous studies that were applied to South African data suggest that the forecasting performance of small closed-economy DSGE models is usually inferior to that of other forecasting models (such as those that employ a VAR structure). Hence, if we find that the forecasting performance of this nonlinear model is superior to that of other models, it would suggest that there could be important nonlinear features in the underlying South African data-generating process for this emerging market economy.

As noted above, the majority of DSGE models that are used for forecasting purposes would usually make use of a first-order linear approximation of the theoretical model that incorporate several nonlinear features and a number of forward-looking expressions.<sup>4</sup> After applying such a log-linear approximation, one is able derive the model solution, before making use of the Kalman filter to approximate the likelihood function of the model (which may include several unobserved variables).<sup>5</sup> While this procedure has been successfully applied to many problems, as noted above, a first-order linear approximation may exclude important nonlinearities and the possibility of large deviations from the steady-state of the respective variables.

The use of DSGE models that are estimated with higher-order approximations and nonlinear filters is not as widespread when used for forecasting purposes.<sup>6</sup> An (2008) and Del Negro and Schorfheide (2011) suggest that one reason for this may be the computational complexities that are involved in the

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<sup>4</sup>The proliferation of forecasting models that make use of first-order approximations has been facilitated by the development of the excellent software platform, *Dynare*. Further details of which can be found in Adjemian *et al.* (2011).

<sup>5</sup>Solution methods for linear rational expectations problems are provided by, Blanchard and Kahn (1980), Klein (2000), Sims (2001), and Uhlig (1999), among others.

<sup>6</sup>Early advocates of this procedure for parameter estimation include, Fernández-Villaverde and Rubio-Ramírez (2005), An and Schorfheide (2007), An (2008), Primiceri and Justiniano (2008), and DeJong and Dave (2007). These studies would usually employ the second-order solution method proposed by Schmitt-Grohé and Uribe (2004), before utilising a particle filter to evaluate the likelihood function.



estimation of these models, when both the state and measurement equations are nonlinear.<sup>7</sup> In addition, Andreasen *et al.* (2014), Den Haan and De Wind (2012) and Kim *et al.* (2008) have noted that the use of higher-order approximations may result in an unstable model solution, since it would consider additional points around which the approximate solution may be unstable.<sup>8</sup> These reasons also motivate for the use of a relatively simple closed-economy modelling framework when evaluating the relative forecasting potential of these models.

To the best of our knowledge, the current literature does not include an example of a nonlinear DSGE model that is applied to the macroeconomic data of an emerging market economy.<sup>9</sup> Such an investigation would be of interest, as one would expect that this data would incorporate larger deviations from the steady-state (as well as potentially more complex nonlinear relationships). Hence, it may be the case that when applied to an emerging market economy, the nonlinear DSGE model may provide a superior out-of-sample fit, when compared with its linear counterpart. In addition, it also may have the potential to outperform other reduced-form forecasting models.

In this chapter we estimate a linear and nonlinear DSGE model (as well as a large selection of competing forecasting models) for the South African economy. The competing forecasting models include classical vector-autoregressive (VAR) models and a number of BVAR varieties. These BVAR models have been estimated with various forms of the Minnesota prior and stochastic variable selection (SVS) techniques.<sup>10</sup> Some of the BVAR models that employ SVS have been extended to allow for time-varying parameters, endogenous structural breaks, and least absolute shrinkage and selection operators.

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<sup>7</sup>Nonlinear filters have been applied in many settings to model various features of time-series data. The forms that some of these filters take is discussed in Kitagawa and Gersch (1996), Doucet *et al.* (2000), DeJong and Dave (2011), and others. While most of the research that makes use of nonlinear DSGE models utilise a particle filter to derive the likelihood function, DeJong and Dave (2011) suggest that using the Efficient-Information-Sampling filter may lead to improved results when applied to structural macroeconomic models.

<sup>8</sup>The latest version of *Dynare*, version 4.2.2, can be used to estimate parameters in a nonlinear model with the aid of the methods that were applied in Fernández-Villaverde and Rubio-Ramírez (2005). However, at the time of writing, these routines do not allow for the generation of forecasts with the particle filter. It is hoped that the results in this chapter will motivate those who are involved with this impressive project to include second-order forecasting options in future versions of *Dynare*, while encouraging further efforts that consider more efficient algorithms for nonlinear models, such as those that are considered in Andreasen *et al.* (2014) and Maliar *et al.* (2013).

<sup>9</sup>In addition, Pichler (2008) is the only example of a forecasting study that makes use of a nonlinear DSGE model that has been applied to a developed-world economy.

<sup>10</sup>The specification of BVAR models with a Minnesota prior is discussed in Litterman (1986a), Litterman (1986b), Doan *et al.* (1984) and Sims and Zha (1998). The application of SVS techniques in a BVAR model is described in Koop and Korobilis (2010) and Korobilis (2011).

The results of this investigation suggest that the nonlinear DSGE model appears to outperform its linear counterpart for all variables in most instances. In addition, the findings suggest that these improvements are statistically significant when forecasting consumer inflation and interest rates over the medium to long horizon, as well as output over short to medium horizons.<sup>11</sup> The nonlinear DSGE model also appears to outperform the VAR and BVAR models when forecasting consumer inflation.<sup>12</sup> In addition, when forecasting output over longer horizons, the predictive ability of the nonlinear DSGE model would appear to be superior, while over a shorter horizon, there are a few cases where a BVAR model generates better forecasts. The forecasts for interest rates are all fairly similar; however, one of the BVAR models with the Minnesota prior is able to outperform the nonlinear DSGE over the medium to long horizon.

The remainder of this paper takes the following form. Section 2 describes the theoretical structure and empirical techniques that are employed to estimate the DSGE models. Section 3 considers the specification of the wide selection of VAR and BVAR models. In section 4, we describe the data that is used in this study and the parameter estimates from the DSGE model, before we discuss the results in section 5. The final section comprises of the conclusion.

## 11.2 Dynamic Stochastic General Equilibrium Models

### 11.2.1 Theoretical structure

The structure of the DSGE models is consistent with the New Keynesian framework in that it incorporates features that describe monopolistic competition, capital accumulation, capital adjustment costs and various other nominal and real rigidities. In many respects, this model may be considered as a simplified version of Fernández-Villaverde and Rubio-Ramírez (2006), which has been used in several investigations that relate to nonlinear DSGE models.<sup>13</sup> Therefore, the economic environment is described by the actions of households,

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<sup>11</sup>These forecasts are evaluated after calculating the relative-root-mean squared errors and the Diebold and Mariano (1995) statistics, which consider the significance of any observed improvement.

<sup>12</sup>The BVAR with Minnesota prior appears to provide the second-best results in this instance.

<sup>13</sup>In contrast to the models that were discussed in part one, this model includes a detailed description of the steady-state conditions, which may differ from zero. This would also imply that in contrast to the models that have been discussed earlier, the data that is applied to this model is not demeaned. In addition, the model that is used in this chapter is notably smaller than those that were discussed previously, mainly due to the computational complexities and time taken to estimate these models.

intermediate producers, final good producers, and the central bank.<sup>14</sup>

Households maximise utility for different measures of consumption, real money balances, and leisure activities, such that after incorporating separable preferences, their utility function can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left[ \Theta_t \left( \frac{c_t^{1-\tau} - 1}{1-\tau} \right) + \chi_m \log \left\{ \frac{M_t}{P_t} \right\} + \eta_h (1 - h_t) \right] \quad (11.2.1)$$

where,  $\beta$  represents the subjective time discount factor,  $\Theta$  represents the effect of a demand shock,  $c_t$  represents consumption,  $\tau$  represents the households preference for consumption,  $M_t/P_t$  represents real monetary balances,  $\chi_m$  represents the households preference for monetary holdings,  $(1 - h_t)$  represents leisure, and  $\eta_h$  represents the households preference for leisure. It is assumed that the demand shock follows an autoregressive structure, such that

$$\log \Theta_{t+1} = \rho_{\Theta} \log \Theta_t + \epsilon_{\Theta,t+1}, \quad \text{where } \epsilon_{\Theta,t+1} \sim N(0, \sigma_{\Theta}^2) \quad (11.2.2)$$

where  $\rho_{\Theta}$  represents the persistence in the aggregate demand shock and  $\epsilon_{\Theta,t+1}$  represents the stochastic demand shock. The household's utility function is then subject to a budget constraint that incorporates capital adjustment costs,

$$\frac{M_{t-1} + B_{t-1} + W_t h_t + Q_t k_t + D_t + L_t}{P_t} \geq c_t + x_t + \frac{\psi_k}{2} \left( \frac{x_t}{k_t} - \delta \right)^2 k_t + \frac{B_t/i_t + M_t}{P_t} \quad (11.2.3)$$

where,  $B_t$  represents bond holdings,  $W_t$  represents the wage rate,  $Q_t$  represents the rate of return on capital,  $k_t$  represents productive capital,  $D_t$  represents dividend payments,  $L_t$  represents lump-sum transfers from government,  $x_t$  represents investment, and  $i_t$  represents the gross nominal interest rate. The  $\delta$  parameter represents the depreciation rate of capital, and  $\psi_k$  is the parameter for the adjustment cost of capital. The inclusion of  $\psi_k$ , which regulates the extent to which changes to the current cost of capital are indexed to past values, is similar to Fernández-Villaverde and Rubio-Ramirez (2006), where certain costs would be imposed when the rate of investment deviates from the rate that would result in a balanced growth path.

The capital stock is then assumed to evolve according to the expression

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<sup>14</sup>The structure of this model follows Pichler (2008). This allows for a comparison between the results of the models that have been applied to the macroeconomic data of a developed and emerging market economy. Hence, if the predictive ability of this nonlinear model is superior to that of its linear counterpart, then the result could be attributed to the underlying characteristics of the data-generating process in the emerging market economy, given the results that are reported in Pichler (2008).

$$k_{t+1} = (1 - \delta)k_t + x_t \quad (11.2.4)$$

The firms that are involved in the production of finished goods make use of constant returns-to-scale production technology where the  $j$  intermediate goods,  $y_t(j)$ , serve as the only inputs. Hence, the quantity of finished goods that are produced is determined by the expression

$$y_t = \left[ \int_0^1 y_t(j) \frac{(\theta - 1)}{\theta} dj \right]^{\frac{\theta}{(\theta - 1)}} \quad (11.2.5)$$

where  $\theta$  represents the elasticity of substitution between intermediate inputs. The gross markup over marginal costs that monopolistic competitive intermediate firms charge would then be equivalent to  $\theta/(\theta - 1)$ .<sup>15</sup>

The price of these goods is then given by

$$P_t = \left[ \int_0^1 P_t(j) (\theta - 1) dj \right]^{\frac{1}{(1 - \theta)}} \quad (11.2.6)$$

where the demand for each intermediate good is given by

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} y_t \quad (11.2.7)$$

Firms that are involved in intermediate production face a Cobb-Douglas production function with labour augmenting technology change, where  $\alpha$  represents capital's share of output and  $a_t$  is the technology shock. This could be expressed as

$$y_t(j) = k_t(j)^\alpha \left( a_t h_t(j) \right)^{1 - \alpha} \quad (11.2.8)$$

The technology shock is then assumed to follow an autoregressive process

$$\log a_{t+1} = (1 - \rho_a) \log \bar{a} + \rho_a \log a_t + \epsilon_{a,t}, \quad \text{where } \epsilon_{a,t} \sim N(0, \sigma_a^2) \quad (11.2.9)$$

where the steady-state of  $a_t$  is denoted by  $\bar{a}$  and the persistence in the technology shock is captured by  $\rho_a$ . The stochastic cost-push shock is then represented by  $\epsilon_{a,t}$ . Sticky-prices are introduced through quadratic functions

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<sup>15</sup>See Alpana *et al.* (2010a) for further details of the application of Rotemberg (1982) pricing to South African data.

that describe the cost of adjusting prices. The specification follows the cost of price adjustment mechanism of Rotemberg (1982),

$$PAC_t(j) = \frac{\psi_p}{2} \left[ \frac{P_t(j)/P_{t-1}(j)}{\bar{\pi}} - 1 \right]^2 y_t P_t \quad (11.2.10)$$

where  $\bar{\pi}$  denotes the steady-state value for inflation, and  $\psi_p$  represents the size of the adjustment costs of prices. At the end of period  $t$  the firms distribute profits to the respective households through dividend payments, where

$$D_t(j) = P_t(j)y_t(j) - W_t h_t(j) - Q_t K_t(j) - \frac{\psi_p}{2} \left[ \frac{P_t(j)/P_{t-1}(j)}{\bar{\pi}} - 1 \right]^2 y_t P_t(j) \quad (11.2.11)$$

The objective of each firm is then to maximise their total market value, for which we construct the optimisation problem

$$\max_{h_t(j), k_t(j), P_t(j)} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \frac{D_t(j)}{P_t} \quad (11.2.12)$$

where  $\lambda$  is the Lagrangian multiplier. Finally, to close the model, we assume that the central bank conducts monetary policy by following a variant of the Taylor rule that may be described as

$$\log \frac{i_t}{\bar{i}} = \phi_i \log \frac{i_{t-1}}{\bar{i}} + \phi_y \log \frac{y_t}{\bar{y}} + \phi_\pi \log \frac{\pi_t}{\bar{\pi}} + \epsilon_{i,t}, \quad \text{where } \epsilon_{i,t} \sim N(0, \sigma_i^2) \quad (11.2.13)$$

where  $\bar{i}$  and  $\bar{y}$  refer to the steady-state values of interest rates and output. The  $\phi_i$  parameter would then refer to the degree of interest rate smoothing, while  $\phi_y$  and  $\phi_\pi$  refer to the response of the central bank to deviations from the steady-state values of output and inflation. The term  $\epsilon_{i,t}$  is the stochastic shock to interest rates, which is assumed to be i.i.d.<sup>16</sup>

### 11.2.2 Model solution, likelihood functions and parameter estimates

The nonlinear DSGE model is solved using second-order perturbation methods, as described by Schmitt-Grohé and Uribe (2004). For comparative purposes we make use of the first order approximation method of Klein (2000), which

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<sup>16</sup>In contrast to the model of Fernández-Villaverde and Rubio-Ramírez (2006), this model does not include a government sector, wage rigidities, or shocks to labour supply and investment-specific technology.

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is used to approximate a linear DSGE model.<sup>17</sup> The likelihood function may then be constructed with the following measurement equation in a state-space representation

$$\mathcal{Y}_t = g(\mathcal{X}_t, v_t, \Xi) \quad (11.2.14)$$

where  $\mathcal{Y}_t$  represents the observed variables, which are related to the full set of variables in  $\mathcal{X}_t$ . The measurement errors are contained in  $v_t \sim N(0, \sigma_v^2)$ , where it is assumed that the off-diagonal elements in  $\sigma_v^2 = 0$ . The parameters in the model are contained in the  $\Xi$  vector. To describe the transition of the variables in the model, we may use the state equation

$$\mathcal{X}_t = \xi(\mathcal{X}_{t-1}, \epsilon_t; \Xi) \quad (11.2.15)$$

where  $\epsilon_t$  is a vector that contains the three stochastic shocks. The function  $\xi$  would then define how the variables evolve over time, which is dependent on previous values for the variables, current realisations of shocks, and the parameters in the model. When describing the evaluation the likelihood function of the nonlinear model, we refer to the function  $\tilde{\xi}$ , while for the linear model we use the notation  $\bar{\xi}$ . We make use of Monte Carlo methods and the particle filter that was used in Fernández-Villaverde and Rubio-Ramirez (2005), which would imply that the parameters in the model are random variables, such that  $\Xi = [\mu, \Sigma_v]$ . The specification of this filter for the nonlinear model may then be expressed as

$$\ell(\mathcal{Y}_t^T | \tilde{\xi}, \Xi) = \prod_{t=1}^T \frac{1}{\mathcal{N}} \sum_{n=1}^{\mathcal{N}} p(\mathcal{Y}_t | \tilde{x}_{t|t-1}^n; \tilde{\xi}, \Xi) \quad (11.2.16)$$

where  $p$  denotes the probability density and  $\sum_{n=1}^{\mathcal{N}} \tilde{x}_{t|t-1}^n$  represents draws from each density in the sequence  $\prod_{t=1}^T p(\mathcal{X}_t | \mathcal{Y}_t^{T-1}; \Xi)$ . The specification of the likelihood function for the linear model,  $\bar{\xi}$ , makes use of a traditional Kalman filter that is provided by Hamilton (1994), such that

$$\begin{aligned} \ell(\mathcal{Y}_t^T | \bar{\xi}, \Xi) &= \prod_{t=1}^T p(\mathcal{Y}_t | \mathcal{Y}_t^{T-1}; \bar{\xi}, \Xi) \\ &= \prod_{t=1}^T \int p(\mathcal{Y}_t | \mathcal{X}_t, \mathcal{Y}_t^{T-1}; \bar{\xi}, \Xi) p(\mathcal{X}_t | \mathcal{Y}_t^{T-1}; \bar{\xi}, \Xi) d\mathcal{X}_t \end{aligned} \quad (11.2.17)$$

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<sup>17</sup>Once again, we make use of the approach that was followed in Pichler (2008) to derive the model solution and parameter estimates, to allow for a comparison of the results for an emerging market economy with those for a developed-world economy.

It is worth noting that these likelihood functions are extremely complex, where portions of them are flat and the possibility of several local minima and maxima would often arise.<sup>18</sup> Furthermore, when using the particle filter for the nonlinear model, the likelihood function is not continuous with respect to the parameter vector  $\Xi$ . In cases such as this, Judd (1998) suggests that traditional gradient-based numerical optimisation techniques may be of little use when seeking to maximise the likelihood function. As an alternative, we employ the simulated annealing global optimisation approach, as in Fernández-Villaverde and Rubio-Ramirez (2005), which provide much-improved results.

After estimating the parameters in the model, we are then able to generate the respective  $h$ -step ahead forecasts,  $E_t[\mathcal{Y}_{t+h}]$ , given the most recent values of all variables,  $\mathcal{Y}_t$ , and the estimated parameters values,  $\hat{\Xi}$ . When seeking to generate values for the linear model, one is able to use the Kalman filter to derive the expected values,  $E_t[\mathcal{Y}_{t+h}|\mathcal{X}_t; \bar{\xi}, \hat{\Xi}]$ . Similarly, for the nonlinear model, one is able to make use of either Monte Carlo methods or numerical integration to generate the future expected values,  $E_t[\mathcal{X}_{t+h}]$ , with the aid of the particle filter. In this case we follow Pichler (2008) and make use of numerical integration to derive  $E_t(\mathcal{Y}_{t+h}|\mathcal{X}_t; \tilde{\xi}, \hat{\Xi})$ .

## 11.3 Vector Autoregressive Models

The competing forecasting models make use of various VAR representations for the variables that are observed.<sup>19</sup> These models include classical unrestricted VAR models and restricted BVAR models.

### 11.3.1 Classical unrestricted VAR models

The classical unrestricted VAR models make use of the structure

$$z_t = c + \varphi_1 z_{t-1} + \varphi_2 z_{t-1} + \cdots + \varphi_s z_{t-s} + \varepsilon_t \quad (11.3.1)$$

where the lag length,  $s$ , was determined by the Schwartz-Bayes information criterion. After estimating successive models for the respective end-of-sample periods, 2000Q1 to 2011Q4, we found that each model suggested an optimal lag length of two periods. This maximum lag length was also applied to all the reduced-form models that are discussed below.

<sup>18</sup>See, Canova and Sala (2009) for more on the complexity of the likelihood functions in DSGE models. Such complexities may result in potential difficulties with parameter identification.

<sup>19</sup>The observed dataset for the reduced-form and DSGE models include measures of output, inflation, and interest rates. Further details of the data are provided in section 11.4.

## 11.3.2 Bayesian VAR models

### 11.3.2.1 Minnesota shrinkage priors

The first group of BVAR models make use of shrinkage priors that follow the work of Litterman (1986a), Litterman (1986b), Doan *et al.* (1984), and Sims and Zha (1998). These models make use of a small selection of hyperparameters that impose various prior restrictions on the model for the estimated coefficient mean,  $\hat{\varphi}$ , tightness,  $\zeta$ , decay,  $\kappa$ , and variation due to other-lagged variables,  $\omega$ .<sup>20</sup>

Since the data in all the BVAR models is assumed to be stationary, the prior means follow the specification for white-noise, where all these hyperparameters (including the first own-lag) are set to zero. Hence, in the above model the prior is set such that  $\hat{\varphi} = 0$ .

When seeking to specify the variance-covariance elements,  $\nu_{ij,\varphi}$ , we impose the following specification for the priors of variable  $i$  in equation  $j$  and lag  $\varphi$ :

$$\nu_{ij,\varphi} = \begin{cases} \zeta / \kappa \varphi & \text{if } i = j \\ \frac{\zeta \omega \sigma_i^2}{\kappa \varphi \sigma_j^2} & \text{if } i \neq j \end{cases}$$

The selection of hyperparameters for the Minnesota prior in this chapter is based on the general practice of Bańbura *et al.* (2010) and Korobilis (2011), who search over a grid of values to identify those values that may provide a superior model fit (based on the preliminary results of a training sample).<sup>21</sup> Gupta and Steinbach (2013) apply a similar strategy when seeking to forecast South African data. However, as Gupta and Steinbach (2013) are primarily concerned with identifying the model that performs best in absolute terms, they evaluate the forecasts that are provided by each of the different Minnesota priors. We follow the practice of Gupta and Steinbach (2013) and evaluate a total of thirteen different BVAR models that employ different hyperparameters to specify the Minnesota prior.

In this specification, the values for the  $\zeta$  hyperparameter control the degree to which the coefficient of the first lag of the dependent variable is believed to be concentrated around zero. Various values for this tightness parameter have been used, from between 0.1 and 2.0, where small values will force the own-lags of the dependent variable to be close to the prior mean.

Since it is assumed that the coefficients for more immediate lags are possibly going to be more influential, we assume that the variance for these coefficients will decrease with an increasing lag length,  $\varphi$ . Once again, we make use of various values for the decay, where  $\kappa$  ranges between 0 and 2.

<sup>20</sup>Of course, the posterior estimates may override these prior restrictions if the data provides strong evidence that the prior is inappropriate.

<sup>21</sup>As an alternative Giannone *et al.* (forthcoming) consider using an optimisation routine that is applied to these hyperparameters to improve the in-sample fit of the model.



In addition, it is assumed that most of the variation in each of the respective variables may be explained by the variation in their respective lags. Therefore, for the explanatory variables that are not lagged dependent variables, a smaller variance is assigned (in relative terms) by choosing a value for  $\omega$  that is between 0 and 1, where the ratio  $\sigma_i^2/\sigma_j^2$  accounts for the differences in the variability of the respective variables.

To ensure that the models that we have considered do not favour any of these prior specifications, we consider the effects of a relatively large number of values for  $\zeta$ ,  $\kappa$ , and  $\omega$ . Similar investigations have also been performed in other forecasting studies for the South African economy, as in Gupta *et al.* (2010).

### 11.3.2.2 BVAR models with stochastic variable selection

The specification of the BVAR models with SVS follow Koop and Korobilis (2010) and Korobilis (2011). Using the formulation of an unrestricted vector autoregressive model in equation (11.3.1), we allow for the respective coefficient matrices,  $\varphi$ , to be multiplied by the indicator matrix,  $\gamma_{i,j}$ , which has elements that take on a Bernoulli distribution. Essentially, these indicator parameters determine whether or not the variable should be included in the final representation that will be used to generate the forecasts. Using a Bayesian framework, these parameters are treated as random variables, for which we assign a prior that is taken to the likelihood function to derive the final posterior values.<sup>22</sup>

In addition to the basic model that employs these variable selection techniques, we also include a model where  $\gamma_{i,j} = 1$  to investigate whether or not these techniques make a significant difference to the forecasting performance.<sup>23</sup> Thereafter, we include a model that makes use of priors which impose hierarchical Bayesian shrinkage using least absolute shrinkage and selection operators (LASSO). Several studies have suggested that these priors have outperformed other types of hierarchical Bayesian shrinkage estimates (such as the Normal-Jeffreys priors), while providing comparable forecasting results to the models that employ a Minnesota prior.<sup>24</sup> When specifying this model, we condition the coefficient matrix by assuming that the off-diagonal elements of the covariance matrix are zero.

We have also included the results of a BVAR model with SVS and time-varying parameters. In this case the coefficients in equation (11.3.1) may be

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<sup>22</sup>These posterior values are generated from simulation techniques that make use of a Gibbs sampler.

<sup>23</sup>This model would take the form of a traditional BVAR model, which is estimated with the aid of a Gibbs sampler and a flat prior. The results from this model could be compared to the VAR, which is estimated with frequentist techniques.

<sup>24</sup>See Korobilis (2011) for further details on the comparative performance of LASSO and Normal-Jeffreys priors.

expressed as,  $\varphi_{i,t} = \varphi_{i,t-1} + \vartheta_{i,t}$ , where  $\vartheta_{i,t} \sim N(0, \sigma_{\vartheta}^2)$ . Hence, this model would allow for a degree of stochastic variation in each of the coefficients. The final BVAR model with SVS allows for an endogenous structural break. This is achieved by incorporating a restricted Markov chain, where the respective processes are able to move to a second regime (during a structural break). However, in contrast with a traditional regime-switching model that makes use of a Markov chain, the process is not able to move back into the initial regime, and as such, it starts afresh from the breakpoint in the time series.

## 11.4 Data and Parameter Estimates

### 11.4.1 Data

The respective DSGE and reduced-form models make use of three observed variables, namely detrended output (in logarithms),  $y_t$ , quarter-on-quarter consumer price inflation,  $\pi_t$ , and a measure of the nominal interest rate,  $i_t$ . The data is measured at a quarterly frequency from 1960Q1 to 2011Q4, with the start and end date of the sample being governed by data availability. To derive a measure of detrended output, we took the logarithm of real gross domestic product, from which we removed the linear trend.<sup>25</sup> The data on the seasonally adjusted real gross domestic product at constant prices (for the year 2005) was obtained from the South African Reserve Bank.<sup>26</sup> The data for the interest rate relates to the three-month Treasury bill, which was obtained from the International Financial Statistics (IFS) database, that is maintained by the International Monetary Fund (IMF). Consumer price inflation was derived from the first difference of the logarithm of the consumer price index. The data on the seasonally-unadjusted Consumer Price Index was obtained from the Global Financial Database.<sup>27</sup> The seasonal was removed with the aid of the X-12 procedure, which has been developed by the Department of Commerce, U.S. Census Bureau.

As discussed above, we select the time period 1960Q1 through 2011Q4 for our analysis. This gives a total sample of 208 observations on each series, where the first 160 observations (1960Q1 through 1999Q4) were used for the initial in-sample analysis, following the existing literature on forecasting for South Africa. The remaining 48 observations (2000Q1 through 2011Q4) were used for the out-of-sample forecasting evaluation, for which the models are recursively estimated by increasing the size of the in-sample by one observation to produce

<sup>25</sup>When applying South African data to a DSGE model, Alpanda *et al.* (2010a) apply the same transformation to output.

<sup>26</sup>For the BVAR models, we had to use the growth rates of detrended output, due to issues of convergence. So, after generating all the forecasts from the BVARs, we transform this variable back into detrended output, before calculating the evaluation statistics.

<sup>27</sup>Data from the Global Financial Database can be obtained from <http://www.globalfinancialdata.com>.

one- to eight-step ahead forecasts. Note, the choice of the starting point of the out-of-sample period coincides with South Africa's decision to move formally to an inflation-targeting regime in February of 2000. When evaluating the forecasts we transform the variables to consider quarter-on-quarter consumer price inflation, the log-level of output, and an annual measure of the nominal interest rate.

### 11.4.2 Parameter Estimates of DSGE model

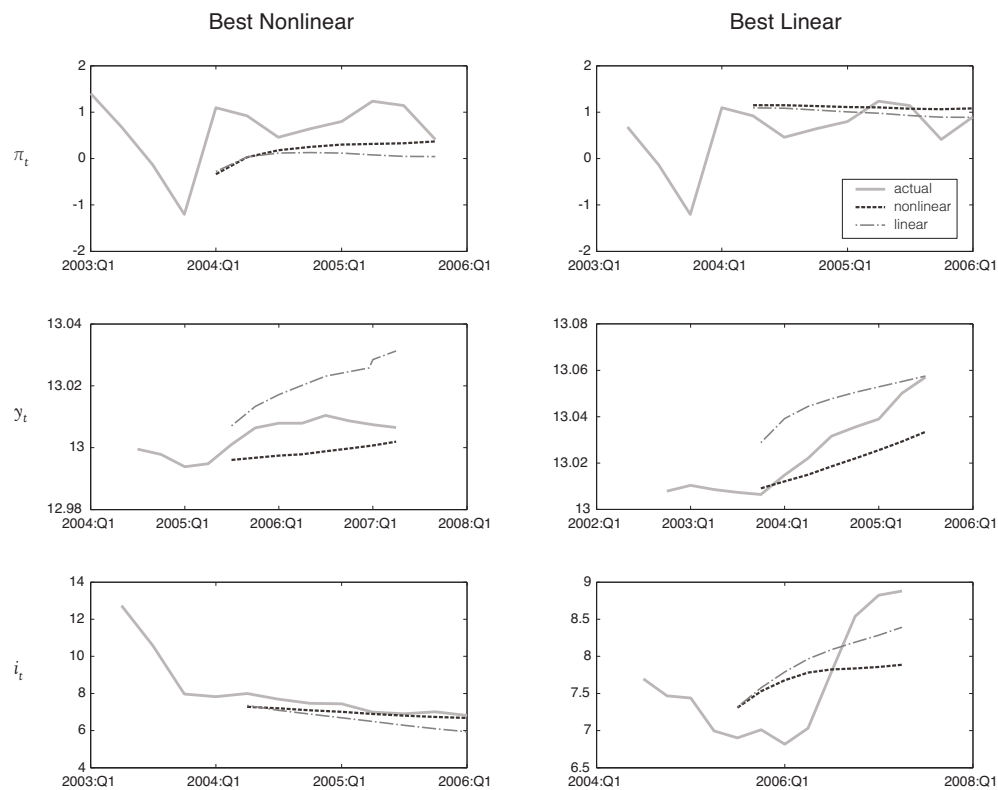
Despite seeking to estimate as many of the parameters as possible, it was necessary to calibrate certain parameters in the DSGE model. In cases where the parameters are not successfully identified, we follow Liu *et al.* (2009), and set the elasticity of output with respect to capital to 0.26, the depreciation rate was set to 0.019, the capital adjustment costs parameter was set to 10, and the elasticity of substitution between intermediate goods was set to 6, as in Alpanda *et al.* (2010a). Furthermore, the parameters corresponding to leisure and real money balances in the utility function, were fixed at values that assume that households spend 30% of their time working in the steady-state (to match the steady-state ratio between real balances and quarterly output). Finally, as in Pichler (2008), the measurement error variances were calibrated to be 10% of the variance of the respective data series.<sup>28</sup>

The results from the maximum likelihood estimates of the remaining parameters are contained in table (11.1). Note that the shocks in the nonlinear model are all much larger, where in the case of interest rates, the shock is over 75% greater than the linear estimate. This is in strong contrast to the findings of Pichler (2008), where the size of the shocks were similar in both linear and nonlinear specifications, when applied to macroeconomic data for the U.S. economy. In addition, the coefficients for the central banks reaction function are all much smaller than in the linear model. Importantly, in the nonlinear model the monetary authority reaction following a change to inflation, relative to its response to output, is four times greater than in the linear case. The preference for consumption is also significantly lower in the nonlinear model, while the remaining parameter estimates are fairly similar.

To show how the forecasts of the linear and nonlinear DSGE models differ, figure (11.1) contains the results of forecasts for each variable, which were generated when the difference in the forecasting error was greatest (as measured at an eight step-ahead horizon). Note that the horizontal axis for time differs for each of these graphs, as we are purely interested in displaying the results where the difference is greatest. The left column of graphs is then used to show those forecasts which favour the nonlinear model and the right column shows those forecasts that favour the linear specification.

<sup>28</sup>Real money balances is measured by M2 deflated by the consumer price inflation, and covers 1965Q1-2011Q4, since M2 is only available from 1965Q1.

Parameter	Description	Linear	Nonlinear
$\beta$	Time discount factor	0.9914	0.993
$\psi_p$	Adjustment cost of prices	33.4637	32.4482
$\bar{\pi}$	Steady-state of inflation	1.0066	1.0085
$\bar{a}$	Steady-state of technology	6824.7289	7034.5183
$\rho_\Theta$	Persistence in demand shock	0.981	0.980
$\rho_a$	Persistence in technology shock	0.9743	0.978
$\phi_i$	Interest rate smoothing	0.8471	0.7507
$\phi_y$	Central bank reaction of output	0.002	0.0006
$\phi_\pi$	Central bank reaction of inflation	0.3443	0.2561
$\tau$	Preference for consumption	2.5241	1.874
$\sigma_i$	Shock to interest rates	0.0018	0.0032
$\sigma_\Theta$	Shock to aggregate demand	0.0343	0.0449
$\sigma_a$	Shock to technology	0.0127	0.0175

**Table 11.1:** Parameter estimates - linear & nonlinear DSGE model**Figure 11.1:** Comparative linear and nonlinear DSGE eight step-ahead forecasts

## 11.5 Out-of-Sample Results

In total we make use of twenty-two models that have been estimated forty-one times times to generated forecasts for the period 2000Q1 to 2011Q4. When comparing these models, we consider the results for each of the  $1, 2, \dots, 8$  step-ahead forecasts over the entire out-of-sample period. In addition, we briefly discuss the results from the average of the one- through eight-step ahead forecast that were generated at each time period.<sup>29</sup>

In the following sub-sections we investigate the forecasting performance for each of the respective variables. The respective root-mean squared errors for each step ahead have also been summarised in figure (11.2), where we show the results for the linear and nonlinear DSGE model, along with the results from a random walk, classical VAR and best BVAR models that employ either a Minnesota prior or stochastic variable selection techniques.<sup>30</sup>

Additional details relating to these results have been included in tables (11.2) through (11.4) for all the of the models that were considered in this study. The first line of these tables contains the root-mean squared error for the nonlinear DSGE model. This statistic is then used to calculate the relative root-mean squared error for the other models.<sup>31</sup> The Diebold and Mariano (1995) statistic is used to determine whether this difference in forecasting performance is statistically significant.

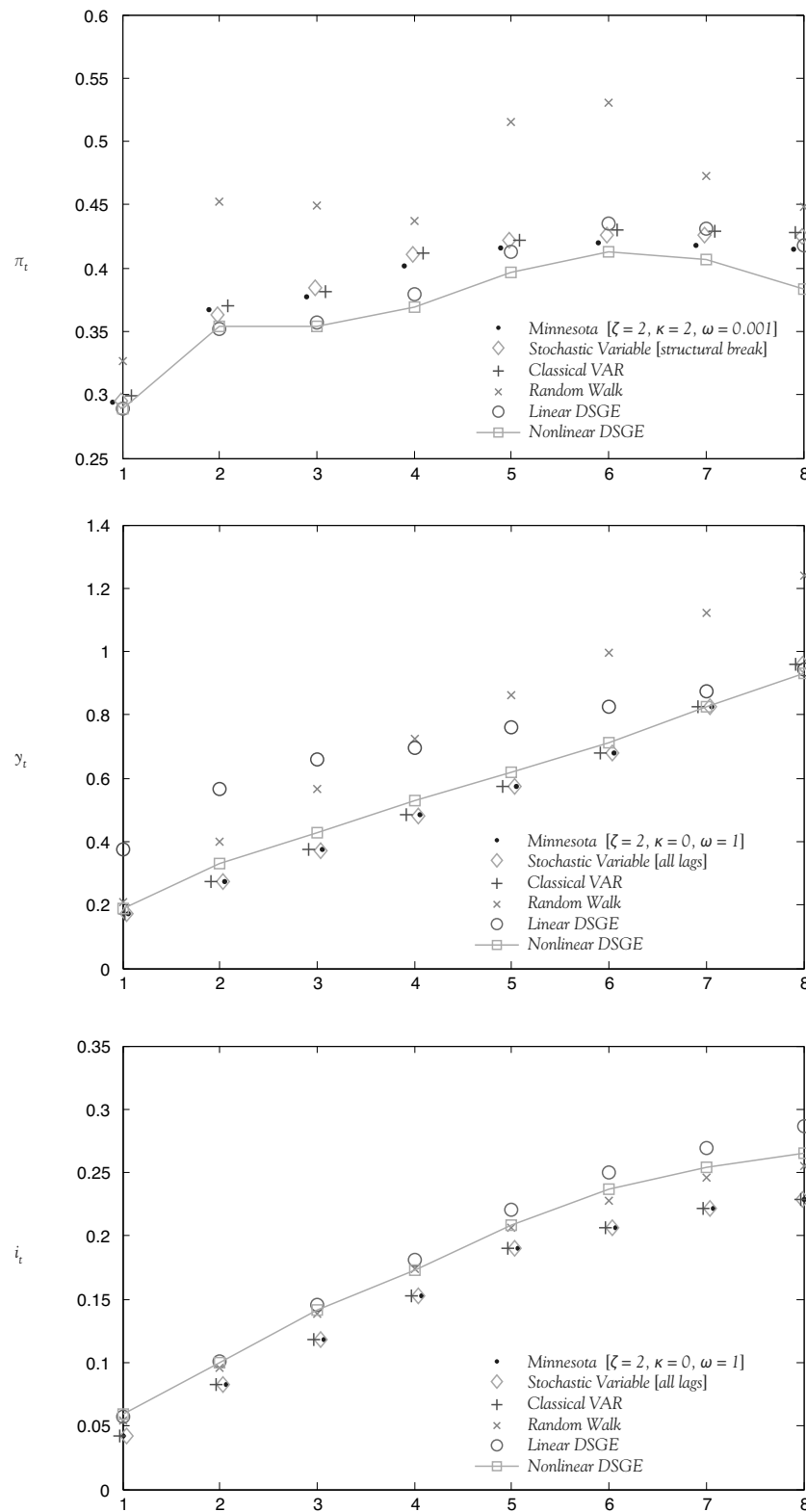
Figures (11.3) through (11.5) have then been used to illustrate how these forecasts performed over different periods of time. For this calculation we consider the average root-mean squared error from the one- to eight-step ahead forecasts that arise at each point in time. The results from this exercise are discussed in relation to the business cycle phases that have been identified in the South African Reserve Bank Quarterly Bulletin 2014, where it is noted that South Africa experienced an upward phase between September 1999 and November 2007. Over this extended period of time, the rate of economic growth would appear to have undergone a significant increase towards the end of 2003. This phase drew to a close in December 2007, following the onset of the Global Financial Crisis, during which the economy experienced a downward phase that lasted 21 months. In September 2009 the economy entered an upward phase once again, which has continued until early 2014.

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<sup>29</sup>Further details of the forecasting results are included in the appendix, under §(A.3.2), where additional details relating to different horizons are also considered.

<sup>30</sup>To identify the best BVAR models we calculate the sum of all step-ahead forecasts over time.

<sup>31</sup>For example, the relative root-mean squared-error for the classical-VAR at a one-step ahead forecasting horizon is calculated as  $[(\text{RMSE}_{\text{classical-VAR}}/\text{RMSE}_{\text{nonlinear-DSGE}}) - 1] \times 100$ , where RMSE is the one-step ahead root-mean squared-error over 2000Q1 to 2011Q4.



**Figure 11.2:** One through eight step-ahead forecasts (average root-mean squared-error over all time periods)

### 11.5.1 Consumer price inflation

In the top panel of figure (11.2) we note that the nonlinear DSGE model is able to provide forecasts that are at least equivalent to those of other forecasting models, when we consider the root-mean squared errors for the respective models. In addition, this graph could also suggest that the nonlinear model would possibly outperform all of the models at longer forecasting horizons. The results also show that the random walk model would appear to provide relatively poor forecasts.

When considering the detailed results for the  $\pi_t$  forecasts that are contained in table (11.2), we note that there are only 7 negative relative root-mean squared errors, and the largest negative value is  $-0.34$ . In contrast, there are 161 instances where a positive relative root-mean squared error is observed. The largest positive value is 27.97. This would suggest that in the vast majority of cases, the nonlinear DSGE model is able to provide superior forecasts

The models that are able to provide a comparatively lower root-mean squared error (when compared with the nonlinear DSGE model) are the first three BVAR models with Minnesota prior. These instances occur at the six and seven step ahead forecasts.<sup>32</sup> The Diebold-Mariano statistic for these 7 negative relative root-mean squared errors are all particularly small, and as such, it is not surprising to note that there are no occasions where any of the competing models are able to generate statistically significant improvements.

Furthermore, what is also worth noting is that when we compare the nonlinear DSGE model with the linear variant, the nonlinear model provides a lower root-mean squared error at all but the one-step ahead forecast horizon. However, at the longer forecasting horizon, the nonlinear DSGE is clearly superior, as the six, seven and eight step-ahead Diebold-Mariano statistics are all well below negative two.<sup>33</sup>

To ensure that these results have not been overly influenced by an outlying forecast that may have been generated for a particular point in time, we also count the number of significant Diebold-Mariano statistics that were generated for each forecast from 2000Q1 to 2011Q4. In this case, the number of significant Diebold-Mariano statistics at the two step-ahead horizon continues to favour the nonlinear DSGE model.<sup>34</sup> Over longer horizons the results of the nonlinear DSGE are more impressive, which would confirm that it is responsible for significantly smaller forecasting errors at longer horizons.<sup>35</sup>

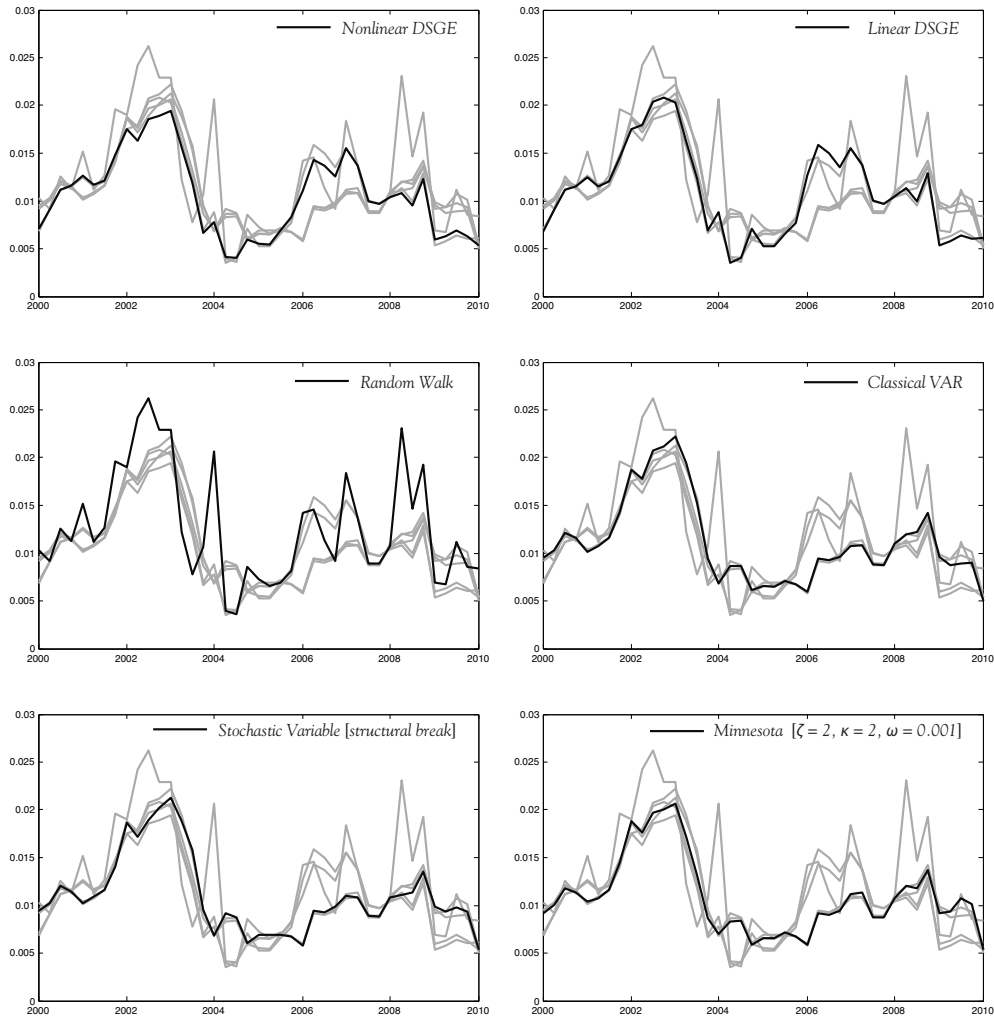
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<sup>32</sup>The only exception arises at a one-step ahead forecasting horizon, where the root-mean squared error of the linear DSGE model is slightly lower.

<sup>33</sup>The nonlinear DSGE model is also clearly superior to that of a Random-Walk.

<sup>34</sup>Further details of this analysis are contained in the supplementary material. When using this method of evaluation, the most impressive of the competing vector autoregressive models is the BVAR with Minnesota prior, which has 12 significant statistics, whereas the nonlinear DSGE has 14. Over this horizon, the results of the linear and nonlinear DSGE models are almost equivalent.

<sup>35</sup>The nonlinear DSGE model is responsible for four additional significant Diebold-



**Figure 11.3:** Inflation forecasts over time (average one through eight step-ahead root-mean squared-error)

Mariano statistics, when compared with the vector autoregressive models at the four step-ahead horizon. At the eight step-ahead horizon, the nonlinear DSGE generates five additional significant Diebold-Mariano statistics. The results for the linear and nonlinear model are similar at the four step-ahead horizon, but the difference in the number of significant Diebold-Mariano statistics at the eight step-ahead horizon is eight, which is in favour of the nonlinear model.



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	1 step	2 step	3 step	4 step	5 step	6 step	7 step	8 step
Nonlinear	0.0091	0.011	0.0113	0.0116	0.0123	0.0124	0.0121	0.0115
Linear	-0.28 [0.29]	0.24 [-0.21]	1.32 [-1.09]	2.36 [-1.41]	3.98 [-1.83]	5.73 [-2.3]**	6.9 [-2.53]**	8.77 [-2.8]**
RW	10.18 [-1.73]	18.27 [-2.46]**	20.61 [-2.44]**	21.33 [-2.52]**	27.2 [-3.49]**	27.97 [-2.94]**	24.7 [-2.28]**	17.67 [-1.76]
VAR	3.88 [-1.48]	3.39 [-0.89]	4.09 [-0.73]	5.94 [-0.92]	3.18 [-0.47]	2.44 [-0.33]	3.38 [-0.41]	6.3 [-0.65]
BVAR	3.87 [-1.49]	3.43 [-0.91]	4 [-0.72]	5.86 [-0.91]	3.13 [-0.47]	2.34 [-0.32]	3.21 [-0.39]	6.18 [-0.64]
Minnes1	3.11 [-1.52]	1.92 [-0.61]	1.72 [-0.4]	3.01 [-0.56]	0.94 [-0.16]	-0.19 [0.03]	-0.23 [0.03]	1.85 [-0.2]
Minnes2	3.05 [-1.4]	1.51 [-0.47]	1.47 [-0.33]	2.89 [-0.53]	0.64 [-0.11]	-0.34 [0.05]	-0.08 [0.01]	2.24 [-0.23]
Minnes3	2.15 [-0.91]	0.77 [-0.21]	1.22 [-0.25]	2.83 [-0.48]	0.24 [-0.04]	-0.34 [0.05]	0.67 [-0.08]	3.32 [-0.32]
Minnes4	1.84 [-0.46]	0 [0]	2.02 [-0.31]	4.17 [-0.58]	0.26 [-0.04]	0.17 [-0.02]	2.28 [-0.25]	5.49 [-0.48]
Minnes5	0.82 [-0.32]	0.31 [-0.08]	1.39 [-0.25]	3.06 [-0.48]	0.11 [-0.02]	-0.06 [0.01]	1.63 [-0.19]	4.44 [-0.41]
Minnes6	1.01 [-0.23]	0.45 [-0.07]	3.15 [-0.43]	5.2 [-0.66]	0.63 [-0.08]	0.58 [-0.07]	2.91 [-0.31]	6.1 [-0.52]
Minnes7	3.56 [-1.24]	2.74 [-0.66]	3.84 [-0.66]	5.91 [-0.9]	2.98 [-0.44]	2.59 [-0.35]	4.04 [-0.48]	7.17 [-0.72]
Minnes8	2.88 [-0.85]	2.15 [-0.44]	4 [-0.63]	6.31 [-0.91]	2.99 [-0.43]	3.02 [-0.4]	5.15 [-0.59]	8.5 [-0.82]
Minnes9	3.83 [-0.68]	2.46 [-0.36]	5.79 [-0.74]	8.29 [-1.05]	3.56 [-0.47]	3.82 [-0.47]	6.62 [-0.72]	10.19 [-0.92]
Minnes10	1.85 [-0.5]	2.07 [-0.37]	4.63 [-0.67]	7 [-0.96]	3.25 [-0.45]	3.58 [-0.46]	6.19 [-0.69]	9.58 [-0.89]
Minnes11	3.33 [-0.55]	3.25 [-0.42]	6.93 [-0.82]	9.2 [-1.1]	3.86 [-0.49]	4.07 [-0.5]	6.95 [-0.74]	10.48 [-0.93]
Minnes12	3.86 [-1.48]	3.39 [-0.89]	4.09 [-0.73]	5.94 [-0.92]	3.18 [-0.47]	2.44 [-0.33]	3.39 [-0.41]	6.31 [-0.65]
Minnes13	3.62 [-1.04]	1.88 [-0.42]	3.26 [-0.55]	5.19 [-0.81]	2.06 [-0.32]	1.69 [-0.24]	2.96 [-0.36]	5.6 [-0.56]
SVS	1.35 [-0.53]	1.27 [-0.31]	2.5 [-0.45]	4.53 [-0.71]	1.65 [-0.25]	1.24 [-0.17]	2.5 [-0.29]	5.46 [-0.52]
SVS-TVP	1.69 [-0.67]	1.87 [-0.46]	3.19 [-0.57]	4.36 [-0.69]	0.72 [-0.11]	0.81 [-0.11]	2 [-0.24]	4.51 [-0.43]
SVS-SB	1.95 [-0.8]	2 [-0.51]	2.57 [-0.48]	4.11 [-0.68]	1.5 [-0.24]	0.55 [-0.08]	1.91 [-0.24]	5.05 [-0.52]
SVS-LAS	1.77 [-0.68]	1.86 [-0.45]	3.1 [-0.55]	5.08 [-0.78]	1.91 [-0.29]	1.49 [-0.2]	3.05 [-0.36]	6.13 [-0.59]

**Table 11.2:** Consumer inflation - root mean square errors and Diebold and Mariano Statistics (average 2000Q1-2011Q4). Nonlinear DSGE model absolute RMSE and competing models relative RMSE with corresponding DM statistics in parenthesis. \*\* significant in favour of Nonlinear DSGE. \* significant in favour of competing model. Model acronyms described in footnote 37.

The results from the root-mean squared-errors that were generated at each point of time, which are displayed in figure (11.3) would appear to suggest that over the period that preceded the Global Financial Crisis, the reduced-form models appear to outperform the DSGE models. However, after the onset of the crisis, the DSGE models would appear to provide better forecasting results. Similar results are also evident following the sudden change in the rate of inflation at the end of 2003, where the DSGE model would appear to outperform the reduced-form models during 2004.<sup>36</sup>

### 11.5.2 Output

In the middle panel of figure (11.2) we note that over shorter horizons, the reduced-form models would appear to provide superior forecasts, although the difference is relatively small. At longer horizons there would appear to be little difference between the reduced-form and nonlinear DSGE models. At most forecasting horizons the nonlinear model clearly outperforms its linear counterpart, while once again, the random-walk would appear to perform relatively poorly.

The relative root-mean square errors for the forecasts of  $y_t$ , which are contained in table (11.3), describe these results in more detail. In this case, the nonlinear DSGE would once again model provide lower root-mean squared errors at the longer horizon, when compared with the various vector autoregressive and random walk models.<sup>37</sup>

Furthermore, there are a number of occasions where the Diebold-Mariano statistic suggests that the nonlinear model provides significantly better results when the forecasting horizon is at least a year.<sup>38</sup> However, at the shorter horizon, some of the BVAR models with Minnesota prior (as well as the BVAR model without stochastic variable selection) appear to provide slightly better forecasts.<sup>39</sup>

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<sup>36</sup>Model acronyms: Nonlinear - Nonlinear DSGE model; Linear - Linear DSGE model; RW - Random Walk model; VAR - classical VAR; BVAR - Bayesian VAR; Minnes1 - BVAR with Minnesota prior ( $\zeta = 2, \kappa = 2, \omega = 0.001$ ); Minnes2 - ( $\zeta = 0.3, \kappa = 0.5, \omega = 0.001$ ); Minnes3 - ( $\zeta = 0.2, \kappa = 1, \omega = 0.001$ ); Minnes4 - ( $\zeta = 0.1, \kappa = 1, \omega = 0.001$ ); Minnes5 - ( $\zeta = 0.2, \kappa = 2, \omega = 0.001$ ); Minnes6 - ( $\zeta = 0.1, \kappa = 2, \omega = 0.001$ ); Minnes7 - ( $\zeta = 0.3, \kappa = 0.5, \omega = 0.05$ ); Minnes8 - ( $\zeta = 0.2, \kappa = 1, \omega = 0.05$ ); Minnes9 - ( $\zeta = 0.1, \kappa = 0.1, \omega = 0.05$ ); Minnes10 - ( $\zeta = 0.2, \kappa = 2, \omega = 0.05$ ); Minnes11 - ( $\zeta = 0.1, \kappa = 2, \omega = 0.05$ ); Minnes12 - ( $\zeta = 2, \kappa = 0, \omega = 1$ ); Minnes13 - ( $\zeta = 0.3, \kappa = 0.5, \omega = 0.5$ ); SVS - BVAR with SVS; SVS-TVP - BVAR with SVS and time varying parameters; SVS-SB - BVAR with SVS and endogenous structural break; SVS-LAS - BVAR with SVS and LASSO prior.

<sup>37</sup>At the eight step-ahead horizon the vector autoregressive and random-walk models all have positive relative root-mean squared errors.

<sup>38</sup>Indeed, there are 18 occasions where the Diebold-Mariano statistic indicates that the nonlinear DSGE model is able to significantly improve upon the forecasts of the various vector autoregressive models.

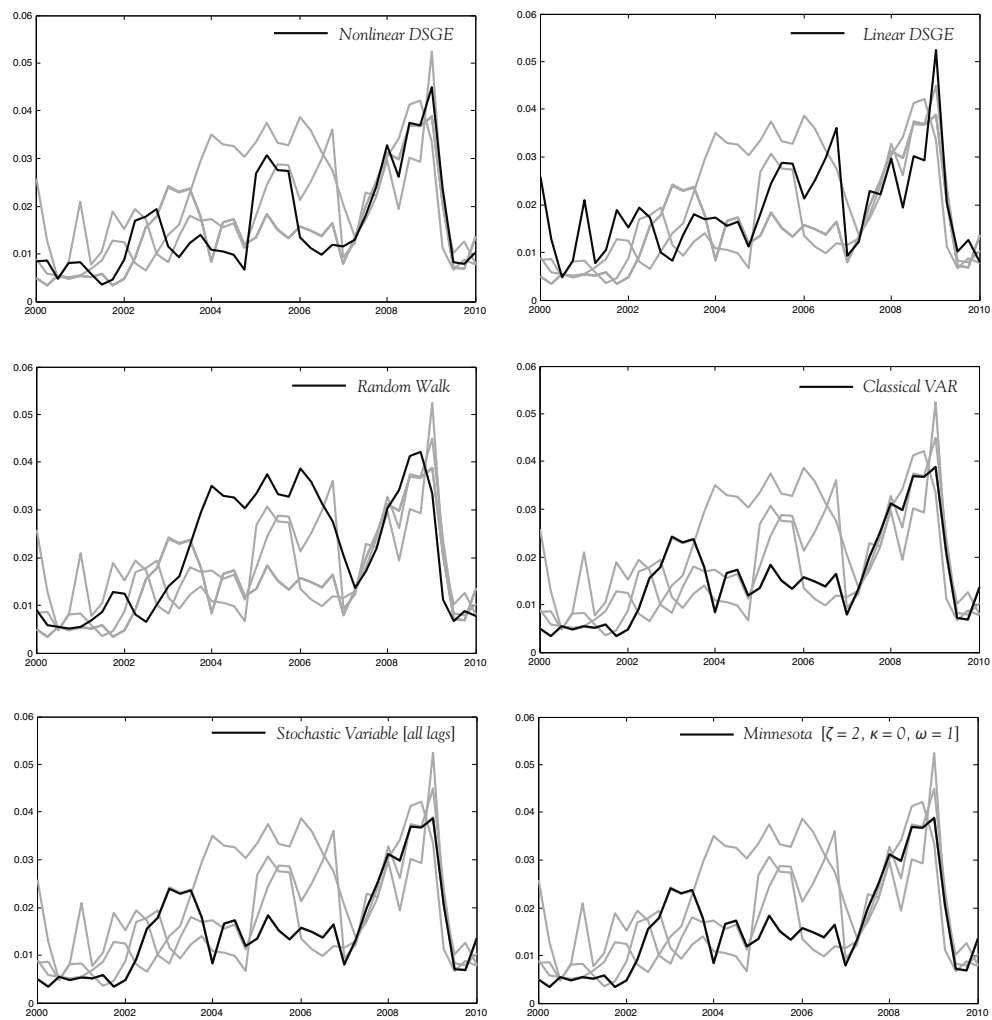
<sup>39</sup>In this case, some of the BVAR models provide statistically significant improvements on six occasions.

Once again, when comparing the nonlinear DSGE model with its linear counterpart, we note that the nonlinear DSGE model would appear to generate a lower loss function on most occasions. However, in this case the root mean squared errors for the nonlinear model are much lower at the short and medium-term horizon, while over the longer term, the results are fairly similar.<sup>40</sup> At the one, two and three step-ahead horizons, the Diebold-Mariano statistics indicate that the forecasts of the nonlinear DSGE model are significantly better.

When considering the results of the root-mean squared-errors at each point of time, we note that the nonlinear DSGE model would appear to detect changes to the rate of output growth relatively quickly. However, when the cycle extends for a protracted period, the VAR model would appear to outperform the DSGE model. Evidence of this is provided in figure (11.4) where we note that the nonlinear forecasts for output from early 2004 are clearly superior. However, as this period of accelerated economic growth continued into 2006, the reduced-form models would appear to provide better forecasts.

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<sup>40</sup> It is also worth noting that when we count the number of significant Diebold-Mariano statistics for the forecasts that were generated at each point in time, we note that 13 statistics favour the nonlinear model, whilst 6 favour the linear model (when we consider the eight step-ahead horizon). At the shorter horizons these statistics favour the nonlinear DSGE model by a larger degree.



**Figure 11.4:** Output forecasts over time (average one through eight step-ahead root-mean squared-error)

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	1 step	2 step	3 step	4 step	5 step	6 step	7 step	8 step
Nonlinear	0.0065	0.0111	0.0144	0.017	0.0194	0.0219	0.0245	0.0272
Linear	77.76 [-4.03] **	54.04 [-3.87]**	33.31 [-3.07]**	20.02 [-1.96]	12.2 [-1.25]	6.24 [-0.7]	2.03 [-0.23]	-1.58 [0.19]
RW	4.18 [-0.47]	13.02 [-1.47]	21.33 [-2.22]**	28.62 [-2.69]**	32.51 [-2.92]**	32.99 [-2.81]**	32.15 [-2.55]**	30.23 [-2.27]**
VAR	-5.88 [1.64]	-9.99 [2.55]	-8.47 [2.45]	-7.06 [1.56]	-5.69 [0.93]	-4.67 [0.62]	-2.31 [0.28]	0.04 [0]
BVAR	-6 [1.69]	-10.21 [2.59]*	-8.71 [2.5]*	-7.26 [1.59]	-5.84 [0.95]	-4.82 [0.64]	-2.45 [0.29]	-0.14 [0.02]
Minnes1	-8.32 [0.82]	-2.68 [0.27]	8.06 [-0.82]	16.08 [-1.46]	20.74 [-1.82]	21.77 [-1.96]	21.43 [-1.99]	19.48 [-1.84]
Minnes2	-7.98 [0.8]	-1.93 [0.2]	8.77 [-0.9]	16.83 [-1.53]	21.46 [-1.89]	22.45 [-2.03]**	22.04 [-2.05]**	20.03 [-1.89]
Minnes3	-7.27 [0.77]	-0.12 [0.01]	10.12 [-1.08]	18.05 [-1.69]	22.39 [-2.03]**	23.18 [-2.16]**	22.57 [-2.14]**	20.51 [-1.94]
Minnes4	-4.8 [0.57]	3.58 [-0.44]	12.9 [-1.43]	20.36 [-1.97]	24.16 [-2.27]**	24.57 [-2.36]**	23.63 [-2.28]**	21.42 [-2.04]**
Minnes5	-5.79 [0.66]	3.07 [-0.37]	12.52 [-1.38]	20.08 [-1.93]	23.95 [-2.24]**	24.41 [-2.34]**	23.51 [-2.26]**	21.31 [-2.03]**
Minnes6	-3.31 [0.42]	6.13 [-0.8]	14.75 [-1.68]	21.8 [-2.14]**	25.26 [-2.41]**	25.42 [-2.48]**	24.29 [-2.36]**	21.98 [-2.1]**
Minnes7	-6.76 [1.84]	-9.97 [2.44]*	-8.15 [2.2]*	-6.27 [1.3]	-4.73 [0.75]	-3.71 [0.49]	-1.49 [0.18]	0.62 [-0.07]
Minnes8	-6.36 [1.72]	-8.71 [2.15]*	-6.61 [1.68]	-4.2 [0.81]	-2.45 [0.37]	-1.43 [0.18]	0.56 [-0.07]	2.34 [-0.26]
Minnes9	-7.41 [1.67]	-6.9 [1.49]	-3.55 [0.72]	0.27 [-0.04]	2.66 [-0.36]	3.83 [-0.46]	5.43 [-0.61]	6.55 [-0.7]
Minnes10	-3.95 [1.18]	-6.16 [1.76]	-4.52 [1.2]	-2.05 [0.4]	-0.34 [0.05]	0.56 [-0.07]	2.32 [-0.27]	3.9 [-0.43]
Minnes11	-5.7 [1.42]	-4.97 [1.17]	-1.88 [0.39]	1.95 [-0.31]	4.28 [-0.58]	5.3 [-0.65]	6.67 [-0.76]	7.6 [-0.82]
Minnes12	-6.02 [1.67]	-10.03 [2.55]*	-8.51 [2.45]*	-7.08 [1.56]	-5.71 [0.93]	-4.69 [0.62]	-2.32 [0.28]	0.03 [0]
Minnes13	-8.4 [1.47]	-8.92 [1.41]	-4.17 [0.72]	-0.6 [0.09]	1.38 [-0.19]	2 [-0.26]	2.95 [-0.37]	3.24 [-0.4]
SVS	-10.1 [1.43]	-5.94 [0.99]	-1.73 [0.29]	2.43 [-0.35]	4.75 [-0.6]	5.57 [-0.64]	6.91 [-0.75]	7.97 [-0.83]
SVS-TVP	-10.59 [1.68]	-5.83 [1.1]	-1.75 [0.31]	2.36 [-0.33]	4.71 [-0.57]	5.47 [-0.61]	6.75 [-0.72]	7.91 [-0.81]
SVS-SB	-9.44 [1.86]	-8.28 [1.74]	-5.33 [1.1]	-2.35 [0.38]	-0.39 [0.05]	0.5 [-0.06]	2.2 [-0.25]	3.89 [-0.41]
SVS-LAS	-10.46 [1.63]	-7.06 [1.29]	-3.29 [0.61]	0.48 [-0.07]	2.7 [-0.36]	3.52 [-0.42]	4.98 [-0.57]	6.15 [-0.67]

**Table 11.3:** Output - root mean square errors and Diebold and Mariano statistics (average 2000Q1-2011Q4). Nonlinear DSGE model absolute RMSE and competing models relative RMSE with corresponding DM statistics in parenthesis. \*\* significant in favour of Nonlinear DSGE. \* significant in favour of competing model. Model acronyms described in footnote 37.

### 11.5.3 Interest rates

In the lower panel of figure (11.2), we note that the forecast errors for interest rates are relatively small, as there are relatively few changes to the interest rates. As such, this is the only case where the random-walk forecast is able to compete with the other models. When forecasting this variable, the DSGE models perform relatively poorly, although the difference is extremely small.

An evaluation of the selected models ability to forecast  $i_t$  is contained in table (11.4). In this case, most of the relative-root-mean squared errors would indicate that competing models would often generate a lower forecasting error. However, with the exception of one of the BVAR models, these improvements are usually insignificant.<sup>41</sup>

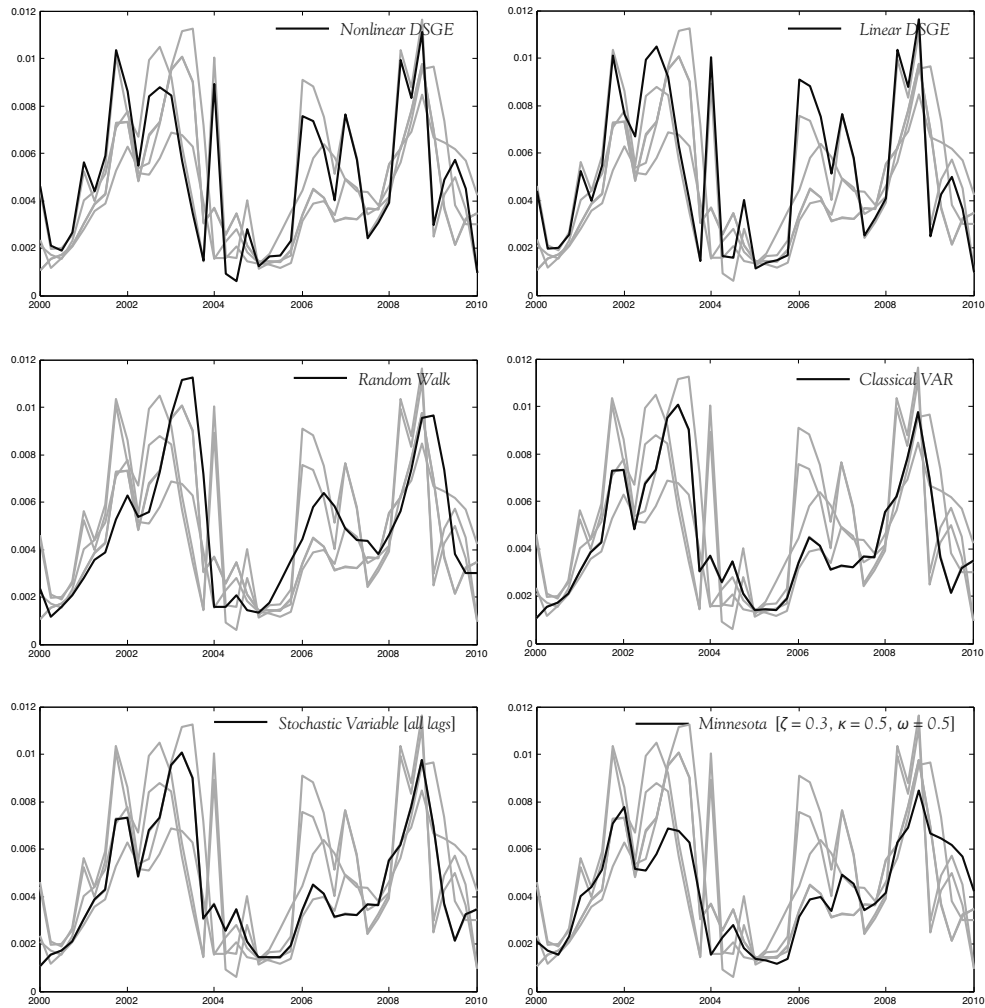
When comparing the results for the nonlinear DSGE model with its linear counterpart, we note that the linear model appears to provide forecasts that are significantly superior at the one step-ahead horizon. However, as the horizon increases the nonlinear model starts to perform significantly better, particularly for six, seven and eight step-ahead forecasts. In addition, when we compare the individual forecasts that were generated for each point in time, we note that the results for the two and four step-ahead horizon are extremely similar, while the results at the eight step-ahead horizon would indicate that the nonlinear model is clearly superior.<sup>42</sup>

When considering the results of the root-mean squared-errors at each point in time, we note once again that the nonlinear DSGE model would appear to detect changes relatively quickly. However, as the cycle continues, the relative strength of the reduced-form models becomes more apparent. Figure (11.5) would suggest that during the extended business cycle, running up to the Global Financial Crisis, the reduced-form models provided better forecasts, while the DSGE model picked up the first signs of relative stability earlier than the other models.

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<sup>41</sup>One of the BVAR models that makes use of a Minnesota prior is able to generate significantly better forecasts when the horizon is at least a year. One reason why so many of these improvements are insignificant is that the forecasting errors for  $i_t$  are mostly extremely small for all models. As such, an improvement of 0.01 over 0.11 generates quite a large relative root-mean squared error, which in most cases is insignificant. The small forecasting errors can largely be attributed to the high degree of persistence in this variable. For example, when the parameters in a Taylor rule are estimated for the South African economy, the smoothing coefficient (which in this case is  $\phi_i$ ) is usually above 0.9. See, Alpanda *et al.* (2010b) for further details on the values of the estimated Taylor rule coefficients in structural macroeconomic models.

<sup>42</sup>On 16 occasions the nonlinear DSGE model generates significantly better forecasts (as measured by the Diebold-Mariano statistic), when considering the eight step-ahead horizon. This contrasts with the 9 occasions where the linear model is responsible for improvements that are statistically significant.



**Figure 11.5:** Interest Rate forecasts over time (average 1 through 8 step-ahead root-mean squared-error)

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	1 step	2 step	3 step	4 step	5 step	6 step	7 step	8 step
Nonlinear	0.0019	0.0032	0.0045	0.0055	0.0064	0.0071	0.0074	0.0075
Linear	-3.36 [2.4]*	-0.97 [0.49]	1.03 [-0.5]	2.7 [-1.16]	4.57 [-1.66]	6.3 [-2.00]**	8.5 [-2.34]**	10.91 [-2.56]**
RW	-0.21 [0.01]	5.01 [-0.24]	0.6 [-0.04]	-3.26 [0.27]	-6.32 [0.59]	-7.74 [0.77]	-6.7 [0.68]	-3.36 [0.35]
VAR	-25.49 [1.61]	-10.34 [0.66]	-9.26 [0.76]	-10.95 [1.18]	-12.55 [1.62]	-13.95 [1.92]	-14.64 [1.97]	-13.62 [1.79]
BVAR	-25.5 [1.61]	-10.4 [0.67]	-9.34 [0.77]	-11.04 [1.18]	-12.58 [1.62]	-13.96 [1.93]	-14.58 [1.97]	-13.55 [1.79]
Minnes1	-20.5 [1.22]	-6.92 [0.4]	-8.05 [0.59]	-11.01 [1.03]	-13.27 [1.44]	-14.74 [1.72]	-14.96 [1.75]	-13.07 [1.52]
Minnes2	-17.96 [1.02]	-5.65 [0.31]	-7.38 [0.52]	-10.48 [0.94]	-12.96 [1.35]	-14.43 [1.6]	-14.28 [1.6]	-12.03 [1.35]
Minnes3	-12.57 [0.67]	-2.5 [0.13]	-5.39 [0.36]	-8.93 [0.77]	-11.83 [1.17]	-13.4 [1.41]	-12.95 [1.38]	-10.38 [1.12]
Minnes4	-0.92 [0.05]	2.77 [-0.14]	-2.83 [0.19]	-7.53 [0.63]	-11.25 [1.07]	-13.18 [1.32]	-12.65 [1.28]	-9.94 [1.03]
Minnes5	-7.27 [0.38]	0.51 [-0.03]	-3.4 [0.22]	-7.31 [0.61]	-10.47 [1.01]	-12.09 [1.23]	-11.45 [1.18]	-8.65 [0.9]
Minnes6	-0.88 [0.04]	2.8 [-0.14]	-2.81 [0.19]	-7.51 [0.63]	-11.23 [1.07]	-13.16 [1.32]	-12.63 [1.28]	-9.92 [1.03]
Minnes7	-24.09 [1.42]	-10.31 [0.61]	-9.79 [0.76]	-11.5 [1.17]	-13.17 [1.59]	-14.35 [1.85]	-14.39 [1.84]	-12.7 [1.61]
Minnes8	-20.27 [1.12]	-8.16 [0.45]	-8.64 [0.63]	-10.68 [1.03]	-12.67 [1.43]	-13.82 [1.65]	-13.39 [1.61]	-11.17 [1.35]
Minnes9	-7.74 [0.4]	-1.78 [0.09]	-5.46 [0.38]	-9.02 [0.81]	-12.07 [1.24]	-13.66 [1.47]	-13.08 [1.42]	-10.53 [1.16]
Minnes10	-16.97 [0.91]	-5.96 [0.32]	-7.15 [0.51]	-9.46 [0.88]	-11.66 [1.26]	-12.8 [1.47]	-12.1 [1.4]	-9.57 [1.11]
Minnes11	-7.68 [0.4]	-1.55 [0.08]	-5.31 [0.37]	-8.95 [0.8]	-12.05 [1.22]	-13.66 [1.46]	-13.05 [1.41]	-10.47 [1.15]
Minnes12	-25.49 [1.61]	-10.35 [0.66]	-9.27 [0.76]	-10.96 [1.18]	-12.56 [1.62]	-13.96 [1.92]	-14.64 [1.97]	-13.61 [1.79]
Minnes13	-10.95 [0.73]	-12.11 [0.92]	-13.9 [1.45]	-15.83 [2.03]*	-17.96 [2.46]*	-19.41 [2.68]*	-19.88 [2.6]*	-18.97 [2.37]*
SVS	-20.87 [1.25]	-7.4 [0.43]	-8.68 [0.64]	-11.75 [1.11]	-14.06 [1.54]	-15.61 [1.83]	-15.9 [1.86]	-14.11 [1.65]
SVS-TVP	-18.49 [1.08]	-5.02 [0.28]	-6.51 [0.46]	-10.89 [0.95]	-14.3 [1.39]	-15.54 [1.61]	-15.32 [1.6]	-13.14 [1.35]
SVS-SB	-21.04 [1.24]	-7.28 [0.41]	-8.3 [0.6]	-11.21 [1.02]	-13.43 [1.41]	-14.69 [1.64]	-14.58 [1.63]	-12.47 [1.38]
SVS-LAS	-21.25 [1.26]	-7.83 [0.45]	-8.86 [0.66]	-11.75 [1.12]	-13.99 [1.55]	-15.47 [1.84]	-15.71 [1.87]	-13.93 [1.65]

**Table 11.4:** Interest rates - root mean square errors and Diebold and Mariano statistics (average 2000Q1-2011Q4). Nonlinear DSGE model absolute RMSE and competing models relative RMSE with corresponding DM statistics in parenthesis. \*\* significant in favour of Nonlinear DSGE. \* significant in favour of competing model. Model acronyms described in footnote 37.



## 11.6 Conclusion

The results suggest that the forecasting performance of the nonlinear DSGE model is at least comparable, and in many cases superior, to that of an equivalent linear model for South African macroeconomic data. The improvements are statistically significant at the medium to longer horizons for inflation and interest rates, and at shorter horizons for output growth. Hence, given the improved out-of-sample fit of the nonlinear DSGE model (over its linear counterpart), these results suggest that there are important nonlinear features (or relatively large departures from the steady state) in the underlying data generating process of this emerging market economy.

This finding is in contrast to the result of Pichler (2008), which suggests that the incorporation of nonlinear features in the model solution does not improve upon the out-of-sample fit of such models, when applied to data from the United States economy. Indeed, our results suggest that when seeking to make use of a structural macroeconomic model to inform policy in an emerging market economy, one should possibly seek to incorporate potentially important nonlinearities, or the effect of large shocks, in the model solution.

In addition, the results of this investigation also suggest that the nonlinear DSGE model is also able to improve upon the forecasting performance of most reduced-form vector autoregressive and random-walk models. However, there are a number of instances where certain BVAR models may provide significant improvements (when forecasting output and interest rates). This usually occurs at the shorter horizon when forecasting output. When forecasting inflation at most horizons and output at longer horizons, the nonlinear DSGE model is often superior. The results also suggest that the nonlinear DSGE forecasts may possibly be attributed to their ability to detect changes in the cycle at a relatively early point, when compared with the other reduced-form models.

# Part V

## Summary

## Chapter 12

### Summary

## 12.1 Summary

This dissertation considered the theoretical construction and use of Dynamic Stochastic General Equilibrium (DSGE) models for South African macroeconomic data. The first two chapters seek to provide some perspective, by incorporating a brief review of the stylised features of the data and the early use of reduced-form macroeconometric models that were used to analyse the business cycle. The various shortcomings of this approach lead to a new direction in the development of macroeconometric models, which were based on micro-founded theoretical structures that describe the invariant behaviour of economic agents. Therefore, it has been suggested that these models satisfy the Lucas (1976) critique, which implies that they could be used to investigate the effects of a change in policy.

To describe the development of theoretical macroeconometric models, chapter three provides details on the endeavours of real business cycle (RBC) theorists. It includes an overview of the work of Kydland and Prescott (1982), Prescott (1986), King *et al.* (1988a), and Cooley and Prescott (1995), who were largely responsible for the developing the initial theoretical models that were used to analyse various features of the data. One of the important features of these models is that the representative firm is characterised as perfectly competitive, and as such inflation would adjust instantaneously to any monetary policy shock. The result of this condition is that changes to monetary policy would not influence the real variables in the model, which is contrary to what is observed in the data.

The inclusion of rigidities in the pricing mechanism allowed for the characterisation of a firm that is monopolistically competitive, following the work of Galí (2002). This additional feature of the model is consistent with the view of the New Keynesian theorists who emphasise the importance of rigidities in the economy. In addition, this condition facilitated an explicit role for monetary policy as the nominal interest rate would influence the behaviour of the real economic variables in the model. Additional features of the model that have facilitated a better description of economic behaviour (which is consistent with the stylised features of the business cycle) include the specification of habits in consumption and money in the utility function. A description of these features is included in chapter four.

The final chapter in part one considers the manner in which the New Keynesian model may be extended to incorporate features that characterise a small open-economy. It includes a discussion of two methods that may be used to close the open economy features of the model. The first of these makes use of the risk-sharing conditions and the second makes use of a debt-elastic risk-premium. Both of these conditions were incorporated in the RBC model of

Schmitt-Grohe and Uribe (2003), which was calibrated to describe the essential features of the United States economy. A detailed analysis of the importance of these conditions, when applied to South African data, is undertaken in chapter nine of this dissertation.

The second part of this dissertation considers the properties of South African macroeconomic data that may be incorporated in the model. Chapter six initiates this part of the thesis and contains an investigation into the methods that may be used to identify the respective trend and the cyclical components of economic output. The use of the Hodrick-Prescott filter for the extraction of the business cycle is then considered in greater detail, where we make use of various methods to identify an optimal smoothing coefficients for this filter. The results of this investigation suggest that there is very little difference between the various measures of the business cycle that are provided by the filter. As such, the standard smoothing coefficient is utilised for the purposes of identifying the South African business cycle. This chapter also includes a description of the transformations that may be applied to the other variables in the model.

Chapter seven considers the identification of structural breaks in the first two moments of the data. After making use of a stochastic volatility model with a time-varying mean and a non-Gaussian smoothing algorithm that allows for jumps in either the mean or the volatility of each variable, we are able to identify periods where a significant change may have occurred. The results of this investigation are then compared to those of multivariate models, before we conclude that the most significant structural break in many of the variables occurred between 1986 and 1987, which was at the height of apartheid. This is used to establish the starting date for the data that is utilised in part 3.

In the final chapter of this part we seek to identify a consistent measure of core inflation, which seeks to account for underlying inflationary pressure. When considering the effects of inflation, policy-makers often refer to the behaviour of core-inflation, and it would be possible to make use of this variable in the model if it is appropriately identified. The various techniques that are used to derive a measure of core inflation in this chapter include a number of trimmed means estimates, several dynamic factor models and a selection of wavelet decompositions. After comparing the in-sample and out-of-sample results, the wavelets estimate was found to provide a most suitable measure of this important variable.

The third part of this dissertation makes use of the dataset that was identified in part two, which is then applied to a DSGE model that incorporates features of a small open-economy, which were described in part one. The model is estimated with the aid of Bayesian techniques in chapter nine and the posterior parameter estimates are discussed in relation to previous findings from macroeconomic models that were applied to South African data. There-

after, the results of impulse response functions are considered to ensure that the effects of the shocks in the model are consistent with the stylised features of the macroeconomic data. This chapter also considers the application of the different methods that may be used to close the open-economy features in the model. When using the risk-sharing condition, we find that it is not possible to include the nominal exchange rate as an observed variable, as the volatility in the nominal exchange rate is so much greater than the difference in the demand shocks between the domestic and foreign economy. However, in the case of the model that makes use of debt-elastic risk-premium conditions we are able to include the exchange rate as an observed variable, where the shocks to the exchange rate populate the term for the stochastic risk-premium. When comparing the historical variance decompositions of the two models, it is noted that the model with the risk-sharing condition would suggest that economy behaves similar to that of a closed-economy, where external shocks have a very small effect on each of the key observed variables. In contrast with these findings, the historical variance decompositions of the model that makes use of the debt-elastic risk-premium suggest that the sum of the risk-premium and external shocks influence most of the key observed variables. In this case, the use of the debt-elastic risk-premium would allow for the variables in the model to be influenced by events in the foreign country, which would imply that it behaves as if it were an open-economy model.

Chapter ten then proceeds to make use of a policy investigation that seeks to identify optimal values for the parameters that define the central banks reaction function. Such a policy investigation is consistent with the premise of Lucas (1980), and may be employed when the parameters are structural. After considering the effect on economic volatility of a change in each of the individual parameters in the policy reaction function (with the aid of a grid-search technique), an optimisation routine is employed to identify the joint parameter values that would minimise aggregate economic volatility in all of these variables. The results of this investigation suggest that the central bank could reduce economic volatility with a slightly stronger response to changes in the rate of inflation. To ascertain the current efficiency of monetary policy and whether there is scope for a change in policy to result in lower volatility in both output and inflation, we construct an efficiency curve that follows the methodology of Taylor (1979). These results suggest that the central bank could reduce output and inflation volatility by a relatively large margin.

The final part of this dissertation considers more recent advances that have been applied to DSGE models for the South African economy. It includes an example of a nonlinear DSGE forecasting model that is solved with the second-order perturbation technique of Schmitt-Grohé and Uribe (2004). The values for the unobserved variables are then generated with the aid of a nonlinear particle filter, which is also applied to obtain values for the respective forecasts. Most of the parameters in this model are estimated using maximum likelihood

techniques. For comparative purposes, we include an equivalent linear model that makes use of a first-order perturbation technique and a Kalman filter. When comparing the parameter estimates of these two models, it is noted that the size of the shocks in the nonlinear model are much larger, which is consistent with theory. In addition, the forecasts of these models are compared to a wide selection of vector autoregressive (VAR) models that are estimated with classical and Bayesian techniques. The Bayesian VAR models include those that employ techniques for stochastic-variable-selection, time-varying parameters, endogenous structural breaks and various forms of prior-shrinkage, where the Minnesota prior is included as a special case. The models are applied to the South African macroeconomic data for an initial in-sample period of 1960Q1 to 1999Q4. This data is used to generate the first eight-step ahead forecast, before the models are then estimated recursively, by extending the in-sample period by a quarter (to generate successive forecasts over the out-of-sample period, 2000Q1 to 2011Q4). The results of this exercise suggest that the forecasting performance of the nonlinear DSGE model is almost always superior to that of the linear counterpart; particularly over longer forecasting horizons. In addition, the nonlinear DSGE model would also appear to outperform the selection of VAR models in most cases.

In terms of future directions of this research, the indications from the final chapter would appear to indicate that nonlinear DSGE model may be used to good effect in an emerging market. To extend this model to allow for open-economy features, it is going to be necessary to employ recent developments, such as those in Andreasen *et al.* (2014) or Kim *et al.* (2008), to prevent the model from being evaluated in an indeterminate parameter space. In addition, the use of Bayesian techniques may also provide a number of advantages in this regard, although they generally require additional computational time (which is already substantial). As most of the computational time is spent evaluating the likelihood function with the aid of a nonlinear filter, it would also be worthwhile to consider the use of other sequential Monte-Carlo methods, such as those of DeJong *et al.* (2013). After incorporating many of these methods into a single model, it may be possible to estimate a nonlinear small open-economy model, where one would hope to observe significant improvements in forecasting performance.

There are also other nonlinear methods that may be employed within a DSGE framework, such as those that characterise regime-switching behaviour. Examples of these models have been included in Farmer *et al.* (2011) and when applied to South African data, we have obtained interesting (unpublished) results with the use of a more elaborate regime-switching small open-economy model. This research project could be extended to consider the use of a regime-switching DSGE framework in a forecasting application. Such an extension may incorporate an evaluation of the forecasting densities and the

methods that may be used to evaluate such densities.<sup>1</sup> In addition, when considering the existence of regime-switching behaviour one could also move onto topics that consider changes into the underlying dynamics of the models that may have implications for whether or not the parameters in the model are indeed structural. An interesting line of research in this regard is considered in Fernández-Villaverde and Rubio-Ramírez (2008), among others.

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<sup>1</sup>While point estimates of various forecasts are of importance, it is also important to evaluate the confidence with which these forecast are made.



# Appendices

# Appendix A

## Additional Details

## A.1 Mathematical properties

### A.1.1 Properties of Perpetuities

Consider the expression for a particular variable,  $x_t$ ,

$$X_t = \sum_{k=0}^{\infty} (\xi)^k x_t$$

Provided that  $\xi < 1$  we will place less weight on subsequent values, such that  $\xi^\infty \sim 0$ . In addition, as long as the values of  $x_t$  do not grow over time we can write,

$$\begin{aligned} X_t &= x_t + \xi^1 x_t + \xi^2 x_t + \xi^3 x_t + \dots + \xi^\infty x_t \\ &= x_t(1 + \xi^1 + \xi^2 + \xi^3 + \dots + \xi^\infty) \end{aligned}$$

Multiplying both sides by  $\xi$ ,

$$X_t \xi = x_t(\xi^1 + \xi^2 + \xi^3 + \xi^4 + \dots + \xi^{\infty+1})$$

Now taking  $X_t - X_t \xi$  we have,

$$\begin{aligned} X_t - X_t \xi &= x_t(1 - \xi^{\infty+1}) \\ X_t &= \frac{x_t(1 - \xi^{\infty+1})}{1 - \xi} \end{aligned}$$

where  $\xi^{\infty+1} \sim 0$  we have,

$$X_t = \frac{x_t}{1 - \xi}$$

### A.1.2 Properties of Infinite Summation Operators

A relatively simple way of removing an infinite sum of the variable  $x_{t+k}$ , could be derived using the properties of difference equations,

$$X_t = \sum_{k=0}^{\infty} (\xi)^k x_{t+k}$$

where  $X_t$  and  $X_{t+1}$  could be written as,

$$\begin{aligned} X_t &= x_t + \xi^1 x_{t+1} + \xi^2 x_{t+2} + \xi^3 x_{t+3} + \dots \\ X_{t+1} &= x_{t+1} + \xi^1 x_{t+2} + \xi^2 x_{t+3} + \xi^3 x_{t+4} + \dots \end{aligned}$$

Note that after multiplying  $X_{t+1}$  by  $\xi^1$ , we can summarise all the terms that fall on the right hand-side of  $X_t$ , with the exception of  $x_t$ . Hence we could summarise this expression as,

$$X_t = x_t + \xi^1 X_{t+1}$$

## A.2 Data Description

### A.2.1 Trends and Structural Changes in South African Macroeconomic Data - Data Description

Description	Acronym	Code	Trans.
Gross domestic product at market prices	gdp	KBP6006D	1
Final consumption expenditure by households: Total	cons	KBP6007D	1
Final consumption expenditure by households: Durable goods	cdur	KBP6050D	1
Final consumption expenditure by households: Semi-durable goods	csdu	KBP6055D	1
Final consumption expenditure by households: Non-durable goods	cndu	KBP6061D	1
Final consumption expenditure by households: Services	cser	KBP6068D	1
Final consumption expenditure by general government	gov	KBP6008D	1
Gross fixed capital formation	gfcf	KBP6009D	1
Gross fixed capital formation: Residential buildings - Total	gfcrr	KBP6110D	1
Gross fixed capital formation: Non-residential buildings - Total	gfcnr	KBP6114D	1
Change in inventories	inv = inv / gdp	KBP6010D	2
Gross domestic expenditure	gde	KBP6012D	1
Exports of goods & services	exp	KBP6013D	1
Imports of goods & services	imp	KBP6014D	1
Gross value added at basic prices of primary sector	gvap	KBP6630D	1
Gross value added at basic prices of secondary sector	gvas	KBP6633D	1
Gross value added at basic prices of tertiary sector	gvat	KBP6637D	1
Gross value added at basic prices of all industries	gva	KBP6645D	1
Total employment in the non-agricultural sectors	emp	KBP7009Q	2
Total remuneration per worker in the non-agric. sector	wage	KBP7013L	1
Total consumer prices (Metropolitan areas)	cpi	P0141.1*	1
Deflator: Gross domestic product at current and market prices	$\text{def} = \frac{\text{gdp}_{\text{[nom]}}}{\text{gdp}_{\text{[real]}}}$	KBP6006L	1
Manufacturing : Volume of production of non-durable goods	prn	KBP7084N	1
Manufacturing : Volume of production of durable goods	prdu	KBP7083N	1
Manufacturing : Total volume of production	prod	KBP7085N	1
Manufacturing: Labour productivity	lpr	KBP7079L	1
Manufacturing: Unit labour costs	lct	KBP7080L	1
Notice deposits with clearing banks : 32 days	noti	KBP1414M	2
Discount rates on 91-day Treasury Bills	tbil	KBP1405W	2
Yield on loan stock traded on the bond exchange: Government stock	govs	KBP2003M	2
Yield on loan stock traded on the bond exchange: Eskom stock	esk	KBP2004M	2
Foreign exchange rate : SA cent per USA dollar (R1 = 100 cents)	exch	KBP5339M	1

**Table A.1:** Data sources, acronyms, transformations and description

In the above table the transformation codes refer to:

1. First difference of the logarithm.
2. First difference.

\* Combination of month-on-month inflation rates for the series that ended in December 2008 (CPI: All items - Index 2005=100), and the new series that was measured from January 2008 (CPI: Headline Inflation - Index 2005=100). Data provided by Statistics South Africa.

## A.3 Additional Results

### A.3.1 Measures of Core Inflation

	Actual	DFM		Wavelet		Trim - Abs		Trim - Asym	
	RMSE	RRMSE	DM	RRMSE	DM	RRMSE	DM	RRMSE	DM
2003M1	7.13	0.31	-0.4	37.6	-3.3	-28.66	3.45	-32.64	3.37
2003M2	6.83	12.92	-2.99	-30.26	3.04	-28.77	3.55	-28.66	3.59
2003M3	6.58	19.27	-3.67	-52.8	3.19	-19.9	3.82	-25.52	3.63
2003M4	6.94	12.14	-3.25	-29.08	3.9	-25.67	4.87	-18.89	5.2
2003M5	6.31	19.85	-4.16	-23.02	3.66	-21.78	5.23	-6.63	6.95
2003M6	6.21	16.87	-5.01	-41.78	4.29	-21.07	5.55	-4.29	6.36
2003M7	5.82	11.21	-5.48	-38.58	4.47	-18.39	6.11	-2.33	6.92
2003M8	4.9	15.78	-7.91	-60.29	5.55	-9.31	4.55	15.18	-7.37
2003M9	5.12	3.55	-9.32	-57.78	7.31	-13.68	6.2	0.48	-0.77
2003M10	4.33	3.7	-3.51	-62.63	7.41	-14.68	5.08	8.25	-8.25
2003M11	2.36	43.54	-16.68	-49.12	3.87	-4.58	1.12	88.53	-21.37
2003M12	0.79	197.96	-7.19	160.31	-1.98	95.45	-4.43	415.73	-9.79
2004M1	0.76	122.03	-4.36	240	-2.3	91.5	-3.52	327.25	-6.98
2004M2	0.69	88.52	-3.44	277.61	-2.43	21.89	-1.75	279.79	-5.56
2004M3	0.88	45.36	-4.13	124.01	-1.75	9.22	-0.3	256.99	-6.92
2004M4	0.6	71.57	-2.04	248.78	-2.74	44.19	-1.65	376.21	-6.05
2004M5	0.74	-7.93	0.27	195.24	-3.36	57.15	-2.91	192.6	-3.82
2004M6	0.66	-9.59	0.45	258.18	-3.94	119.93	-3.99	182.92	-3.5
2004M7	0.75	4.36	-0.15	192.99	-4.27	168.15	-4.81	134.15	-3.04
2004M8	0.62	16.84	-0.75	-6.35	1.21	139.43	-4.06	53.6	-1.48
2004M9	0.89	-23.89	2.11	14.95	-1.73	129.01	-8.13	-36.57	1.35
2004M10	0.87	-14.71	2.43	136.92	-4.73	157.69	-8.68	-31.53	1.41
2004M11	0.49	36.76	-1.22	152.78	-3.84	219.48	-5.17	19.62	-1.21
2004M12	1.02	-23.7	1.67	138.04	-3.47	55.18	-2.22	-52	3.7
2005M1	0.47	97.47	-1.68	710.49	-2.9	292.44	-5.03	-6.93	0.33
2005M2	0.36	112.57	-1.71	17.21	-1.1	210.2	-3.6	-0.77	0.06
2005M3	0.41	59.07	-1.17	332.08	-4.11	216.17	-5.59	8.61	-0.92
2005M4	0.41	77.5	-1.77	83.31	-2.19	85.07	-2.18	31.43	-1.21
2005M5	0.4	106.13	-1.51	26.9	-1.22	77.59	-2.17	64.35	-1.38
2005M6	0.5	65.18	-1.85	20.04	-1.77	74.71	-1.9	64.99	-1.32
2005M7	0.71	-3.97	0.19	13.53	-1.3	76.94	-5.8	62.66	-4.18
2005M8	0.54	23.18	-1.09	58.98	-1.26	125.13	-2.52	79.68	-1.66
2005M9	0.64	-2.4	0.11	-6.67	0.45	86.68	-1.96	83.24	-1.89
2005M10	0.77	-25.98	2.42	202.84	-4.49	55.68	-1.25	9.63	-0.33
2005M11	0.67	-25.73	2.71	94.87	-3.62	153.78	-2.33	78.65	-2.26
2005M12	1.02	-55	2.18	3.18	-1.2	65.72	-4.44	59.1	-6.45

**Table A.2:** Relative root-mean squared Error and Diebold and Mariano statistics (2003M1-2012M12)

	Actual	DFM		Wavelet		Trim - Abs		Trim - Asym	
	RMSE	RRMSE	DM	RRMSE	DM	RRMSE	DM	RRMSE	DM
2006M1	0.79	-25.86	2.04	92.83	-2.61	161.3	-3.55	58.64	-4.04
2006M2	0.99	-49.49	2.84	35.06	-2.62	138.01	-4.14	90.22	-5.29
2006M3	0.95	-54.89	3.29	27.07	-3.1	131.65	-4.45	136.99	-6.54
2006M4	1.59	-54.51	4.06	41.44	-3.95	68.55	-9.14	49.55	-12.64
2006M5	1.95	-43.04	3.71	38.54	-3.93	41.38	-9.58	34.62	-22.57
2006M6	1.55	-26.04	1.98	4.27	-6.42	81.26	-8.61	70.12	-22.01
2006M7	1.21	-42.34	1.74	-48.89	1.88	93.51	-7.73	95.71	-16.61
2006M8	1.27	-51.42	2.13	-27.65	1.5	115.6	-6.22	76.87	-10.3
2006M9	1.22	-70.56	2.47	-57.79	2.07	86.76	-6.48	71.16	-7
2006M10	1.31	-55.34	2.89	-70.52	3.15	82.17	-7.09	35.07	-6.99
2006M11	1.45	-40.35	2.57	-9.53	2.66	46.14	-5.62	37.38	-4.84
2006M12	1.74	-36	2.43	-48.43	3.11	52.58	-6.32	24.68	-4.79
2007M1	1.68	-25.52	1.81	-32.67	2.56	63.81	-5.81	60.08	-6.89
2007M2	1.92	-31.59	2.07	-56.55	2.47	67.26	-5.65	42.48	-6.05
2007M3	2.43	-27.33	2.27	-26.62	3.05	47.18	-7.51	49.13	-8.52
2007M4	2.65	-24.08	2.08	-17.45	2.7	45.45	-5.79	27.95	-6.01
2007M5	2.18	-14.06	1.6	-26.85	2.22	84.54	-4.76	38.47	-2.81
2007M6	2.71	-25.82	2.34	-77.79	2.23	59.31	-5.17	7.89	-2.85
2007M7	2.99	-24.77	2.61	-62.41	2.65	45.36	-5.06	8.85	-2.83
2007M8	3.4	-21.24	2.73	-36.77	2.93	35.5	-5.84	16.99	-3.55
2007M9	4.05	-19.78	3.2	-10.57	4.26	26.68	-6.95	12.54	-4.47
2007M10	4.1	-14.41	2.67	-1.05	1.51	26.83	-8.23	16.03	-5.07
2007M11	3.68	-7.82	1.74	-21.55	2.99	48.92	-7.8	21.92	-5.47
2007M12	3.24	-3.67	0.96	-39.86	3.58	60.51	-8.87	32.11	-5.78
2008M1	2.93	-12	2.14	-58.47	3.34	57.4	-10.37	54.89	-8.4
2008M2	2.53	207.28	-11.13	-28.07	0.89	69.58	-8.34	51.89	-7.31
2008M3	1.9	318.83	-7.74	1.74	-0.04	96.92	-6.1	109.29	-7.1
2008M4	1.22	533	-7.25	61.95	-1.14	163.19	-3.85	241.68	-6.14
2008M5	1.31	417.79	-7.66	103.45	-1.71	78.38	-2.02	117.09	-3.12
2008M6	2.71	110	-4.89	16.99	-2.24	-20.24	0.76	-4.24	0.11
2008M7	3.11	40.67	-1.78	5.85	-2.32	-29.28	1.2	-26.98	0.79
2008M8	4.75	13.01	-0.46	13.14	-2.72	-39.69	2.6	-56.65	1.92
2008M9	5.29	-10.84	0.41	14.28	-2.95	-43.95	3.04	-63.73	2.55
2008M10	5.14	-23.51	0.96	-2.65	2.04	-25.11	4.36	-60.41	2.91
2008M11	4.69	-16.14	0.64	-29.47	2.9	-37.71	3.44	-60.75	2.7
2008M12	4.04	-17.03	0.68	-11.72	3.1	-31.7	3.8	-29.31	3.99

**Table A.3:** Relative root-mean squared error and Diebold and Mariano statistics (2003M1-2012M12)

	Actual	DFM		Wavelet		Trim - Abs		Trim - Asym	
	RMSE	RRMSE	DM	RRMSE	DM	RRMSE	DM	RRMSE	DM
2009M1	3.2	-9.64	0.36	-33.69	2.93	-23.85	2.97	-47.32	2.19
2009M2	2.54	-2.89	0.08	-66.65	2.81	-2.12	0.55	-28.95	2.6
2009M3	3.79	-56.87	2.77	-63.09	4.42	7.09	-1.67	-36.26	4.8
2009M4	3.83	-66.51	4.15	-42.12	4.88	12.23	-3.37	-11.93	6.05
2009M5	3.29	-59.35	4.3	-13.09	3.24	-2.54	6.92	-10.39	3.51
2009M6	2.74	-62.03	4.16	-61.03	4.11	2.9	-9.69	6.14	-4.39
2009M7	1.84	-61.76	2.53	-72.78	2.9	41.57	-5.67	26.35	-9.43
2009M8	1.73	-47.31	1.56	-65.52	2.1	46.34	-6.57	56	-7.76
2009M9	1.66	-33.77	1.01	-58.8	1.75	34.65	-6.28	65.79	-6.43
2009M10	1.62	-39.59	1.35	-64.37	2.12	21.34	-4.09	37.87	-5.81
2009M11	1.84	-46.41	1.77	-48.82	2.67	-15.98	2.27	30.14	-6.27
2009M12	1.86	-51.86	2.17	-62.55	2.98	-2.91	0.96	35.08	-5.37
2010M1	2.19	-68.84	3.74	-25.27	4.7	-14.8	4.79	3.03	-4.77
2010M2	2.03	-74.12	5.19	46.27	-3.95	-32.72	6.23	14.94	-5.39
2010M3	2	-61.36	5.99	15.16	-7.24	-41.53	6.36	-0.04	0.16
2010M4	1.37	-45.62	5.56	-20.96	3.45	-72.17	5.18	-16.88	6.17
2010M5	1.23	-43.31	4.93	-56.12	3.96	-70.9	4.48	-50.87	5.04
2010M6	1.09	-35.41	3.72	8.96	-0.26	-63.33	4.28	-49.13	5.52
2010M7	0.81	-37.67	2.68	47.05	-0.87	6.64	-0.17	-40.68	3.82
2010M8	0.49	-29.71	1.45	261.61	-2.19	243.07	-2.93	1.7	-0.36
2010M9	0.57	-41.69	1.14	233.69	-2.52	206.17	-3.31	-0.53	0.02
2010M10	0.93	-73.78	2.07	96.16	-2.79	99.12	-4.62	-41.25	2.21
2010M11	0.91	-19.44	1.38	67.7	-2.6	139.47	-3.38	-18.33	1.7
2010M12	1.19	-14.73	1.45	43.93	-2.84	70.41	-3.56	-18.95	2.6
2011M1	1.59	-19.91	2.65	14.71	-3.17	33.6	-4.34	-38.29	3.14
2011M2	1.34	-2.46	0.93	19.18	-3.5	74.97	-3.78	-22.33	3.7
2011M3	1.71	-27.34	5.53	9.5	-3.99	23.69	-4.54	-20.92	4.86
2011M4	1.33	-40.96	3.53	-8.47	3.36	39.07	-4.45	-12.38	5.77
2011M5	1.54	-51.86	4.51	-25.56	3.98	22.52	-6.47	-1.57	0.89
2011M6	1.12	-55.36	3.35	-55.01	3.52	63.88	-5.54	-1.97	4.37
2011M7	0.85	-33.62	1.38	16.73	-0.37	58.74	-6.1	-29.34	4.38
2011M8	0.6	-50.32	2.34	91.36	-1.2	67.16	-3.29	10.73	-1.39
2011M9	0.7	-65.49	3.5	29.42	-0.59	42.02	-3.49	-12.11	1.91
2011M10	0.55	-55.63	3.13	28.11	-1.15	38.45	-1.48	6.66	-0.39
2011M11	0.62	-16.88	0.91	78	-2.43	-21.74	0.7	-21.87	0.69
2011M12	0.51	21.82	-1.48	203.64	-3.01	13.74	-2.03	-10.52	0.39
2012M1	1	-29.01	1.9	64.57	-3.5	-57.2	2.74	-45.77	3.16

**Table A.4:** Relative root-mean squared error and Diebold and Mariano statistics (2003M1-2012M12)

	Actual	DFM		Wavelet		Trim - Abs		Trim - Asym	
	RMSE	RRMSE	DM	RRMSE	DM	RRMSE	DM	RRMSE	DM
1 step	0.62	139.03	-3.18	7.54	-1.2	132.56	-5.6	153.56	-5.17
2 step	1.09	87.61	-2.87	-9.64	1.17	50.17	-3.72	65.99	-4.21
3 step	1.48	65.5	-2.64	-17.55	2.01	26.72	-2.52	37.78	-3.31
4 step	1.84	50.16	-2.4	-19.79	2.26	15.41	-1.66	22.89	-2.4
5 step	2.18	37.91	-2.16	-18.93	2.19	8.99	-1.07	13.61	-1.64
6 step	2.51	27.44	-1.9	-16.69	2.02	4.62	-0.6	6.89	-0.95
7 step	2.82	18.07	-1.56	-15.32	1.92	2.26	-0.32	2.05	-0.31
8 step	3.07	10.55	-1.1	-15.41	1.95	1.42	-0.21	-1.22	0.19
9 step	3.31	4.34	-0.51	-13.8	1.8	0.61	-0.09	-3.9	0.59
10 step	3.53	-0.93	0.11	-10.45	1.4	-0.15	0.02	-6.16	0.9
11 step	3.7	-5.66	0.69	-6.89	0.93	-0.37	0.06	-7.86	1.11
12 step	3.83	-9.75	1.13	-4.63	0.63	-0.26	0.04	-9.41	1.29

**Table A.5:** Relative root-mean squared error and Diebold and Mariano statistics (1-12 step ahead)



### **A.3.2 Nonlinear DSGE Models and their Forecasting Potential**

	8-step Ave	4-step Ave	2-step Ave	8-step	7-step	6-step	5-step	4-step	3-step	2-step	1-step
BVAR Min 1	0.44	0.41	0.36	0.41	0.41	0.41	0.41	0.40	0.37	0.36	0.29
BVAR Min 2	0.44	0.41	0.36	0.41	0.41	0.42	0.41	0.40	0.37	0.36	0.29
BVAR Min 3	0.45	0.41	0.36	0.41	0.42	0.42	0.41	0.40	0.38	0.36	0.29
BVAR Min 4	0.45	0.42	0.36	0.42	0.43	0.43	0.42	0.42	0.39	0.36	0.29
BVAR Min 5	0.45	0.41	0.36	0.42	0.42	0.42	0.42	0.41	0.38	0.36	0.29
BVAR Min 6	0.46	0.42	0.36	0.42	0.43	0.43	0.43	0.42	0.40	0.36	0.29
BVAR Min 7	0.45	0.42	0.36	0.43	0.43	0.43	0.42	0.41	0.38	0.36	0.29
BVAR Min 8	0.46	0.42	0.36	0.43	0.43	0.43	0.42	0.42	0.38	0.36	0.29
BVAR Min 9	0.47	0.43	0.36	0.44	0.44	0.44	0.44	0.43	0.40	0.36	0.29
BVAR Min 10	0.46	0.42	0.36	0.44	0.44	0.44	0.43	0.42	0.39	0.36	0.29
BVAR Min 11	0.47	0.43	0.36	0.44	0.44	0.44	0.44	0.44	0.41	0.37	0.29
BVAR Min 12	0.45	0.42	0.37	0.42	0.42	0.42	0.42	0.41	0.38	0.36	0.29
BVAR Min 13	0.45	0.42	0.36	0.42	0.43	0.43	0.42	0.41	0.38	0.36	0.29
BVAR SVS	0.45	0.42	0.36	0.42	0.43	0.43	0.42	0.41	0.39	0.36	0.29
BVAR No SVS	0.45	0.42	0.37	0.42	0.42	0.42	0.42	0.41	0.38	0.36	0.29
BVAR SVS TVP	0.45	0.42	0.36	0.42	0.42	0.43	0.42	0.41	0.39	0.36	0.29
BVAR SVS STB	0.45	0.41	0.36	0.42	0.42	0.42	0.42	0.41	0.38	0.36	0.29
BVAR SVS Lasso	0.45	0.42	0.36	0.42	0.43	0.43	0.42	0.42	0.39	0.36	0.29
Classic VAR	0.45	0.42	0.37	0.42	0.42	0.42	0.42	0.41	0.38	0.36	0.29
R Walk	0.52	0.47	0.42	0.43	0.47	0.53	0.52	0.44	0.45	0.45	0.33
Linear DSGE (Kalman)	0.45	0.39	0.35	0.41	0.43	0.43	0.41	0.37	0.35	0.35	0.28
Nonlinear DSGE (Part)	0.43	0.39	0.35	0.38	0.40	0.41	0.39	0.36	0.35	0.35	0.28

Table A.6: Consumer Inflation - sum of root mean square errors (2000Q1-2011Q4)

	8-step Ave	4-step Ave	2-step Ave	8-step	7-step	6-step	5-step	4-step	3-step	2-step	1-step
BVAR Min 1	0.75	0.44	0.26	1.06	0.96	0.85	0.74	0.60	0.46	0.30	0.18
BVAR Min 2	0.75	0.44	0.26	1.06	0.97	0.85	0.74	0.61	0.47	0.31	0.18
BVAR Min 3	0.75	0.45	0.26	1.06	0.96	0.86	0.75	0.62	0.48	0.32	0.18
BVAR Min 4	0.76	0.46	0.27	1.07	0.97	0.87	0.76	0.63	0.49	0.33	0.18
BVAR Min 5	0.76	0.46	0.27	1.07	0.97	0.87	0.76	0.63	0.49	0.33	0.18
BVAR Min 6	0.77	0.46	0.28	1.07	0.97	0.88	0.77	0.64	0.50	0.34	0.18
BVAR Min 7	0.64	0.36	0.23	0.96	0.82	0.68	0.58	0.49	0.37	0.27	0.17
BVAR Min 8	0.65	0.37	0.24	0.97	0.84	0.71	0.60	0.50	0.39	0.28	0.17
BVAR Min 9	0.68	0.40	0.24	1.01	0.88	0.75	0.64	0.53	0.41	0.29	0.17
BVAR Min 10	0.67	0.39	0.25	0.99	0.86	0.73	0.62	0.52	0.41	0.29	0.18
BVAR Min 11	0.70	0.41	0.25	1.03	0.90	0.76	0.65	0.55	0.43	0.30	0.17
BVAR Min 12	0.63	0.36	0.23	0.96	0.82	0.68	0.57	0.48	0.37	0.27	0.17
BVAR Min 13	0.65	0.38	0.24	0.95	0.84	0.71	0.61	0.51	0.40	0.27	0.16
BVAR SVS	0.70	0.41	0.25	1.04	0.91	0.78	0.66	0.56	0.43	0.30	0.16
BVAR No SVS	0.63	0.36	0.23	0.95	0.82	0.67	0.57	0.48	0.37	0.27	0.17
BVAR SVS TVP	0.69	0.40	0.25	1.03	0.90	0.77	0.65	0.54	0.42	0.29	0.16
BVAR SVS STB	0.66	0.38	0.23	0.98	0.85	0.72	0.61	0.51	0.39	0.28	0.16
BVAR SVS Lasso	0.69	0.40	0.24	1.01	0.89	0.76	0.64	0.54	0.42	0.29	0.16
Classic VAR	0.63	0.36	0.23	0.96	0.82	0.68	0.57	0.48	0.37	0.27	0.17
R Walk	0.87	0.53	0.33	1.24	1.12	1.00	0.86	0.72	0.57	0.40	0.21
Linear DSGE (Kalman)	0.77	0.60	0.48	0.94	0.87	0.82	0.76	0.69	0.65	0.56	0.37
Nonlinear DSGE (Part)	0.65	0.40	0.27	0.93	0.82	0.71	0.61	0.53	0.42	0.33	0.18

Table A.7: Output - sum of root mean square errors (2000Q1-2011Q4)

	8-step Ave	4-step Ave	2-step Ave	8-step	7-step	6-step	5-step	4-step	3-step	2-step	1-step
BVAR Min 1	0.17	0.11	0.07	0.22	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR Min 2	0.18	0.11	0.07	0.23	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR Min 3	0.18	0.12	0.07	0.23	0.23	0.21	0.19	0.16	0.13	0.09	0.04
BVAR Min 4	0.18	0.12	0.08	0.24	0.23	0.21	0.19	0.17	0.13	0.09	0.05
BVAR Min 5	0.18	0.12	0.07	0.24	0.23	0.21	0.19	0.16	0.13	0.09	0.04
BVAR Min 6	0.18	0.12	0.08	0.24	0.23	0.21	0.19	0.17	0.13	0.09	0.05
BVAR Min 7	0.17	0.11	0.06	0.23	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR Min 8	0.18	0.11	0.06	0.23	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR Min 9	0.18	0.12	0.07	0.24	0.23	0.21	0.19	0.16	0.13	0.09	0.04
BVAR Min 10	0.18	0.11	0.07	0.24	0.23	0.21	0.19	0.16	0.12	0.08	0.04
BVAR Min 11	0.18	0.12	0.07	0.24	0.23	0.21	0.19	0.16	0.13	0.09	0.04
BVAR Min 12	0.17	0.11	0.06	0.22	0.22	0.20	0.18	0.15	0.11	0.08	0.04
BVAR Min 13	0.17	0.11	0.07	0.21	0.21	0.19	0.18	0.15	0.12	0.09	0.05
BVAR SVS	0.17	0.11	0.07	0.22	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR No SVS	0.17	0.11	0.06	0.22	0.22	0.20	0.18	0.15	0.11	0.08	0.04
BVAR SVS TVP	0.17	0.11	0.07	0.22	0.21	0.20	0.18	0.15	0.12	0.09	0.04
BVAR SVS STB	0.17	0.11	0.07	0.23	0.22	0.20	0.19	0.15	0.12	0.08	0.04
BVAR SVS Lasso	0.17	0.11	0.07	0.22	0.22	0.20	0.19	0.15	0.12	0.08	0.04
Classic VAR	0.17	0.11	0.06	0.22	0.22	0.20	0.18	0.15	0.11	0.08	0.04
R Walk	0.20	0.13	0.08	0.30	0.25	0.23	0.21	0.17	0.14	0.10	0.05
Linear DSGE (Kalman)	0.21	0.13	0.08	0.28	0.26	0.25	0.22	0.18	0.14	0.10	0.05
Nonlinear DSGE (Part)	0.20	0.13	0.08	0.26	0.25	0.23	0.20	0.17	0.14	0.09	0.05

Table A.8: Interest rates - sum of root mean square errors (2000Q1-2011Q4)

	8-step		7-step		6-step		5-step		4-step		3-step		2-step	
	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other
BVAR Min1	14	3	14	4	14	4	13	6	13	5	13	7	16	11
BVAR Min2	15	3	14	4	13	4	14	6	13	5	11	7	14	12
BVAR Min3	13	3	14	4	14	4	15	5	15	5	13	7	15	12
BVAR Min4	10	5	11	3	11	4	10	6	12	6	15	9	14	12
BVAR Min5	11	3	12	2	13	3	13	5	13	5	15	9	17	12
BVAR Min6	10	5	10	6	10	7	11	7	11	7	14	11	16	14
BVAR Min7	15	4	15	5	14	4	15	6	15	6	14	8	17	14
BVAR Min8	16	3	14	4	14	3	15	5	14	5	14	9	16	13
BVAR Min9	14	4	15	4	16	4	14	6	14	6	15	7	17	12
BVAR Min10	14	3	14	3	15	3	15	5	14	5	15	9	17	13
BVAR Min11	13	5	13	4	13	4	13	5	13	5	15	8	16	12
BVAR Min12	15	3	14	5	15	4	15	6	15	6	14	8	16	13
BVAR Min13	15	3	15	4	14	3	14	5	14	5	13	8	17	14
BVAR SVS	14	3	13	3	14	3	15	5	14	5	16	9	17	12
BVAR no SVS	15	3	14	5	15	4	15	6	15	6	14	8	16	13
BVAR SVS TVP	13	3	12	3	14	3	15	5	15	5	15	8	17	13
BVAR SVS STB	14	3	13	5	14	4	15	5	13	5	13	8	17	12
BVAR SVS Lasso	14	3	13	3	14	3	14	5	15	5	14	9	17	14
Classic VAR	15	3	14	5	15	4	15	6	15	6	14	8	16	13
R Walk	17	3	18	2	18	2	19	4	18	4	23	6	28	6
Linear DSGE	12	4	9	4	9	6	8	5	8	7	11	10	15	16

Table A.9: Consumer Inflation - number of significant Diebold and Mariano statistics (2000Q1-2011Q4)

	8-step		7-step		6-step		5-step		4-step		3-step		2-step	
	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other
BVAR Min1	17	10	15	10	14	13	13	12	13	10	12	12	14	19
BVAR Min2	17	10	16	10	14	13	14	12	13	11	12	12	13	18
BVAR Min3	17	10	15	10	15	12	15	11	14	12	13	12	15	17
BVAR Min4	19	8	17	9	15	10	15	9	15	9	13	10	17	15
BVAR Min5	18	8	16	9	15	10	15	9	15	9	12	10	17	14
BVAR Min6	18	7	16	9	16	9	16	9	15	9	15	10	17	13
BVAR Min7	10	14	9	14	11	17	8	16	13	16	12	18	13	20
BVAR Min8	12	14	11	14	13	16	10	14	13	16	12	18	13	20
BVAR Min9	14	14	13	14	12	16	12	14	14	15	12	17	13	23
BVAR Min10	14	14	12	14	12	16	10	14	12	16	12	17	14	21
BVAR Min11	14	14	13	14	12	16	12	14	13	15	12	18	14	23
BVAR Min12	9	14	8	15	9	17	7	16	12	15	12	18	13	21
BVAR Min13	14	14	14	15	12	16	11	16	12	16	10	16	11	18
BVAR SVS	14	13	14	12	13	15	12	14	14	15	12	14	13	21
BVAR no SVS	9	15	8	15	9	17	7	16	12	15	11	18	12	21
BVAR SVS TVP	14	13	14	13	13	15	13	13	13	16	10	17	12	21
BVAR SVS STB	14	13	13	15	12	16	11	14	13	15	11	17	12	22
BVAR SVS Lasso	14	13	14	14	13	16	14	14	15	16	12	17	13	20
Classic VAR	9	14	8	15	9	17	7	16	12	15	12	18	12	21
R Walk	21	5	20	5	21	8	22	7	22	9	20	9	23	13
Linear DSGE	13	6	17	5	21	5	22	5	24	4	25	5	28	6

**Table A.10:** Output - number of significant Diebold and Mariano statistics (2000Q1-2011Q4)

	8-step		7-step		6-step		5-step		4-step		3-step		2-step	
	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other	Nonlinear	Other
BVAR Min1	10	18	10	15	11	15	10	16	11	17	10	17	12	21
BVAR Min2	11	16	10	15	12	15	10	16	11	16	11	16	12	21
BVAR Min3	11	15	11	14	13	15	12	16	13	16	13	16	13	20
BVAR Min4	10	15	11	13	13	15	13	16	15	15	15	16	15	19
BVAR Min5	12	15	11	14	13	15	12	16	14	15	14	18	14	20
BVAR Min6	10	15	11	13	13	15	13	16	15	15	15	16	15	19
BVAR Min7	9	18	10	15	11	16	10	17	10	17	10	18	12	22
BVAR Min8	9	17	10	14	11	15	10	16	12	16	11	18	12	22
BVAR Min9	11	15	11	14	12	15	13	16	14	15	15	18	14	20
BVAR Min10	10	16	10	13	11	15	11	17	12	16	12	18	12	23
BVAR Min11	10	15	11	13	12	15	13	16	14	15	15	18	14	20
BVAR Min12	9	19	10	16	10	16	9	17	8	18	8	21	10	24
BVAR Min13	9	17	9	15	11	15	11	14	12	16	12	18	16	19
BVAR SVS	10	18	10	15	11	16	10	17	11	17	10	17	11	21
BVAR no SVS	9	19	10	16	10	16	9	17	8	18	8	21	10	24
BVAR SVS TVP	10	17	10	13	13	14	11	16	12	16	11	16	11	21
BVAR SVS STB	10	17	10	14	11	15	10	16	11	17	10	17	11	21
BVAR SVS Lasso	10	18	10	16	11	15	10	16	11	17	10	17	11	21
Classic VAR	9	19	10	16	10	16	9	17	8	18	8	21	10	24
R Walk	12	17	11	15	13	16	14	16	14	16	15	19	14	20
Linear DSGE	16	9	14	9	12	8	12	9	11	10	11	12	14	15

Table A.11: Interest rates - number of significant Diebold and Mariano statistics (2000Q1-2011Q4)

## Appendix B

### Log-linearising with Local Approximations



## B.1 Introduction

Many economic problems require the use of approximation methods that are able to express nonlinear functional relationships with a linear representation. Possibly one of the crudest methods for approximating nonlinear functions is by way of a *local* approximation. In this case, one would make use of information about a function and its derivatives (at a local point), to construct a function that matches the properties of the function around that point. One of the most popular forms of local approximation is through the use of a Taylor series expansion. After obtaining the linear approximation of a nonlinear function we could then make use of one of the many methods that have been developed to solve systems of linear difference rational expectations equations.<sup>1</sup>

Since economists often want to derive unit-free measures to express their results in terms of elasticities or rates of substitution, they may seek to *log-linearise* an equation at a local point. Where the local point is the steady-state of a variable, this form of approximation would convert a nonlinear equation into an equation that is linear in terms of the log-deviations of the associated variables from the steady-state. For small deviations from the steady-state, log-deviations can be manipulated so that they can be interpreted as the percentage deviation from the steady-state.

## B.2 Taylor series approximation

The essential postulate of Taylor's theorem is that we can differentiate any nonlinear function  $n$ -times to form the polynomial approximation,

$$\begin{aligned} f(X_t) \approx & f(\bar{x}) + f'(\bar{x})(X_t - \bar{x}) + f''(\bar{x})\frac{(X_t - \bar{x})^2}{2!} + \dots \\ & + f^{(n)}(\bar{x})\frac{(X_t - \bar{x})^n}{n!} \end{aligned} \quad (\text{B.2.1})$$

and the resulting error will be smaller than any of the terms. The expression in (B.2.1) would represent an  $n$ th order Taylor series approximation near the value of  $\bar{x}$ .<sup>2</sup> In this course we restrict ourselves to a first-order Taylor series approximation, where,

$$f(X_t) \approx f(\bar{x}) + f'(\bar{x})(X_t - \bar{x}) \quad (\text{B.2.2})$$

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<sup>1</sup>Popular methods for solving systems of linear difference rational expectations equations include, Blanchard and Kahn (1980), Uhlig (1999), Klein (2000), and Sims (2001).

<sup>2</sup>Note that when we move from  $\bar{x}$ , the accuracy of this approximation may become rather poor and may be no better than an approximation at any other point (Judd, 1998, p. 197).

This would imply that to take a first-order Taylor series approximation of a nonlinear function all we need to do is calculate the first-order derivative of the function, which is multiplied by the deviation from the steady-state. The resulting calculation would then need to be added to the steady value. With this in mind, it is worth recalling a few important properties of derivatives,

1. *For the exponential function:*

$$\begin{aligned} f(X_t) &= \beta e^{\alpha X_t} \\ f'(\bar{x}) &= \alpha \beta e^{\alpha \bar{x}} \end{aligned}$$

2. *For the logarithmic function:*

$$\begin{aligned} f(X_t) &= \beta \log X_t \\ f'(\bar{x}) &= \frac{\beta}{\bar{x}} \end{aligned}$$

3. *For an arbitrary constant:*

$$\begin{aligned} f(X_t) &= \alpha + \beta X_t \\ f'(\bar{x}) &= \beta \end{aligned}$$

Hence by way of example, consider the following expression,

$$f(A_t) = (1 + \alpha)A_t$$

In this case, we may express the first-order derivative at point  $\bar{a}$  is,

$$f'(\bar{a}) = (1 + \alpha)$$

where the first-order Taylor series approximation would be given by,

$$f(A) \approx (1 + \alpha)\bar{a} + (1 + \alpha)(A_t - \bar{a})$$

If we assume that the variables in the model are demeaned, then  $\bar{a} = 0$ , and this expression could then be summarized as,

$$f(A_t) \approx (1 + \alpha)A_t$$

Note that in this case, the first-order Taylor series approximation of a linear expression does not differ to the initial expression, which should not come as a surprise to anyone (as the original expression is linear).

### B.2.1 Multivariate extensions

When there is more than one variable in the expression, as in the case where we have  $f(X, Y)$ , then the first-order local approximation about the points  $\bar{x}$  and  $\bar{y}$  would be,

$$f(X_t, Y_t) \approx f(\bar{x}, \bar{y}) + f'_x(\bar{x}, \bar{y})(X_t - \bar{x}) + f'_y(\bar{x}, \bar{y})(Y_t - \bar{y}) \quad (\text{B.2.3})$$

where  $f'_x$  and  $f'_y$  denote the derivatives with respect to  $x$  and  $y$ , respectively. This result may be explained by way of example, where we consider the expression,

$$f(A_t, C_t) = (1 + \alpha)A_t + C_t$$

The first-order derivatives at point  $\bar{a}$  and  $\bar{c}$  are,

$$\begin{aligned} f'(\bar{a}) &= (1 + \alpha) \\ f'(\bar{c}) &= 1 \end{aligned}$$

Such that we would be left with the Taylor series approximation,

$$f(A_t, C_t) \approx [(1 + \alpha)\bar{a} + \bar{c}] + (1 + \alpha)(A_t - \bar{a}) + (1)(C_t - \bar{c})$$

### B.3 Log-linearising

To log-linearise an expression simply take the natural logarithm of all the variables before taking a Taylor series approximation of the function. Note that if we are to apply the above result for derivatives of logarithmic functions, then the first-order Taylor series approximation of  $\log f(X_t)$  would be given as,

$$\log f(X_t) \approx \log f(\bar{x}) + \frac{f'(\bar{x})}{f(\bar{x})}(X_t - \bar{x})$$

Similarly, for the multivariate case we could derive expressions from,

$$f(X_t) = f(Y_t) \cdot f(Z_t)$$

where after taking the natural logarithm,

$$\log f(X_t) = \log f(Y_t) + \log f(Z_t)$$

In this case we can use the derivatives,

$$f'(\bar{x}) = \frac{f'(\bar{x})}{f(\bar{x})}, \quad f'(\bar{y}) = \frac{f'(\bar{y})}{f(\bar{y})}, \quad f'(\bar{z}) = \frac{f'(\bar{z})}{f(\bar{z})}$$

which would leave us with the first-order Taylor series approximations for the linear representation,

$$\log f(\bar{x}) + \frac{f'(\bar{x})}{f(\bar{x})}(X_t - \bar{x}) \approx \log f(\bar{y}) + \log f(\bar{z}) + \dots$$

$$\frac{f'(\bar{y})}{f(\bar{y})}(Y_t - \bar{y}) + \frac{f'(\bar{z})}{f(\bar{z})}(Z_t - \bar{z})$$

Note that since it is given that  $\log f(X_t) = \log f(Y_t) + \log f(Z_t)$ , we may summarize this expressions as,

$$\frac{f'(\bar{x})}{f(\bar{x})}(X_t - \bar{x}) \approx \frac{f'(\bar{y})}{f(\bar{y})}(Y_t - \bar{y}) + \frac{f'(\bar{z})}{f(\bar{z})}(Z_t - \bar{z})$$

Whilst this answer is often used when constructing models, some authors may choose to express these results in terms of percentage deviations of  $X_t$  about  $\bar{x}$ , which is given as  $(X_t - \bar{x})/\bar{x}$ . Therefore, the notation  $x_t = (X_t - \bar{x})/\bar{x}$  seeks to describe deviations from the steady state. Hence by multiplying and dividing through by  $\bar{x}$  the left-hand term could be expressed as,

$$\frac{\bar{x}}{f(\bar{x})} \frac{f'(\bar{x})}{\bar{x}} (X_t - \bar{x}) = \frac{\bar{x}}{f(\bar{x})} x_t$$

To summarize, the procedure of log-linearising an expression would require one to take the natural logarithm of the original expression, before making use of a Taylor expansion around a local point (usually the steady-state). To end off this discussion, take a look at the following examples, which include expressions for the Cobb-Douglas production function, accounting identity, capital accumulation equation, consumption Euler equation, amongst others.<sup>3</sup>

### B.3.1 Example: Production Function

Consider a Cobb-Douglas production function,  $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$ , for which we can take the natural logarithm,

$$\log Y_t = \log A_t + \alpha \log K_t + (1 - \alpha) \log N_t$$

This expression would have the derivatives,

$$f'(\bar{y}) = \frac{1}{\bar{y}}, \quad f'(\bar{a}) = \frac{1}{\bar{a}}$$

$$f'(\bar{k}) = \frac{\alpha}{\bar{k}}, \quad f'(\bar{n}) = \frac{(1 - \alpha)}{\bar{n}}$$

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<sup>3</sup>It is worth noting that Galí (2008) makes use of the method proposed by Uhlig (1999) to derive log-linear approximations of functions. In the examples that we consider, these methods are equivalent, up to an arbitrary constant. In a model that is primarily concerned with deviations from the steady-state, the inclusion of an arbitrary constant is not going to affect the results.

Hence, putting it all together, we could express the log-linear expression as,

$$\log(\bar{y}) + \frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \log(\bar{a}) + \alpha \log(\bar{k}) + (1 - \alpha) \log(\bar{n}) + \dots$$

$$\frac{1}{\bar{a}}(A_t - \bar{a}) + \frac{\alpha}{\bar{k}}(K_t - \bar{k}) + \frac{(1 - \alpha)}{\bar{n}}(N_t - \bar{n})$$

Note that the since  $\log Y_t = \log A_t + \alpha \log K_t + (1 - \alpha) \log N_t$  it will be the case that,  $\log \bar{y} = \log \bar{a} + \alpha \log \bar{k} + (1 - \alpha) \log \bar{n}$ , so we could cancel these terms to derive,

$$\frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \frac{1}{\bar{a}}(A_t - \bar{a}) + \frac{\alpha}{\bar{k}}(K_t - \bar{k}) + \frac{(1 - \alpha)}{\bar{n}}(N_t - \bar{n})$$

Now if we express  $y_t$  as a percentage deviation from the steady-state, such that,

$$y_t = \frac{(Y_t - \bar{y})}{\bar{y}}$$

and likewise for the other variables, then we are able to summarize the log-linear expression as,

$$y_t \approx a_t + \alpha k_t + (1 - \alpha) n_t$$

### B.3.2 Example: Accounting Identity (I)

Consider the accounting identity,  $Y_t = C_t$ , for which we can take the natural logarithm,

$$\log Y_t = \log C_t$$

This expression would have the derivatives,

$$f'(\bar{y}) = \frac{1}{\bar{y}} \quad , \quad f'(\bar{c}) = \frac{1}{\bar{c}}$$

Hence, putting it all together, we could express the log-linear expression as,

$$\log(\bar{y}) + \frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \log \bar{c} + \frac{1}{\bar{c}}(C_t - \bar{c})$$

Note that the since  $\log \bar{y} = \log \bar{c}$ , we could cancel these terms to derive,

$$\frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \frac{1}{\bar{c}}(C_t - \bar{c})$$

Now if we express  $y_t$  as a percentage deviation from the steady-state, such that,

$$y_t = \frac{(Y_t - \bar{y})}{\bar{y}}$$

Using similarly percentage deviations for the steady-state variables, would allow us to derive,

$$y_t \approx c_t$$

### B.3.3 Example: Accounting Identity (II)

Consider the accounting identity,  $Y_t = C_t + I_t$ , for which we can take the natural logarithm,

$$\log Y_t = \log(C_t + I_t)$$

This expression would have the derivatives,

$$f'(\bar{y}) = \frac{1}{\bar{y}}, \quad f'(\bar{c}) = \frac{1}{\bar{c} + \bar{i}}, \quad f'(\bar{i}) = \frac{1}{\bar{c} + \bar{i}}$$

Hence, putting it all together, we could express the log-linear expression as,

$$\log(\bar{y}) + \frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \log(\bar{c} + \bar{i}) + \frac{1}{\bar{c} + \bar{i}}(C_t - \bar{c}) + \frac{1}{\bar{c} + \bar{i}}(I_t - \bar{i})$$

Note that the since  $\log \bar{y} = \log(\bar{c} + \bar{i})$ , we could cancel these terms to derive,

$$\frac{1}{\bar{y}}(Y_t - \bar{y}) \approx \frac{1}{\bar{c} + \bar{i}}(C_t - \bar{c}) + \frac{1}{\bar{c} + \bar{i}}(I_t - \bar{i})$$

Now if we express  $y_t$  as a percentage deviation from the steady-state, such that,

$$y_t = \frac{(Y_t - \bar{y})}{\bar{y}}$$

To derive a similar expression for  $c_t = \frac{(C_t - \bar{c})}{\bar{c}}$ , we have,

$$\begin{aligned} \frac{(C_t - \bar{c})}{\bar{c} + \bar{i}} &= \frac{(C_t - \bar{c})}{\bar{c} + \bar{i}} \frac{\bar{c}}{\bar{c}} \\ &= \frac{(C_t - \bar{c})}{\bar{c}} \frac{\bar{c}}{\bar{c} + \bar{i}} \\ &= \frac{(C_t - \bar{c})}{\bar{c}} \frac{\bar{c}}{\bar{y}} = \frac{\bar{c}}{\bar{y}} c_t \end{aligned}$$

Using similarly percentage deviations for  $i_t$ , would allow us to derive,

$$y_t \approx \frac{\bar{c}}{\bar{y}} c_t + \frac{\bar{i}}{\bar{y}} i_t$$

### B.3.4 Example: Capital Accumulation

Consider the capital accumulation equation,  $K_{t+1} = I_t + (1 - \delta)K_t$ , for which we can take the natural logarithm,

$$\log K_{t+1} = \log (I_t + (1 - \delta) K_t)$$

This expression would have the derivatives,

$$f'(\bar{k}_{+1}) = \frac{1}{\bar{k}} \quad , \quad f'(\bar{i}) = \frac{1}{\bar{i} + (1 - \delta)\bar{k}} \quad , \quad f'(\bar{k}) = \frac{1 - \delta}{\bar{i} + (1 - \delta)\bar{k}}$$

Hence, after putting it all together we could express the log-linear expression as,

$$\begin{aligned} \log(\bar{k}) + \frac{1}{\bar{k}}(K_{t+1} - \bar{k}) &\approx \log \left( \frac{1}{\bar{i} + (1 - \delta)\bar{k}} \right) + \dots \\ &\quad \frac{1}{\bar{i} + (1 - \delta)\bar{k}}(I_t - \bar{i}) + \frac{(1 - \delta)}{\bar{i} + (1 - \delta)\bar{k}}(K_t - \bar{k}) \end{aligned}$$

Note that since  $\log K_{t+1} = \log (I_t + (1 - \delta) K_t)$ , we could cancel these terms,

$$\frac{1}{\bar{k}}(K_{t+1} - \bar{k}) \approx \frac{1}{\bar{i} + (1 - \delta)\bar{k}}(I_t - \bar{i}) + \frac{1 - \delta}{\bar{i} + (1 - \delta)\bar{k}}(K_t - \bar{k})$$

Now at the steady-state,  $\bar{k} = (\bar{i}_t + (1 - \delta) \bar{k}_t)$ , which allows for the expression,

$$\frac{1}{\bar{k}}(K_{t+1} - \bar{k}) \approx \frac{1}{\bar{k}}(I_t - \bar{i}) + \frac{1 - \delta}{\bar{k}}(K_t - \bar{k})$$

We can then express  $k_t = \frac{(K_t - \bar{k})}{\bar{k}}$  and after multiplying investment by  $\frac{\bar{i}}{\bar{i}}$ , we can derive,

$$\begin{aligned} k_{t+1} &\approx \frac{\bar{i}}{\bar{k}} \frac{(I_t - \bar{i})}{\bar{i}} + (1 - \delta)k_t \\ &\approx \frac{\bar{i}}{\bar{k}} i_t + (1 - \delta)k_t \end{aligned}$$

### B.3.5 Example: Intertemporal Consumption

The log-linear expression for  $Q_t = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$ :

Take the natural logarithm,

$$\log Q_t = \log \beta - \sigma \log E_t [C_{t+1}] + \sigma \log C_t + \log P_t - \log E_t [P_{t+1}]$$

This expression would have the derivatives,

$$f'(\bar{c}) = \frac{\sigma}{\bar{c}}, \quad f'(\bar{p}) = \frac{1}{\bar{p}}$$

It is further given that,  $\log Q_t \approx -i_t$ ,  $\log \beta \equiv \rho$  and  $\pi_t = p_{t+1} - p_t$ .

Hence, after putting it all together we could express the log-linear expression as,

$$\begin{aligned} -i_t \approx & -\rho - \log(\bar{c}) + \log(\bar{c}) - \frac{\sigma}{\bar{c}}(E_t[C_{t+1}] - \bar{c}) + \frac{\sigma}{\bar{c}}(C_t - \bar{c}) \dots \\ & + \log(\bar{p}) - \log(\bar{p}) + \frac{1}{\bar{p}}(P_t - \bar{p}) - \frac{1}{\bar{p}}(E_t[P_{t+1}] - \bar{p}) \end{aligned}$$

which may be summarised as,

$$\begin{aligned} -\frac{\sigma}{\bar{c}}(C_t - \bar{c}) \approx & -\frac{\sigma}{\bar{c}}(E_t[C_{t+1}] - \bar{c}) + i_t - \rho \dots \\ & + \frac{1}{\bar{p}}(P_t - \bar{p}) - \frac{1}{\bar{p}}(E_t[P_{t+1}] - \bar{p}) \end{aligned}$$

where the variables are expressed as deviations from steady state,

$$\begin{aligned} -\sigma c_t & \approx -\sigma E_t[c_{t+1}] + i_t - \rho + p_t - E_t[p_{t+1}] \\ c_t & \approx E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \rho - E_t[p_{t+1} - p_t]) \\ & \approx E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \rho - E_t[\pi_{t+1}]) \end{aligned}$$

### B.3.6 Example: Relative Price of Leisure

Consider the labour-supply condition,  $W_t/P_t = C_t^\sigma N_t^\gamma$ , for which we can take the natural logarithm,

$$\log W_t - \log P_t = \sigma \log C_t + \gamma \log N_t$$

This expression would have the derivatives,

$$\begin{aligned} f'(\bar{w}) &= \frac{1}{\bar{w}}, & f'(\bar{p}) &= \frac{1}{\bar{p}} \\ f'(\bar{c}) &= \frac{\sigma}{\bar{c}}, & f'(\bar{n}) &= \frac{\gamma}{\bar{n}} \end{aligned}$$

Hence, putting it all together, we could express the log-linear expression as,

$$\begin{aligned} \log(\bar{w}) + \frac{1}{\bar{w}}(W_t - \bar{w}) - \log(\bar{p}) - \frac{1}{\bar{p}}(P_t - \bar{p}) \approx & \dots \\ \sigma \log \bar{c} + \frac{\sigma}{\bar{c}}(C_t - \bar{c}) + \gamma \log \bar{n} + \frac{\gamma}{\bar{n}}(N_t - \bar{n}) \end{aligned}$$



Note that since  $\log W_t - \log P_t = \sigma \log C_t + \gamma \log N_t$ , we could cancel these terms to derive,

$$\frac{1}{\bar{w}}(W_t - \bar{w}) - \frac{1}{\bar{p}}(P_t - \bar{p}) \approx \frac{\sigma}{\bar{c}}(C_t - \bar{c}) + \frac{\gamma}{\bar{n}}(N_t - \bar{n})$$

Now if we express  $w_t$  as a percentage deviation from the steady-state, such that,

$$w_t \approx \frac{(W_t - \bar{w})}{\bar{w}}$$

and likewise for the other variables, then we are able to summarize the log-linear expression as,

$$w_t - p_t = \sigma c_t + \gamma n_t$$

### B.3.7 Example: Cost of Production

Consider the firms optimal use of labour,  $W_t/P_t = (1 - \alpha)A_tN_t^{-\alpha}$ , for which we can take the natural logarithm,

$$\log W_t - \log P_t = \log(1 - \alpha) + \log A_t - \alpha \log N_t$$

This expression would have the derivatives,

$$\begin{aligned} f'(\bar{w}) &= \frac{1}{\bar{w}}, & f'(\bar{p}) &= \frac{1}{\bar{p}} \\ f'(\bar{a}) &= \frac{1}{\bar{a}}, & f'(\bar{n}) &= \frac{\alpha}{\bar{n}} \end{aligned}$$

Hence, putting it all together, we could express the log-linear expression as,

$$\begin{aligned} \log(\bar{w}) + \frac{1}{\bar{w}}(W_t - \bar{w}) - \log(\bar{p}) - \frac{1}{\bar{p}}(P_t - \bar{p}) &\approx \dots \\ \log(1 - \alpha) + \log(\bar{a}) + \frac{1}{\bar{a}}(A_t - \bar{a}) - \alpha \log(\bar{n}) - \frac{\alpha}{\bar{n}}(N_t - \bar{n}) \end{aligned}$$

Note that since  $\log W_t - \log P_t = \log(1 - \alpha) + \log A_t - \alpha \log N_t$ , we could cancel these terms to derive,

$$\frac{1}{\bar{w}}(W_t - \bar{w}) - \frac{1}{\bar{p}}(P_t - \bar{p}) \approx \frac{1}{\bar{a}}(A_t - \bar{a}) - \frac{\alpha}{\bar{n}}(N_t - \bar{n})$$

Now if we express the variables in terms of a percentage deviation from the steady-state, then we are able to summarize the log-linear expression as,

$$w_t - p_t \approx a_t - \alpha n_t$$

### B.3.8 Example: Demand for Real Monetary Balances

Consider the demand for real monetary balances,  $\left(\frac{M_t}{P_t}\right)^{-v} = C_t^{-\sigma} [1 - Q_t]$ , for which we can take the natural logarithm,

$$-v \log M_t + v \log P_t = -\sigma \log C_t + \log [1 - Q_t]$$

This expression would have the derivatives,

$$f'(\bar{m}) = \frac{v}{\bar{m}}, \quad f'(\bar{p}) = \frac{v}{\bar{p}}, \quad f'(\bar{c}) = \frac{\sigma}{\bar{c}}$$

where we again have,  $\log Q_t \approx -i_t$ , which allows us to state,  $\log [1 - Q_t] \approx i_t$ . Hence, putting it all together, we could express the log-linear expression as,

$$-\log(\bar{m}) - \frac{v}{\bar{m}}(M_t - \bar{m}) + \log(\bar{p}) + \frac{v}{\bar{p}}(P_t - \bar{p}) \approx -\sigma \log \bar{c} - \frac{\sigma}{\bar{c}}(C_t - \bar{c}) + i_t$$

Note that the since,  $-v \log M_t + v \log P_t = -\sigma \log C_t + \log [1 - Q_t]$ , we could cancel these terms to derive,

$$\frac{v}{\bar{m}}(M_t - \bar{m}) - \frac{v}{\bar{p}}(P_t - \bar{p}) \approx +\frac{\sigma}{\bar{c}}(C_t - \bar{c}) - i_t$$

Now if we express the variables in terms of a percentage deviation from the steady-state, then we are able to summarize the log-linear expression as,

$$\begin{aligned} vm_t - vp_t &\approx \sigma c_t - i_t \\ m_t - p_t &\approx \frac{\sigma}{v} c_t - \frac{1}{v} i_t \end{aligned}$$

### B.3.9 Example: Price setting with the Calvo (1983) mechanism

Consider the price setting behaviour,

$$\Phi_t^{1-\epsilon} = \theta + (1 - \theta) [\Phi_t^*]^{1-\epsilon}$$

To log-linearise this expression we start by taking the natural logarithm of both sides. Hence for the expression,

$$\begin{aligned} \Phi_t^{1-\epsilon} &= \theta + (1 - \theta) [\Phi_t^*]^{1-\epsilon} \\ (1 - \epsilon) \log \Phi_t &= \log \{ \theta + (1 - \theta) [\Phi_t^*]^{1-\epsilon} \} \\ &= \log \{ \theta + (1 - \theta) \exp [(1 - \epsilon) \log \Phi_t^*] \} \end{aligned}$$

The steady-state derivatives may then be expressed as,

$$f'(\bar{\Phi}) = \frac{1 - \epsilon}{\bar{\Phi}} \quad \text{and} \quad f'(\bar{\Phi}^*) = \frac{(1 - \theta)(1 - \epsilon) \exp [(1 - \epsilon) \log \bar{\Phi}^*]}{\bar{\Phi}^*}$$

If we are to assume that there is no reason for prices to change at the steady-state, even for those producers that have the opportunity to change prices, then  $\bar{\Phi}^* = \bar{P}^*/\bar{P} = 1$  and  $\bar{\Phi} = \bar{P}/\bar{P} = 1$ . This enables us to write,

$$\begin{aligned} f'(\bar{\Phi}^*) &= \frac{(1-\theta)(1-\epsilon) \exp[(1-\epsilon) \log(1)]}{\bar{\Phi}^*} \\ &= \frac{(1-\theta)(1-\epsilon)}{\bar{\Phi}^*} \end{aligned}$$

Putting this all together, the log-linear expression may be given as,

$$\begin{aligned} (1-\epsilon) \log(\bar{\Phi}) + \frac{(1-\epsilon)}{\bar{\Phi}} (\Phi_t - \bar{\Phi}) \approx \dots \\ \log [\theta + (1-\theta) \exp \{(1-\epsilon) \log(\bar{\Phi}^*)\}] + \frac{(1-\theta)(1-\epsilon)}{\bar{\Phi}^*} (\Phi_t^* - \bar{\Phi}^*) \end{aligned}$$

Removing the expression,

$$(1-\epsilon) \log(\bar{\Phi}) = \log [\theta + (1-\theta) \exp \{(1-\epsilon) \log(\bar{\Phi}^*)\}]$$

Leaves us with,

$$\frac{(1-\epsilon)}{\bar{\Phi}} (\Phi_t - \bar{\Phi}) \approx \frac{(1-\theta)(1-\epsilon)}{\bar{\Phi}^*} (\Phi_t^* - \bar{\Phi}^*)$$

Substituting in the steady-state values provides the final log-linear representation,

$$(1-\epsilon)(\Phi_t - 1) \approx (1-\theta)(1-\epsilon)(\Phi_t^* - 1)$$

### B.3.10 Example: The behaviour of a New-Keynesian Firm

Given the expression for the behaviour of the firm,

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ C_{t,t+k}^{-\sigma} C_t^{\sigma} P_t P_{t,t+k}^{-1} Y_{t+k|t} \left( \Phi_t^* - \frac{\epsilon}{\epsilon-1} MC_{t+k|t}^r \Phi_{t-1,t+k} \right) \right] = 0$$

We may write this in terms of left-hand and right-hand side expressions that can be linearised separately. Hence,

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\theta\beta)^k [C_{t,t+k}^{-\sigma} C_t^{\sigma} P_t P_{t,t+k}^{-1} Y_{t+k|t} (\Phi_t^*)] = \dots \\ E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[ C_{t,t+k}^{-\sigma} C_t^{\sigma} P_t P_{t,t+k}^{-1} Y_{t+k|t} \left( \frac{\epsilon}{\epsilon-1} MC_{t+k|t}^r \Phi_{t-1,t+k} \right) \right] = 0 \end{aligned}$$

Taking the natural logarithm of the expression within the brackets,

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma \log C_{t,t+k} + \sigma \log C_t + \log P_t \dots \\ - \log P_{t,t+k} + \log Y_{t+k|t} + \log \Phi_t^*]$$

We can then perform the Taylor series approximation to complete the log-linearisation,

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma \log \bar{c} + \sigma \log \bar{c} + \log \bar{p} - \log \bar{p} + \log \bar{y} + \log \bar{\Phi}^* \dots \\ - \frac{\sigma}{\bar{c}} (C_{t,t+k} - \bar{c}) + \frac{\sigma}{\bar{c}} (C_t - \bar{c}) + \frac{1}{\bar{p}} (P_t - \bar{p}) - \frac{1}{\bar{p}} (P_{t,t+k} - \bar{p}) \dots \\ + \frac{1}{\bar{y}} (Y_{t+k|t} - \bar{y}) + \frac{1}{\bar{\Phi}^*} (\Phi_t^* - \bar{\Phi}^*)]$$

Now, as we have seen on many occasions before, the original logarithmic expression will cancel out the natural logarithm at the steady-state, which after expressing the terms as percentage deviations, leaves us with,

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t,t+k} + \sigma c_t + p_t - p_{t,t+k} + y_{t+k|t} + \pi_t^*]$$

Then turning our attention to the right-hand side of the expression, where after taking the natural logarithm of the expression,<sup>4</sup>

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma \log C_{t,t+k} + \sigma \log C_t + \log P_t - \log P_{t,t+k} \dots \\ + \log Y_{t+k|t} + \log MC_{t+k|t}^r + \log \Phi_{t-1,t+k}]$$

We can then perform the Taylor series approximation to complete the log-linearisation,

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma \log \bar{C} + \sigma \log \bar{C} + \log \bar{P} - \log \bar{P} \dots \\ + \log \bar{M}C + \log \bar{\Phi} + \epsilon \log \bar{P} + \log \bar{Y}] \dots \\ - \frac{\sigma}{\bar{c}} (C_{t,t+k} - \bar{c}) + \frac{\sigma}{\bar{c}} (C_t - \bar{c}) + \frac{1}{\bar{p}} (P_t - \bar{p}) - \frac{1}{\bar{p}} (P_{t,t+k} - \bar{p}) \dots \\ + \frac{1}{\bar{y}} (Y_{t+k|t} - \bar{y}) + \frac{1}{\bar{M}C} (MC_{t+k|t}^r - \bar{M}C) + \frac{1}{\bar{\Phi}} (\Phi_{t-1,t+k} - \bar{\Phi})$$

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<sup>4</sup>By now you have probably noted that we can effectively remove any constants when log-linearising, as a constant term would not deviate from it's steady-state.

Now after removing the original logarithmic expression as well as the expressions that relate to the deviations from constants, we are left with,<sup>5</sup>

$$\Rightarrow E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t,t+k} + \sigma c_t + p_t - p_{t,t+k} + y_{t+k|t} + \widehat{m}c_{t+k|t} + \pi_{t-1,t+k}]$$

We may then write the combined expression as,

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t,t+k} + \sigma c_t + p_t - p_{t,t+k} + y_{t+k|t} + \pi_t^*] &= \dots \\ E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t,t+k} + \sigma c_t + p_t - p_{t,t+k} + y_{t+k|t} + \widehat{m}c_{t+k|t} + \pi_{t-1,t+k}] \end{aligned}$$

This may be summarised as,

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\pi_t^*] &= E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{m}c_{t+k|t} + \pi_{t-1,t+k}] \\ E_t \sum_{k=0}^{\infty} (\theta\beta)^k [p_t^* - p_{t-1}] &= E_t \sum_{k=0}^{\infty} (\theta\beta)^k [\widehat{m}c_{t+k|t} + p_{t+k} - p_{t-1}] \end{aligned}$$

## B.4 Conclusion

Whilst this method for obtaining a set of linear difference equations that is extremely convenient, it should be noted that local approximations are only valid around the steady-state. Hence, if a variable experiences relatively large departures from the steady-state, or if these departures are described by a nonlinear process, then it would be better to make use of perturbation or projection methods that can solve for higher-order approximations.<sup>6</sup> The use of these techniques requires more advanced computational methodologies that are beyond the scope of this manuscript.<sup>7</sup> The interested reader is referred to Fernández-Villaverde and Rubio-Ramirez (2005), An and Schorfheide (2007), An (2008), Primiceri and Justiniano (2008), Pichler (2008), DeJong and Dave (2011), DeJong *et al.* (2013), and Balcilar *et al.* (2013).

<sup>5</sup>Hence, we use the notation  $\widehat{m}c_{t+k|t}$  for the percentage deviation from steady-state to simplify on the notation later on.

<sup>6</sup>Schmitt-Grohé and Uribe (2004) describe how to solve second-order rational expectations problems. For more on perturbation techniques, see Judd (1998).

<sup>7</sup>For example, these models usually require the use of symbolic computation to derive the model solution. In addition, a combination of global optimisation techniques and nonlinear filters are normally required for the evaluation of the likelihood function.

## Appendix C

### Introduction to Bayesian Econometrics

## C.1 Introduction

During the year of 1763, the Royal Society of London published a posthumous article that described the thoughts of Reverend Thomas Bayes (born circa 1702; died 1761). The paper, which is titled “An Essay Towards Solving a Problem in the Doctrine of Chances”, established the basis of a method that determines the chance of realising an uncertain event through the accumulation of evidence, using what is now termed *Bayes theorem*.<sup>1</sup> This technique can be used to estimate parameters, in the sense that we can accumulate evidence from data, to determine the chance of realising certain parameter values.

The popularity of the Bayesian approach to econometric modelling continues to enjoy a growing number of followers and many macroeconomists argue that it has many important advantages over the classical (frequentist) approach.<sup>2</sup> Support for this point of view can be found in Schorfheidede and Negro (2011), Fernández-Villaverde *et al.* (2010), and Koop and Korobilis (2010), where it is argued that Bayesian methods provide the following advantages:

- *Identification issues*: Most classical techniques require large data samples if the technique is to generate valid parameters. With the application of Bayesian methods, as long as the prior distributions are suitable, the results will be valid and a lack of identification does not present any conceptual difficulties. This feature is particularly important when dealing with macroeconomic models that make use of many variables and relatively short datasets, as Bayesian methods would not suffer from insufficient degrees of freedom.
- *Combining different sources of data*: One could use the prior to incorporate findings from other studies. In this way macroeconomic models could include micro-level information to good effect.
- *Misspecification & uncertainty*: Since Bayesian methods treat parameters as random variables, it is relatively easy to identify cases where a parameter is not correctly identified by the data (as the likelihood for the parameter will usually be flat). In addition, since the result provides an indication of possible alternative parameter estimates, around the measure of central tendency, one is able to measure uncertainty, which may be an important consideration in various empirical investigations.

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<sup>1</sup>The popular historian, Bill Bryson, has suggested that whilst Bayes was an extremely gifted mathematician, he was a poor preacher.

<sup>2</sup>Although it has been acknowledged for some time, that the Bayesian approach has many useful econometric applications, it is only with the recent advances in computing power that we are able to exploit its true potential.

- *Efficient Computation*: The continued development of efficient Markov-Chain Monte-Carlo methods (and Gibbs sampling techniques) has allowed for Bayesian methods to be applied to many interesting applications. In addition, these techniques are also very adept at dealing with latent (unobserved) variables, which are often included in macroeconomic models.

When using these techniques for the estimation of parameters in macroeconomic models, I would also argue that one of the most important reasons for making use of Bayesian methods is that it provides a useful synthesis between estimation and calibration techniques, as you will see a little later.

In what follows we will consider some of the basic concepts of Bayesian modelling. For those looking for a complete treatment of the application of these techniques, see Geweke (2005), Koop (2003), Koop *et al.* (2007), Lancaster (2004), Poirier (1995) or Zellner (1971).

## C.2 The Bayesian Paradigm

Essentially the Bayesian approach to econometrics is based on a few simple rules of probability that determine whether we are to change our beliefs following the accumulation of evidence. These rules facilitate:

- parameter estimation (e.g. find coefficient values)
- model comparison (e.g. conduct hypothesis testing)
- prediction (e.g. forecast future values or states)

Such a methodology is particularly relevant to most econometric analyses, where we confront an economic model with evidence, where the model usually describes relationships between variables that may be summarised by parameters and the evidence is provided by the data. For example, consider an economic model that describes the hypothetical relationship between consumption ( $C$ ) and income ( $Y$ ).

$$C_t = \alpha + \beta Y_t + \epsilon_t, \quad \epsilon_t \sim [0, \sigma^2] \quad (\text{C.2.1})$$

where  $\alpha$ ,  $\beta$  and  $\sigma^2$  are the parameters that are summarized by  $\theta$ . Since there may be many possible values for  $\theta$  from the population  $\Theta$ , we may use the expression,  $\theta \in \Theta$ . We can then test our initial assertions regarding  $\theta$  using data on  $C$  and  $y$  to determine whether these assertions are consistent with the evidence, or whether other values are more probable. In essence this amounts to the estimation of a probability distribution over possible structures in  $\Theta$ . This distribution would then enable one to determine the most probable values of the parameters for the given data. Thereafter, once we have suitably identified the parameters in the model, we are then able to predict the values



that are not included in the sample set, if the objective is to forecast future macroeconomic values of the variables.

### C.2.1 Bayesian Theory

To illustrate the simplicity of the methodology that underlies this approach, consider two events,  $A$  and  $B$ . The rules of probability would then imply:<sup>3</sup>

$$p(A, B) = p(A|B)p(B) \quad \text{or} \quad p(A, B) = p(B|A)p(A) \quad (\text{C.2.2})$$

where  $p(A, B)$  is the joint probability of  $A$  and  $B$  occurring,  $p(A|B)$  is the conditional probability of  $A$  occurring given that  $B$  occurred, and  $p(B)$  is the marginal probability of  $B$  occurring.

Equating the two expressions for  $p(A, B)$  in (C.2.2) provides us with Bayes' rule for conditional probabilities that underlies the methodology of Bayesian Econometrics:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} \quad \text{or} \quad p(B|A) = \frac{p(A|B)p(B)}{p(A)} \quad (\text{C.2.3})$$

When applying Bayes' rule to an econometric problem we usually conceive that  $p(A)$  refers to a measure of the strength in the belief that  $A$  is true, given  $B$ . In this case, we assume that  $A$  refers to one of many parameter estimates in  $\Theta$ . Hence, we could use this premise to conclude that whilst the parameter estimates in  $\theta_1$  are unlikely, those in  $\theta_2$  are quite probable (where  $\theta_1$  and  $\theta_2$  are both contained in vector  $A$ , which is a sample of  $\Theta$ ).<sup>4</sup>

To determine whether either  $\theta_1$  or  $\theta_2$  is more likely, we need to test them against data, which is contained in vector  $B$ . If we are to presume initially that these sets of parameter estimates are equally probable then our prior belief is that  $p(\theta_1) = p(\theta_2) = 0.5$ . Given the content of the theory we are then able to express our beliefs, which may be formulated as probabilities, that will then be taken to the data to provide evidence to support either of these sets of parameter estimates.<sup>5</sup> If we believe that the data will suggest that  $\theta_1$  is unlikely and  $\theta_2$  is likely then we could write,  $p(B|\theta_1) = 0.1$  and  $p(B|\theta_2) = 0.6$ . This would enable us to calculate:

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<sup>3</sup>The law of conditional probability is usually stated as  $p(A|B) = \frac{p(A \cap B)}{p(B)}$ , where  $p(A \cap B)$  is the joint probability of  $A$  and  $B$  occurring. This law holds for as long as  $p(B) \neq 0$ .

<sup>4</sup>Bayesians apply probability theory more liberally, to all events, whether they are repeatable or not.

<sup>5</sup>Further details of how the data may be expressed as a probability is provided in the following sub-section.

$$p(B) = p(B|\theta_1)p(\theta_1) + p(B|\theta_2)p(\theta_2) = 0.1 \times 0.5 + 0.6 \times 0.5 = 0.35, \quad (\text{C.2.4})$$

$$p(\theta_1|B) = \frac{p(B|\theta_1)p(\theta_1)}{p(B)} = \frac{0.05}{0.35} = \frac{1}{7}, \quad (\text{C.2.5})$$

$$p(\theta_2|B) = \frac{p(B|\theta_2)p(\theta_2)}{p(B)} = \frac{0.30}{0.35} = \frac{6}{7}. \quad (\text{C.2.6})$$

These results suggest that after presenting the data,  $\theta_2$  is six times more probable than  $\theta_1$ . The important point of this exercise is to note that after we subjected our prior beliefs to the data, our understanding changed and that this revision of beliefs would have happened no matter what the value of the priors might have been. The only exceptions to this would be:

- $p(\theta_1)=0$  or  $p(\theta_2)=0$ ; where no credence is given to a structure you will not learn that it is right
- $p(B|\theta_1)=p(B|\theta_2)$ ; where the data is equally probable for the given theories, we will not learn anything from it.

### C.3 Regression Analysis & Coefficient Estimation

Most econometric models seek to identify a vector or matrix of coefficients,  $\theta$ , based on a vector or matrix of data,  $y$ . Bayesians econometricians would thus replace the above  $A$  event with  $\theta$  and the  $B$  event with  $y$  to obtain,

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \quad (\text{C.3.1})$$

This equation basically states that we are trying to learn as much as we can about something that is unknown (i.e. the  $\theta$  parameters) given something that is obtainable (i.e. the data in  $y$ ). According to Bayes, the conditional probability of the *unknown* given the *known* is the best way of summarizing what we wish to learn. It is important to note that in this case, the parameters of interest,  $\theta$ , remain unknown (unobserved) both before and after the data is collected, however, the data is known (observed) once it has been collected.

Having thus established that  $p(\theta)$  is of fundamental interest we can ignore the term  $p(y)$  since it does not involve  $\theta$ . This enables us to rewrite (C.3.1) as:

$$p(\theta|y) \propto p(y|\theta)p(\theta) \quad (\text{C.3.2})$$

where the term  $\propto$  is the sign for ‘*proportional to*’. In this equation:

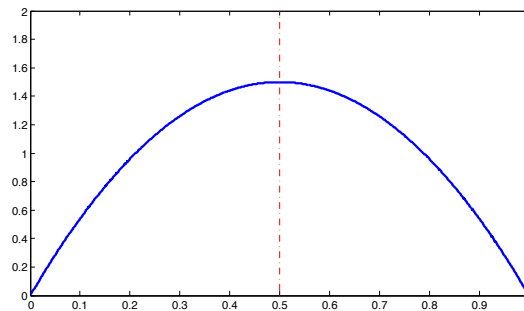


Figure C.1: Beta Distribution

- $p(\theta|y)$  is the **posterior density** (probability attached to particular parameter values after observing the data)
- $p(y|\theta)p(\theta) = p(y, \theta)$  is the **econometric model** (joint probability distribution of the data and the parameters) that consists of:
  - $p(y|\theta)$  is the **likelihood function** (density of the data conditional on the parameters in the model, which describes what the data would look like for particular values of  $\theta \in \Theta$ )
  - $p(\theta)$  is the **prior density** (what we know about  $\theta$  prior to seeing the data)

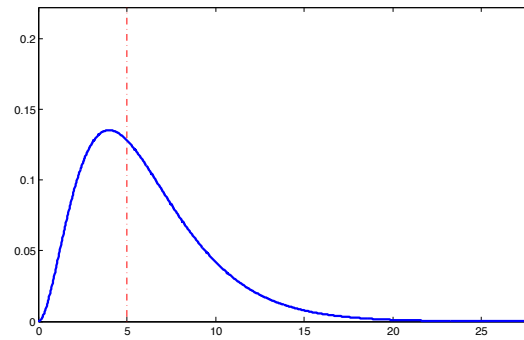
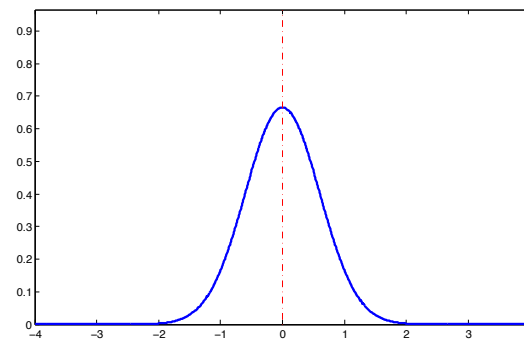
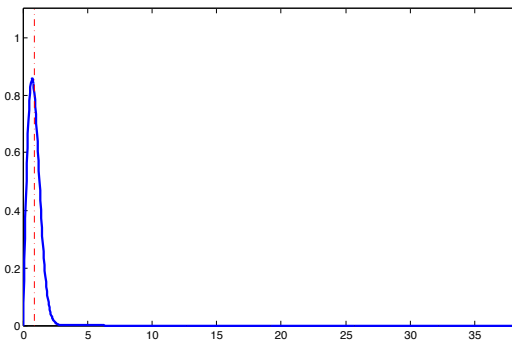
Hence this equation basically states that to estimate  $\theta$  given the data (i.e.  $p(\theta|y)$ ), we need to multiply our prior knowledge of  $\theta$  (i.e.  $p(\theta)$ ) by a likelihood function that is used to describe the data generating process that is conditional on various values of  $\theta$ , (i.e.  $p(y|\theta)$ ).

### C.3.1 The prior [ $p(\theta)$ ]

Bayesian techniques require the specification of a prior distribution for each parameter that is to be estimated.<sup>6</sup> Since the parameters in these models are random variables, the prior should contain information about the first two moments of a distribution (as a minimum). The formulation of priors is a topic around which a great deal has been written. This discussion only provides basic details on how to formulate a prior for most dynamic stochastic general equilibrium models. For those who are looking for additional details on the formulation of priors, see the references that were provided in the introduction.

Most macroeconomic models make use of a combination of beta, gamma, normal, and inverse gamma distributions. These have been included in the diagrams (C.1) through (C.4). Note that for the beta distribution has the limits  $[0, 1]$ , whilst the normal distribution may take on either positive or negative values. In addition, you should note that the gamma distribution is extremely

<sup>6</sup>You could think of the prior as being analogous to the starting values when applying maximum likelihood techniques.

**Figure C.2:** Gamma Distribution**Figure C.3:** Normal Distribution**Figure C.4:** Inverse Gamma Distribution

flexible, whilst the inverse gamma distribution is largely concentrated around zero. Hence, if your parameter relates to a probability (such as the degree of price stickiness) that occurs between the values of 0 and 1, then a beta distribution is appropriate, whilst if we are uncertain whether the parameter should be positive or negative (such as for the degree of risk aversion) then we would make use a normal distribution.<sup>7</sup> In addition, the use of an inverse gamma

<sup>7</sup>In this case we could impose artificial limits to the normal distribution if necessary.

distribution would be particularly useful when specifying the distribution of stochastic errors as we would usually assume that the values of unexplained shocks are quite small.

When specifying these priors in the software platform that we will be using, we also need to specify the first two moments. Obviously the specification of these moments needs to be permissible. For example, one could not make use of a mean of 2 for a parameter that is assumed to take on a beta distribution. When choosing the first moment, we usually make use of those that are contained in the literature. For example, interest rate smoothing is usually around 0.7, the reaction of the central bank to inflation is around 1.5, etc.

When choosing to specify the second moment of these distributions, you should bear in mind that when choosing to specify a very narrow distribution, you are essentially telling the model that you are fairly certain about the chosen prior mean value of this parameter. As such the impact of the likelihood is going to be minimal and such priors are usually termed *dogmatic*. In contrast a *flat* prior is one has a fairly large second moment and the posterior density of this parameter will largely be influenced by the likelihood function.<sup>8</sup>

### C.3.2 The Posterior $[p(\theta|y)]$

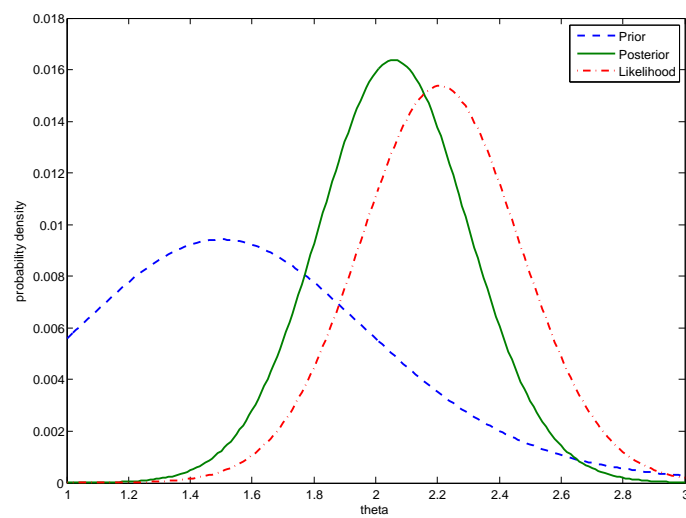
The posterior density represents your beliefs about  $\theta$  given your prior beliefs and the beliefs embodied in the likelihood. This computation may be extremely intensive as it usually involves taking the product of distributions, before sampling the joint distribution to derive the individual parameter estimates. For example, figure (C.5) displays the resulting posterior for a parameter, given the likelihood and prior. Note that the posterior falls within the distributions of the prior and likelihood. In this example, the prior was relatively flat (when compared to the likelihood) and as a result, the posterior approximates the likelihood more closely.

## C.4 Conclusion

This brief note sought to provide some of the basic details that may be useful when seeking to estimate a macroeconomic model with Bayesian techniques. For a formal treatment of the use of Bayesian techniques in dynamic macroeconomic models, see chapter 14 in DeJong and Dave (2011) or chapter 9 in Canova (2007); whilst influential articles include Smets and Wouters (2007), An and Schorfheide (2007), and DeJong *et al.* (2000), amongst many others.

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<sup>8</sup>The use of extremely flat (or uniform) priors in econometric models would derive an equivalent maximum likelihood estimate with the use of MCMC methods. This approach to econometric modelling has been termed quasi-maximum likelihood in some of the literature.



**Figure C.5:** Deriving the posterior from the prior and likelihood

# List of References

- Abel, A.B. (1990). Asset pricing under habit formation and catching up with the joneses. *AEA Papers and Proceedings*, vol. 80, pp. 38–42.
- Adelman, I. and Adelman, F.L. (1959). The dynamic properties of the klein-goldberger model. *Econometrica*, vol. 27(4), pp. 596–625.
- Adjemian, S., Bastani, H., Juillard, M., Karame, F., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. and Villemot, S. (2011). Dynare: Reference manual, version 4. Dynare Working Papers, 1, CEPREMAP.
- Adolfson, M., Laséen, S., Lindé, J. and Villani, M. (2008a). Evaluating an estimated new keynesian small open economy model. *Journal of Economic Dynamics and Control*, vol. 32, no. 8, pp. 2690–2721.
- Adolfson, M., Laséen, S., Lindé, J., Villani, M. and Svensson, L.E.O. (2008b). Optimal monetary policy in an operational medium-sized dsge model. *NBER Working Paper*, vol. Working Paper 14092.
- Ahmed, S., Levin, A. and Wilson, B. (2004). Recent u.s. macroeconomic stability: Good luck, good policies, or good practices? *The Review of Economics and Statistics*, vol. 86, no. 3, pp. 824–832.
- Alpanda, S., Kotzé, K. and Woglom, G. (2010a). The role of the exchange rate in a new keynesian dsge model for the south african economy. *South African Journal of Economics*, vol. 78, no. 2, pp. 170–191.
- Alpanda, S., Kotzé, K. and Woglom, G. (2010b). Should central banks of small open economies respond to exchange rate fluctuations? the case of south africa. *ERSA Working Paper*, vol. No. 174.
- Alpanda, S., Kotzé, K. and Woglom, G. (2011). Forecasting performance of an estimated dsge model for the south african economy. *South African Journal of Economics*, vol. 79, no. 1, pp. 50–67.
- Altig, D., Christiano, L., Eichenbaum, M. and Linde, J. (2011 April). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, vol. 14, no. 2, pp. 225–247.
- Amstad, M. and Fischer, A. (2004). Sequential information flow and real-time diagnosis of swiss inflation: Intra-monthly dcf estimates for a low-inflation environment. CEPR Discussion Paper.

- Amstad, M. and Potter, S. (2009). Real time underlying inflation gauges for monetary policymakers. *FRB of New York Staff Report No. 420*.
- An, S. (2008). Bayesian estimation of dsge models: Lessons from second-order approximations. Tech. Rep., Singapore Management University.
- An, S. and Schorfheide, F. (2007). Bayesian analysis of dsge models. *Econometric Reviews*, vol. 26, pp. 211–219.
- Andreasen, M.M., Fernández-Villaverde, J. and Rubio-Ramirez, J.F. (2014 January). The pruned state-space system for non-linear dsge models: Theory and empirical applications. Tech. Rep. NBER Working Papers 18983, NBER.
- Aron, J. and Muellbauer, J. (2004). Construction of cpix data for forecasting and modelling in south africa. *South African Journal of Economics*, vol. 72, no. 5, pp. 884–912.
- Bai, J. (1997). Estimation of a change point in multiple regression models. *The Review of Economics and Statistics*, pp. 551–563.
- Bai, J., Lumsdaine, R. and Stock, J. (1998). Testing for and dating common breaks in multivariate time series. *The Review of Economics and Statistics*, vol. 65, pp. 395–432.
- Balcilar, M., Gupta, R. and Kotzé, K. (2013). Forecasting south african macroeconomic data with a nonlinear dsge model. University of Pretoria, Working Paper Series.
- Ball, L.M. (1999). Policy Rules for Open Economies. In: *Monetary Policy Rules*, NBER Chapters, pp. 127–156. National Bureau of Economic Research, Inc. Available at: <http://ideas.repec.org/h/nbr/nberch/7415.html>
- Bañbura, M., Giannone, D. and Reichlin, L. (2010). Large Bayesian vector auto regressions. *Journal of Applied Econometrics*, vol. 25, no. 1, pp. 71–92.
- Baqae, D. (2010). Using wavelets to measure core inflation: The case of new zealand. *The North American Journal of Economics and Finance*, vol. 21, no. 3, pp. 241–255.
- Baxter, M. and King, R.G. (1999). Measuring business cycles: Approximate band-pass filters for economic time series. *Review of Economics and Statistics*, pp. 81:575–93.
- Benassy, J.-P. (2011). *Macroeconomic Theory*. Oxford University Press, Oxford.
- Bernanke, B., Gertler, M. and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In: Taylor, J.B. and Woodford, M. (eds.), *Handbook of Macroeconomics*, vol. 1C, pp. 1341–1393. Elsevier Science, North-Holland, Amsterdam.



- Bernanke, B.S. and Gertler, M. (1999). Monetary policy and asset price volatility. *Economic Review*, , no. Q IV, pp. 17–51.
- Binsbergen, J.H., Fernández-Villaverde, J., Koijen, R.S. and Rubio-Ramirez, J.F. (2012). The term structure of interest rates in a dsge model with recursive preferences. *Journal of Monetary Economics*, vol. 59, pp. 634–648.
- Blanchard, O. and Quah, D. (1989). The dynamic effects of aggregate demand and supply disturbances. *American Economic Review*, vol. 79, no. 4, pp. 655–673.
- Blanchard, O.J. (2009). The state of macro. *National Bureau of Economic Research*.
- Blanchard, O.J. and Kahn, C.M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, pp. 48(5):1305–1311.
- Blanchard, O.J. and Simon, J. (2001). The long and large decline in us output volatility. *Brookings Papers on Economic Activity*, vol. 1, pp. 135–174.
- Blignaut, Z., Farrell, G., Munyama, V. and Rangasamy, L. (2009). A note on the trimmed mean measure of core inflation in south africa. *South African Journal of Economics*, vol. 77, no. 4.
- Blinder, A.S. (1997 May/June). Measuring short run inflation for central bankers - commentary. *Federal Reserve Bank of St. Louis Review*, pp. 157–160.
- Boivin, J., Kiley, M.T. and Mishkin, F.S. (2010). How has the monetary transmission mechanism evolved over time? Tech. Rep., National Bureau of Economic Research.
- Bryan, M.F. and Cecchetti, S.G. (1994). Measuring core inflation. In: Mankiw, N.G. (ed.), *Monetary Policy*, pp. 67–123. University of Chicago Press, Chicago.
- Burger, P. (2008). The changing volatility of the south african economy. *South African Journal of Economics*, vol. 76, no. 3, pp. 335–355.
- Burger, P. and Jimmy, C. (2006). Should south africa have a fiscal rule? *South African Journal of Economics*, vol. 74, no. 4, pp. 642–669.
- Burns, A.F. and Mitchell, W.C. (1947). *Measuring Business Cycles*. National Bureau of Economic Research, New York.
- Calvo, G. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, vol. 12, pp. 383–398.
- Canova, F. (2007). *Methods for Applied Macroeconomic Research*. Princeton University Press, Princeton.
- Canova, F. and Sala, L. (2009). Back to square one: Identification issues in dsge models. *Journal of Monetary Economics*, vol. 56, no. 4, pp. 431–449.
- Carabenciov, I., Ermolaev, I., Freedman, C., Juillard, M., Kamenik, O., Korshunov, D., Laxton, D. and Laxton, J. (2008). A small multi-country projection model. Tech. Rep. 08/279, IMF working paper.

- Cecchetti, S.G. (2009). Monetary policy and the measurement of inflation: Prices, wages and expectations. Tech. Rep. BIS Papers No 49, p1-12, Bank of International Settlements.
- Cecchetti, S.G., Flores-Lagunes, A. and Krause, S. (2001). Has monetary policy become more efficient? a cross-country analysis. Unpublished mimeograph, Ohio State University.
- Chari, V.V., Kehoe, P.J. and McGrattan, E.R. (2008). New keynesian models: Not yet useful for policy analysis. *Federal Reserve Bank of Minneapolis*, vol. 409.
- Christ, C. (1995). History of the cowles commission 1932-1952. In: *Econometrics, Macroeconomics and Economic Policy: Selected Papers of Carl Christ*. Elgar, Oxford.
- Christiano, L. and Fitzgerald, T. (2003). The band pass filter. *International Economic Review*, vol. 44, no. 435-465.
- Christiano, L.J., Eichenbaum, M. and Evans, C.L. (1999 October). Monetary policy shocks: What have we learned and to what end? In: Taylor, J.B. and Woodford, M. (eds.), *Handbook of Macroeconomics*, vol. 1 of *Handbook of Macroeconomics*, chap. 2, pp. 65–148. Elsevier.
- Christiano, L.J., Eichenbaum, M. and Evans, C.L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, vol. 113, pp. 1–45.
- Christiano, L.J., Trabandt, M. and Walentin, K. (2010). Introducing financial frictions and unemployment into a small open economy model. *Sveriges Riksbank Working Paper No. 214*.
- Christoffel, K., Coenen, G. and Warne, A. (2010 May). Forecasting with dsge models. Working Paper Series 1185, European Central Bank.
- Clarida, R. and Gali, J. (1994). Sources of real exchange rate fluctuations: how important are nominal shocks? *Carnegie-Rochester Conference Series on Public Policy*, vol. 41, no. 1, pp. 1–56.
- Clarida, R., Gali, J. and Gertler, M. (1999 December). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, vol. 37, no. 4, pp. 1661–1707.
- Cogley, T. (2002). A simple adaptive measure of core inflation. *Journal of Money, Credit and Banking*, vol. 34, no. 1, pp. 94–113.
- Cogley, T. and Sargent, T.J. (2001). Evolving post-world war ii u.s. inflation dynamics. *NBER Macroeconomics Annual*, vol. 16, no. 1, pp. 331–373.
- Cooley, T.F. and Hansen, G.D. (1989). Inflation tax in a real business cycle model. *American Economic Review*, vol. 79(4), pp. 733–748.

- Cooley, T.F. and Prescott, E.C. (1995). Economic growth and business cycles. In: Cooley, T.F. (ed.), *Frontiers of business cycle research*, pp. 1–38. Princeton University Press.
- Cristadoro, R., Forni, M., Reichlin, L. and Veronese, G. (2005). A core inflation indicator for the euro area. *Journal of Money, Credit and Banking*, vol. 37, no. 3, pp. 539–560.
- Crowley, P.M. (2007). A guide to wavelets for economists. *Journal of Economic Surveys*, vol. 21, no. 2, pp. 207–267.
- Cuevas, A. and Topak, S. (2009). Monetary policy and relative price shocks in south africa and other inflation targeters. *IMF Working Papers*, pp. 1–25.
- Daubechies, I. (1992). *Ten Lectures on Wavelets*, vol. 61. SIAM.
- DeJong, D., Ingram, B.F. and H., C. (2000). A bayesian approach to dynamic macroeconomics. *Journal of Econometrics*, vol. 98(2), pp. 203–223.
- DeJong, D., Liesenfeld, R., Moura, G.V., Richard, J.-F. and Dharmarajan, H. (2013). Efficient likelihood evaluation of state-space representations. *Review of Economic Studies*, vol. 80, pp. 538–567.
- DeJong, D.N. and Dave, C. (2007). *Structural Macroeconometrics*. 1st edn. Princeton University Press, Princeton.
- DeJong, D.N. and Dave, C. (2011). *Structural Macroeconometrics*. 2nd edn. Princeton University Press, Princeton.
- De Kock Commission (1978). Interim report of the commission of inquiry into the monetary system and monetary policy in south africa rp 112/1978. Pretoria: Government Printer.
- De Kock Commission (1984). Final report of the commission of inquiry into the monetary system and monetary policy in south africa rp 70/1984. Pretoria: Government Printer.
- Del Negro, M. and Schorfheide, F. (2011). *The Oxford Handbook of Bayesian Econometrics*, chap. Bayesian Macroeconometrics, pp. 293–389. Oxford University Press.
- Den Haan, W.J. and De Wind, J. (2012). Nonlinear and stable perturbation-based approximations. *Journal of Economic Dynamic & Control*, vol. 36, pp. 1477–1497.
- de Wet, G. and Dreyer, J. (1978). A quarterly econometric model of the south african economy (and a simulation exercise on the price effect of the 1975 devaluation). *Journal of Studies in Economics and Econometrics*, vol. 2, pp. 19–51.
- Diebold, F.X. and Mariano, R.S. (1995). Predictive accuracy. *Journal of Business and Economic Statistics*, vol. 13, no. 3, pp. 253–263.
- Doan, T., Litterman, R. and Sims, C. (1984). Forecasting and conditional projection using realistic prior distributions. *Econometric Reviews*, vol. 3, no. 1, pp. 1–100.

- Doucet, A., Godsill, S. and Andrieu, C. (2000). On sequential monte carlo sampling methods for bayesian filtering. *Statistics and computing*, vol. 10, no. 3, pp. 197–208.
- Dowd, K., Cotter, J. and Loh, L. (2010). Us core inflation: A wavelet analysis. *Macroeconomic Dynamics*, vol. 15, no. 4, p. 513.
- Doz, C., Giannone, D. and Reichlin, L. (2011). A two-step estimator for large approximate dynamic factor models based on kalman filtering. *Journal of Econometrics*, vol. 164, pp. 188–205.
- Du Plessis, S. and Kotzé, K. (2012). Trends and structural changes in south african macroeconomic volatility. ERSA Working Paper No. 297.
- Du Plessis, S. (2002). Evaluating the sarb's inflation target. *South African Journal of Economics*, vol. 70, no. 6, pp. 982–1007.
- Du Plessis, S. (2003). *An Institutional Assessment of Inflation Targeting as a Framework for Monetary Policy*. Ph.D. thesis, University of Stellenbosch.
- Du Plessis, S. (2004). Stretching the south african business cycle. *Economic Modelling*, vol. 21, no. 1, pp. 685–701.
- Du Plessis, S. (2006). Reconsidering the business cycle and stabilisation policies in south africa. *Economic Modelling*, vol. 23, no. 5, pp. 761–774.
- Du Plessis, S. and Boshoff, W. (2007). A fiscal rule to produce counter-cyclical fiscal policy in south africa. Stellenbosch: University of Stellenbosch, Stellenbosch working paper 13/2007.
- Du Plessis, S., Smit, B. and Sturzenegger, F. (2007 September). The cyclicity of monetary and fiscal policy in south africa since 1994. *South African Journal of Economics*, vol. 75, no. 3, pp. 391–411.
- Du Plessis, S. and Smit, B.W. (2003). Stabilisation policy. In: *Paper presented at the eight annual conference for econometric modelling in Africa*. Stellenbosch.
- Du Plessis, S., Smit, B.W. and Sturzenegger, F. (2008). Identifying aggregate supply and demand shocks in south africa. *Journal of African Economies*, vol. 17, no. 5, pp. 765–793.
- Durbin, J. and Koopman, S.J. (2001). *Time Series Analysis by State Space Methods*. Oxford University Press, Oxford.
- Du Toit, L. (2008). Optimal hodrick-prescott filtering in south africa. Tech. Rep., University of Stellenbosch.
- Edge, R.M., Kiley, M.T. and Laforte, J.-P. (2010). A comparison of forecast performance between federal reserve staff forecasts, simple reduced-form models, and a dsge model. *Journal of Applied Econometrics*, vol. 25, no. 4, pp. 720–754.

- Edwards, L. and Lawrence, R.Z. (2006). South african trade policy matters: Trade performance and policy. *NBER Technical Working Paper*, vol. 12760.
- Epstein, R. (1987). *A History of Econometrics*. North-Holland, Amsterdam.
- Erceg, C.J., Henderson, D.W. and Levin, A.T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, vol. 46, pp. 281–313.
- Fair, R.C. (1992 February). The cowles commission approach, real business cycle theories, and new keynesian economics. Cowles Foundation Discussion Papers 1004, Cowles Foundation for Research in Economics, Yale University.
- Farmer, R.E.A., Waggoner, D.F. and Zha, T. (2011). Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamic & Control*, vol. 35(12), pp. 2150–2166.
- Farrel, G. and Todani, K.R. (2004 25-26 March). Capital flows, exchange control regulations and exchange rate policy: The south african experience. In: *How to reduce Debt Costs in Southern Africa?*, OECD Seminar. Bond Exchange of South Africa, Johannesburg.
- Fernández-Villaverde, J. (2010 March). The econometrics of DSGE models. *SERIEs*, vol. 1, no. 1, pp. 3–49.  
Available at: <http://ideas.repec.org/a/spr/series/v1y2010i1p3-49.html>
- Fernández-Villaverde, J., Guerron-Quintana, P. and Rubio-Ramírez, J.F. (2010). *The New Macroeconometrics: A Bayesian Approach*. The Oxford Handbook of Applied Bayesian Analysis. Oxford University Press, Oxford.
- Fernández-Villaverde, J., Guerron-Quintana, P., Rubio-Ramírez, J.F. and Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, vol. 101, pp. 2530–2561.
- Fernandez-Villaverde, J. and Rubio-Ramirez, J.F. (2004). Comparing dynamic equilibrium economies to data: A bayesian approach. *Journal of Econometrics*, vol. 123, pp. 153–187.
- Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2005). Estimating dynamic equilibrium economies: Linear versus nonlinear likelihood. *Journal of Applied Econometrics*.
- Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2006 October). A baseline dsge model. Tech. Rep., University of Pennsylvania. Mimeo.
- Fernández-Villaverde, J. and Rubio-Ramírez, J.F. (2008). How Structural Are Structural Parameters? In: *NBER Macroeconomics Annual 2007, Volume 22*, NBER Chapters, pp. 83–137. National Bureau of Economic Research, Inc.  
Available at: <http://ideas.repec.org/h/nbr/nberch/4087.html>

- Frankel, J., Smit, B. and Sturzenegger, F. (2007 April). South africa: Macroeconomic challenges after a decade of success. Cambridge MA: CID Working Paper No. 133.
- Friedman, B.M. and Kuttner, K.N. (1992). Money, income, prices, and interest rates. *American Economic Review*, vol. 82, no. 3, pp. 472–492.
- Friedman, M. (1968). The role of monetary policy. *American Economic Review*, vol. 58, pp. 1–17.
- Friedman, M. and Schwartz, A.J. (1963). *A Monetary History of the United States, 1867-1960*. No. frie63-1 in NBER Books. National Bureau of Economic Research, Inc.
- Gabor, D. (1946). Theory of communication. *Journal of the I.E.E.*, vol. 93(3), pp. 429–457.
- Gali, J. (2002). New perspectives on monetary policy, inflation, and the business cycle. *National Bureau of Economic Research*, vol. 8767.
- Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press, Princeton.
- Gali, J. (2010). The return of the wage phillips curve. *National Bureau of Economic Research*, vol. Working Paper No. 15758.
- Gali, J. and Gambetti, L. (2009). On the sources of the great moderation. *American Economic Journal: Macroeconomics*, vol. 1, no. 1, pp. 26–57.
- Gali, J. and Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, vol. 72.
- Gencay, R., Selcuk, F. and Whitcher, B. (2010). *An Introduction to Wavelet and other Filtering Methods in Finance and Economics*. Academic Press, San Diego.
- Gertler, M. and Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. *unpublished manuscript*.
- Geweke, J. (2005). *Contemporary Bayesian Econometrics and Statistics*. John Wiley & Sons, New York, NY.
- Ghysels, E., Harvey, A. and Renault, E. (1996). Stochastic volatility. Tech. Rep., Cahiers de recherche 9613, Centre interuniversitaire de recherche en économie quantitative, CIREQ.
- Giannone, D. and Matheson, T.D. (2007). A new core inflation indicator for new zealand. *International Journal of Central Banking*, vol. 3, no. 4, pp. 145–180.
- Giannone, D., Reichlin, L. and Primiceri, G. (forthcoming). Prior selection for vector autoregressions. *Review of Economics and Statistics*, , no. 4.
- Gidlow, R. (2002). The history of exchange controls in south africa. *South African Journal of Economic History*, vol. 17, pp. 25–48.

- Goodhart, C. (1989). Has moore become too horizontal? *Journal of post Keynesian economics*, vol. 12, no. 1, pp. 29–34.
- Grossmann, A. and Morlet, J. (1984). Decomposition of hardy functions into square integrable wavelets of constant shape. *SIAM journal on mathematical analysis*, vol. 15, no. 4, pp. 723–736.
- Gumata, N., Kabundi, A. and Ndou, E. (2013). Important channels of transmission of monetary policy shock in south africa. Tech. Rep. WP/2013/06, South African Reserve Bank Working Paper Series.
- Gupta, R. and Kabundi, A. (2010). Forecasting macroeconomic variables in a small open economy: A comparison between small- and large-scale models. *Journal of Forecasting*, vol. 29, no. 1-2, pp. 168–185.
- Gupta, R. and Kabundi, A. (2011). A large factor model for forecasting macroeconomic variables in south africa. *International Journal of Forecasting*, vol. 27, no. 4, pp. 1076–1088.
- Gupta, R., Kabundi, A. and Modise, M.P. (2010). Has the sarb become more effective post inflation targeting? *Economic Change and Restruction*.
- Gupta, R. and Steinbach, R. (2013). A dsge-var model for forecasting key south african macroeconomic variables. *Economic Modelling*, vol. 33, pp. 19–33.
- Haar, A. (1910). Zur theorie der orthogonalen funktionensysteme. *Mathematische Annalen*, vol. 69, no. 3, pp. 331–371.
- Hamilton, J.D. (1994). *Time Series Analysis*. Princeton University Press, Princeton.
- Hansen, B. (1997). Approximating asymptotic  $\rho$  values for structural-change tests. *Journal of Business and Economic Statistics*, vol. 15, no. 1, pp. 60–67.
- Hansen, G.D. (1985 November). Indivisible labor and the business cycle. *Journal of Monetary Economics*, vol. 16, no. 3, pp. 309–327.
- Heckman, J.J. (2000). Causal parameters and policy analysis in economics: A twentieth century retrospective. *Quarterly Journal of Economics*, vol. 115, no. 1, pp. 45–97.
- Heil, C. and Walnut, D.F. (2006). *Fundamental Papers in Wavelet Theory*. Princeton University Press, Princeton, New Jersey.
- Hodrick, R.J. and Prescott, E.C. (1980). Postwar u.s. business cycles: an empirical investigation. *Carnegie Mellon University Discussion Paper*, p. no. 451.
- Hodrick, R.J. and Prescott, E.C. (1997). Postwar u.s. business cycles: an empirical investigation. *Journal of Money, Credit and Banking*, vol. 29, no. 1, pp. 1–16.
- Hubbard, B.B. (1998). *The World According to Wavelets: The Story of a Mathematical Technique in the Making*. 2nd edn. A K Peters/CRC Press.

- International Monetary Fund (2013 October). *World Economic Outlook: Transitions and Tensions*. International Monetary Fund, Washington D.C.
- Ireland, P. (2004a). A method for taking models to data. *Journal of Economic Dynamics and Control*, vol. 28, pp. 1205–1226.
- Ireland, P. (2004b). Technology shocks in the new keynesian model. *Review of Economics and Statistics*, pp. 86:923–36.
- Ireland, P.N. (2001). Sticky price models of the business cycle: Specification and stability. *Journal of Monetary Economics*, vol. 47, pp. 3–18.
- Iskrev, N. (2010). Local identification in dsge models. *Journal of Monetary Economics*, vol. 57, pp. 189–202.
- Judd, K.L. (1998 June). *Numerical Methods in Economics*. MIT Press Books. The MIT Press.
- Justiniano, A. and Preston, B. (2010). Monetary policy and uncertainty in an empirical small open-economy model. *Journal of Monetary Economics*, vol. 25, pp. 93–128. Federal Reserve Bank of Chicago.
- Kahn, B. (2008 29 to 31 October). Challenges of inflation targeting for emerging-market economies: The south african case. In: *Conference Papers on Challenges for Monetary Policy-makers in Emerging Markets*. South African Reserve Bank.
- Keynes, J.M. (1940). On a method of statistical business cycle research: A comment. *Economic Journal*, vol. 50, no. 197, pp. 154–156.
- Khan, B. (1987). Exchange controls and the exchange rate policy in the south african economy. In: Black, P.A. and Dallery, B.E. (eds.), *Leading Issues in South African Macroeconomics: Selected Readings*. Southern Book Publishers, Johannesburg.
- Kim, C.-J. and Nelson, C.R. (1999). Has the us economy become more stable? *The Review of Economics and Statistics*, vol. 81, pp. 608–616.
- Kim, J., Kim, S., Schaumburg, E. and Sims, C.A. (2008). Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics & Control*, vol. 32, pp. 3397–3414.
- King, R., Plosser, C. and Rebelo, S. (2002). Production, growth and business cycles: Technical appendix. *Computational Economics*, vol. 20, no. 1-2, pp. 87–116.
- King, R.G. and Plosser, C.I. (1984). Money, credit, and prices in a real business cycle. *American Economic Review*, vol. 74, no. 3, pp. 363–380.
- King, R.G., Plosser, C.I. and Rebelo, S.T. (1988a). Production, growth and business cycles : I. the basic neoclassical model. *Journal of Monetary Economics*, vol. 21, no. 2-3, pp. 195–232.



- King, R.G., Plosser, C.I. and Rebelo, S.T. (1988*b*). Production, growth and business cycles : II. new directions. *Journal of Monetary Economics*, vol. 21, no. 2-3, pp. 309–341.
- King, R.G. and Rebelo, S.T. (1999). Resuscitating real business cycles. In: Taylor, J.B. and Woodford, M. (eds.), *Handbook of Macroeconomics*, vol. 1 of *Handbook of Macroeconomics*, chap. 14, pp. 927–1007. Elsevier.
- King, R.G. and Watson, M. (1996). Money, prices, interest rates and the business cycle. *Review of Economics and Statistics*, vol. 58(1), pp. 35–53.
- Kitagawa, G. (1987). Non-gaussian state-space modeling of nonstationary time series. *Journal of the American Statistical Association*, vol. 82, no. 400, pp. 1032–1050.
- Kitagawa, G. and Gersch, W. (1996). *Smoothness priors analysis of time series*, vol. 116. Springer Verlag.
- Klein, N. (2011). Measuring the potential output of south africa. IMF African Department No. 11/2, Washington D.C.
- Klein, P. (2000). Using the generalized schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, vol. 24, no. 1, pp. 405–423.
- Koop, G. (2003). *Bayesian Econometrics*. John Wiley & Sons, Chichester.
- Koop, G. and Korobilis, D. (2010). Bayesian multivariate time series methods for empirical macroeconomics. *Foundations and Trends in Econometrics*, vol. 3, pp. 267–358.
- Koop, G., Poirier, D.J. and Tobias, J.L. (2007). *Bayesian Econometric Methods*. Econometric Exercises 7. Cambridge Univ Press, Cambridge.
- Korobilis, D. (2011). Var forecasting using bayesian variable selection. *Journal of Applied Econometrics*.
- Kumhof, M., Laxton, D. and Naknoi, K. (2010). Does the exchange rate belong in monetary policy rules? new answers from a dsge model with endogenous tradability and trade frictions. In: Anderton, R. and Kenny, G. (eds.), *Macroeconomic Performance in a Globalising Economy*, pp. 120–154. Cambridge University Press.
- Kydland, F. and Prescott, E. (1982). Time to build and aggregate fluctuations. *Econometrica*, vol. 50, pp. 1345–1370.
- Kydland, F.E. and Prescott, E.C. (1991). The econometrics of the general equilibrium approach to business cycles. *Scandinavian Journal of Economics*, vol. 93, no. 2, pp. 161–78.  
Available at: <http://ideas.repec.org/a/bla/scandj/v93y1991i2p161-78.html>

- Kydland, F.E. and Prescott, E.C. (1996 Winter). The computational experiment: An econometric tool. *Journal of Economic Perspectives*, vol. 10, no. 1, pp. 69–85. Available at: <http://ideas.repec.org/a/aea/jecper/v10y1996i1p69-85.html>
- Lancaster, A. (2004). *An Introduction to Modern Bayesian Econometrics*. Blackwell, Oxford.
- Leamer, E.E. (1983). Let's take the con out of econometrics. *American Economic Review*, vol. 73(1), pp. 31–43.
- Lim, G. and McNelis, P.D. (2008). *Computational Macroeconomics for the Open Economy*. MIT Press, Cambridge, Massachusetts.
- Litterman, R. (1986a). Forecasting with bayesian vector autoregressions: Five years of experience. *Journal of Business & Economic Statistics*, vol. 4, no. 1, pp. 25–38.
- Litterman, R. (1986b). A statistical approach to economic forecasting. *Journal of Business & Economic Statistics*, vol. 4, no. 1, pp. 1–4.
- Liu, G. and Gupta, R. (2007). A small-scale dsge model for forecasting the south african economy. *South African Journal of Economics*, vol. 75, no. 2.
- Liu, G., Gupta, R. and Schaling, E. (2009). A new-keynesian dsge model for forecasting the south african economy. *Journal of Forecasting*, vol. 28, pp. 387–404.
- Liu, G., Gupta, R. and Schaling, E. (2010). Forecasting the south african economy: A hybrid-dsge approach. *Journal of Economic Studies*, vol. 37, no. 2, pp. 181–195.
- Lubik, T.A. (2007). Non-stationarity and instability in small open-economy models even when they are “closed”. *Economic Quarterly*, , no. Fall, pp. 393–412. Available at: <http://ideas.repec.org/a/fip/fedreq/y2007ifallp393-412nv.93no.4.html>
- Lucas, R. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, pp. 19–46.
- Lucas, R. and Sargent, T. (1978). After the phillips curve: Persistence of high inflation and high unemployment. In: *After Keynesian Economics*. Federal Reserve Bank of Boston Conference volume.
- Lucas, R. and Stokey, N. (1987). Money and interest in a cash-in-advance economy. *Econometrica*, vol. 55, pp. 491–513.
- Lucas, R.E. (1980). Methods and problems in business cycle theory. *Journal of Money, Credit, and Banking*, vol. 12, pp. 696–715.
- Lucas, R.E. (1987). Models of business cycles. In: *Yrjo Jahnsson Lectures series*. Blackwell, London and New York.
- Lucas, R.E. and Prescott, E.C. (1977). Investment under uncertainty. *Econometrica*, vol. 39, pp. 659–81.

- Maliar, L., Maliar, S. and Villemot, S. (2013). Taking perturbation to the accuracy frontier: A hybrid of local and global solutions. *Computational Economics*.
- Mallat, S.G. (1989). A theory for multiresolution signal decomposition: The wavelet representation. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 11, no. 7, pp. 674–693.
- Marcet, A. and Ravn, M.O. (2003). The hp-filter in cross country comparisons. *Working Paper*.
- Marcus, G. (2011). Statement of the monetary policy committee. Tech. Rep., South African Reserve Bank.
- McConnel, M.M. and Perez Quiros, G. (2000). Output fluctuations in the united states: What has changed since the early 1980s? *American Economic Review*, vol. 90, pp. 1464–1476.
- Mishkin, F.S. (1995). Symposium on the monetary transmission mechanism. *The Journal of Economic Perspectives*, vol. 9, no. 4, pp. 3–10.
- Monacelli, T. (2003). Monetary policy in a low pass-through environment. *Journal of Money, Credit and Banking*, vol. 37, pp. 1047–1066.
- Moore, B.J. (1988). The endogenous money supply. *Journal of Post Keynesian Economics*, vol. 10, no. 3, pp. 372–385.
- Moore, B.J. (1989). A simple model of bank intermediation. *Journal of Post Keynesian Economics*, vol. 12, no. 1, pp. 10–28.
- Morgan, M. (1990). *The History of Econometric Ideas*. Cambridge University Press, Cambridge.
- Mukherjee, S. and Bhattacharya, R. (2011). Inflation targeting and monetary policy transmission mechanisms in emerging market economies. Tech. Rep., International Monetary Fund.
- Ortiz, A. and Sturzenegger, F. (2007). Estimating sarbs policy reaction rule. Tech. Rep., Harvard University.
- Pagan, A. (2003). Report on modelling and forecasting at the bank of england. Tech. Rep., Bank of England.
- Pedersen, T.M. (1998). The hodrick-prescott filter, the slusky-effect, and the distortionary effect of filters. Tech. Rep. 9, University of Copenhagen.
- Pedersen, T.M. (2001). The hodrick-prescott filter, the slusky-effect, and the distortionary effect of filters. *Journal of Economic Dynamics and Control*, vol. 25, pp. 1081–1101.
- Pichler, P. (2008). Forecasting with dsge models: The role of nonlinearities. *The B. E. Journal of Macroeconomics*, vol. 8, no. 1.

- Poirier, D.J. (1995). *Intermediate Statistics and Econometrics: A Comparative Approach*. MIT Press, Cambridge, MA.
- Prescott, E.C. (1986). Theory ahead of business cycle measurement. *Quarterly Review: Federal Reserve Bank of Minneapolis*, vol. 10, pp. 9–22.
- Primiceri, G. and Justiniano, A. (2008). The time varying volatility of macroeconomic fluctuations. *American Economic Review*, vol. 98, no. 3, pp. 604–641.
- Quandt, R. (1960). Tests of the hypothesis that a linear regression obeys two separate regimes. *Journal of the American Statistical Association*, vol. 55, pp. 324–330.
- Rabanal, P. and Rubio-Ramirez, J.F. (2005). Comparing new keynesian models in the euro area: A bayesian approach. *Federal Reserve Bank of Atlanta*, vol. Working Paper No. 2003-30a.
- Ramsey, J.B. (2002). Wavelets in economics and finance: Past and future. *Studies in Nonlinear Dynamics & Econometrics*, vol. 6, no. 3.
- Ramsey, J.B. and Lampart, C. (1998 Aprila). The decomposition of economic relationships by time scale using wavelets: Expenditure and income. *Studies in Nonlinear Dynamics and Econometrics*, vol. 3, no. 1, pp. 23–42.
- Ramsey, J.B. and Lampart, C. (1998b). Decomposition of economic relationships by timescale using wavelets. *Macroeconomic Dynamics*, vol. 2, no. 1, pp. 49–71.
- Ramsey, J.B. and Zhang, Z. (1997). The analysis of foreign exchange data using waveform dictionaries. *Journal of Empirical Finance*, vol. 4, no. 4, pp. 341–372.
- Rand, J. and Tarp, F. (2002). Business cycles in developing countries: Are they different? *World Development*, pp. 30(2) 2071–2088.
- Ratto, M., Roeger, W. and in 't Veld, J. (2008 July). Quest iii: an estimated dsge model of the euro area with fiscal and monetary policy. European Economy - Economic Papers 335, Directorate General Economic and Monetary Affairs, European Commission.  
Available at: <http://ideas.repec.org/p/euf/ecopap/0335.html>
- Rotemberg, J.J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, vol. 49, pp. 517–31.
- Ruch, F. and Bester, D. (forthcoming). Towards a measure of core inflation using singular spectrum analysis. *South African Journal of Economics*.
- Rudebusch, G.D. (1998). Do measures of monetary policy in a var make sense? *International Economic Review*, pp. 907–931.
- Ruge-Murcia, F.J. (2007). Methods to estimate dynamic stochastic general equilibrium models. *Journal of Economic Dynamics and Control*, vol. 31, no. 8, pp. 2599–2636.

- Schleicher, C. (2002). An introduction to wavelets for economists. Tech. Rep. Working Paper 2002-3, Bank of Canada.
- Schmitt-Grohe, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, vol. 61, pp. 163–195.
- Schmitt-Grohé, S. and Uribe, M. (2004). Solving dynamic general equilibrium models using a second order approximation of the policy function. *Journal of Economic Dynamics and Control*, vol. 28, pp. 755–75.
- Schorfheide, F. and Negro, M.D. (2011). *Bayesian Macroeconometrics*, pp. 293–389. The Oxford Handbook of Bayesian Econometrics. Oxford University Press, Oxford.
- Sensier, M. and Dijk, D.V. (2004). Testing for volatility changes in u.s. macroeconomic time series. *The Review of Economics and Statistics*, vol. 86, no. 3, pp. 833–839.
- Shephard, N. (1994). Partial non-gaussian state space. *Biometrika*, vol. 81, no. 1, pp. 115–131.
- Shephard, N. (1996). Time series models in econometrics, finance and other fields.
- Shephard, N. (2005). *Stochastic Volatility: Selected Readings*. Oxford University Press, Oxford.
- Shumway, R. and Stoffer, D. (2010). *Time Series Analysis and its Applications*. Springer-Verlag, New York.
- Sims, C.A. (1972 September). Money, income, and causality. *American Economic Review*, vol. 62, no. 4, pp. 540–52.
- Sims, C.A. (1980a). Comparison of interwar and postwar business cycles. *American Economic Review*, vol. 70, no. 2, pp. 250–257.
- Sims, C.A. (1980b). Macroeconomics and reality. *Econometrica*, vol. 48, no. 2, pp. 1–49.
- Sims, C.A. (1992). Interpreting the macroeconomic time series facts: The effect of monetary policy. *European Economic Review*, vol. 36, pp. 975–1011.
- Sims, C.A. (2001). Solving linear rational expectations models. *Computational Economics*, vol. 20, pp. 1–20.
- Sims, C.A. and Zha, T. (1998). Bayesian methods for dynamic multivariate models. *International Economic Review*, vol. 39, no. 4, pp. 949–968.
- Smal, D., Pretorius, C. and Ehlers, N. (2007). The core forecasting model of the south african reserve bank. *South African Reserve Bank Working Paper*, vol. WP/07/02.

- Smets, F. and Wouters, R. (2002). Openness, imperfect exchange rate pass-through and monetary policy. *Journal of Monetary Economics*, vol. 49, pp. 947–981.
- Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, vol. 1, no. 5, pp. 1123–1175.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian approach. *European Central Bank Working Papers Series*, vol. 722.
- Smit, B.W. and Pellissier, G.M. (1997). The ber annual econometric model of the south african economy: A cointegration version. *Journal of Studies in Economics and Econometrics*, pp. 21(1) 1–35.
- Solow, R.M. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, vol. 70, no. 1, pp. 65–94.
- Solow, R.M. (1957). Technical change and the aggregate production function. *Review of Economics and Statistics*, vol. 39, no. 3, pp. 312–320.
- South African Reserve Bank (2014 June). Quarterly bulletin. Tech. Rep., South African Reserve Bank.
- Statistics South Africa (2009 3 February 2009a). Shopping for two: The cpi new basket parallel survey - results and comparisons with published cpi data. Tech. Rep., Statistics South Africa, <http://www.statssa.gov.za/cpi/index.asp>.
- Statistics South Africa (2009 5 February 2009b). The south african cpi sources and methods manual. Tech. Rep. Release v.1, Statistics South Africa, <http://www.statssa.gov.za/cpi/index.asp>.
- Steinbach, R. (2013). Essays on dynamic macroeconomics. Tech. Rep., PhD Dissertation, University of Stellenbosch.
- Steinbach, R., Mathuloe, P. and Smit, B.W. (2009a). An open economy new keynesian dsge model of the south african economy. *South African Journal of Economics*, vol. 77, no. 2, pp. 207–227.
- Steinbach, R., Mathuloe, P. and Smit, B.W. (2009b). An open economy new keynesian dsge model of the south african economy. South African Reserve Bank Working Paper WP/09/01, South African Reserve Bank.
- Stock, J.H. and Watson, M.W. (1996). Evidence on structural instability in macroeconomic time series relations. *Journal of Business and Economic Statistics*, vol. 14, pp. 11–29.
- Stock, J.H. and Watson, M.W. (2001). Vector autoregressions. *Journal of Economic Perspectives*.
- Stock, J.H. and Watson, M.W. (2003 Januarya). Has the business cycle changed and why? In: Gertler, M. and Rogoff, K. (eds.), *NBER Macroeconomics Annual 2002, Volume 17*, pp. 159–230. MIT Press, Cambridge, Ma.

- Stock, J.H. and Watson, M.W. (2003*b*). Has the business cycle changed? evidence and explanations. Tech. Rep., Harvard University and Princeton University.
- Stock, J.H. and Watson, M.W. (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association*, vol. 3, no. 5, pp. 968–1006.
- Strang, G. and Nguyen, T. (1996). *Wavelets and Filter Banks*. Wellesley Cambridge Press.
- Summers, L.H. (1986). Some skeptical observations on business cycle theory. *Quarterly Review: Federal Reserve Bank of Minneapolis*, vol. Fall, pp. 22–27.
- Svensson, L.E. (1997). Inflation forecast targeting: Implementing and monitoring inflation targets. *European Economic Review*, vol. 41, pp. 1111 – 1146.
- Svensson, L.E. (2005). Monetary policy with judgment: Forecast targeting. *International Journal of Central Banking*, vol. 1, no. 1, pp. 1–54.
- Swanepoel, J. (2004). The monetary and fiscal policy mix in south africa. *South African Journal of Economics*, vol. 72, no. 4, pp. 730–758.
- Swanepoel, J. and Schoeman, N. (2003). Countercyclical fiscal policy in south africa: Role and impact of automatic fiscal stabilisers. *South African Journal of Economic and Management Sciences*, vol. 6, no. 4, pp. 802–822.
- Taylor, B. (2009). Using matlab to develop macroeconomic models. Tech. Rep., Mathworks.
- Taylor, J.B. (1979). Estimation and control of a macroeconomic model with rational expectations. *Econometrica*, vol. 47, pp. 1267–1286.
- Taylor, J.B. (1993 December). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, no. 1, pp. 195–214.
- Taylor, J.B. (1999). Monetary policy rules. Chicago.
- Tinbergen, J. (1939). *Statistical Testing of Business Cycle Theories*. Geneva: League of Nations.
- Tobin, J. (1970). Money and income: Post hoc ergo proctor hoc? *Quarterly Journal of Economics*, vol. 84, no. 2, pp. 301–317.
- Tovar, C.E. (2008). Dsge models and central banks. *Bank for International Settlements Working Papers*, vol. No. 258.
- Tovar, C.E. (2009). Dsge models and central banks. *Economics - The Open-Access, Open-Assessment E-Journal*, vol. 3, no. 16, pp. 1–31.
- Uhlig, H. (1999). A toolkit for analyzing nonlinear dynamic stochastic models easily. In: Marimon, R. and Scott, A. (eds.), *Computational Methods for the Study of Dynamic Economies*, pp. 30–61. Oxford University Press.

- Uhlig, H. (2005 March). What are the effects of monetary policy on output? results from an agnostic identification procedure. *Journal of Monetary Economics*, vol. 52, no. 2, pp. 381–419.
- Van der Merwe, E.J. (2004). Inflation targeting in south africa. South African Reserve Bank Occasional Paper No. 19.
- Wallace, N. (2001 November). Whither monetary economics? *International Economic Review*, vol. 42, no. 4, pp. 847–69.  
Available at: <http://ideas.repec.org/a/ier/iecrev/v42y2001i4p847-69.html>
- Walsh, C.E. (2010). *Monetary Theory and Policy*. 3rd edn. MIT Press, Cambridge, Massachusetts.
- Wecker, W. (1979). Predicting the turning points of time series. *Journal of Business*, vol. 52, pp. 35–50.
- Woglom, G. (2003). How has inflation targeting affected monetary policy in south africa. *South African Journal of Economics*, vol. 71, no. 2, pp. 198–210.
- Woglom, G. (2005). Forecasting south african inflation. *South African Journal of Economics*, vol. 73, no. 2, pp. 302–320.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.
- Woodford, M. (2007). Forecast targeting as a monetary policy strategy: Policy rules in practice. Columbia University.
- Yule, G.U. (1926). Why do we sometimes get nonsense-correlations between time series. *Journal of Statistical Society*, vol. 89, pp. 1–64.
- Zellner, A. (1971). *An Introduction to Bayesian Inference in Econometrics*. John Wiley & Sons, New York, NY.